Flow of a Cement Slurry Modeled as a Generalized Second Grade Fluid

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MOTIVATION & OBJECTIVES

Motivation

• Well cementing is the process of placing a cement slurry in the annulus space between the well casing and the surrounding for zonal isolation.
• Rheological behavior of oil well cement slurries is significant in well cementing operation.

Objectives

• Study the impact of constitutive parameters on behavior of cement slurry.
• Perform parametric study for various dimensionless numbers.

MATHEMATICAL MODEL

• In this paper, we assume that the cement slurry behaves as a non-Newtonian fluid.
• We use a constitutive relation for the viscous stress tensor which is based on a modified form of the second grade (Rivlin-Ericksen) fluid [2].

Steady motion and unsteady motion are analyzed with one-dimensional flow.

Conservation of mass

\[ \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = 0 \]

\( \rho \) : density of cement slurry, which is related to \( \rho_0 \) (density of pure fluid);
\( \mathbf{v} \) : velocity vector; \( \rho_0 \); \( \rho \): volume fraction

Conservation of linear momentum

\[ \rho \frac{d\mathbf{v}}{dt} = \text{div} \tau + \rho g \]

d/dt: total time derivative, given by \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \)
\( \tau \): stress tensor; \( \mathbf{g} \): body force vector

Conservation of angular momentum

\[ \mathbf{T} = \mathbf{T} \]

Stress tensor of fluid [3]:

\[ \tau_{ij} = -p I_{ij} + \mu_{eff}(\phi, A_1) A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \]

\( p \): pressure; \( \alpha_1, \alpha_2 \) are the normal stress coefficients; \( \mu_{eff} \) is the effective viscosity, which is dependent on volume fraction and shear rate [4]:

\[ \mu_{eff}(\phi, A_1) = \mu_0 \left( 1 - \frac{\phi}{\phi_m} \right) [1 + \alpha_1 A_1^2 \theta^2] \]

\( \phi_0 \): maximum solid concentration packing;
\( A_n \): n-order Rivlin-Ericksen tensors, where \( A_1 \) is P' + 15P'; \( A_2 \) is \( A_1^2 + A_1 P' + P'A_1 \);

Convection-diffusion equation

\[ \frac{\partial \phi}{\partial t} + \text{div}(\mathbf{v} \phi) = -\text{div} N \]

Particle flux [5]: particles collisions; spatially varying viscosity; Brownian diffusive flux

\[ N = -D_{eff} \nabla \phi (\phi) \cdot \rho \beta \nabla \phi (\phi) = x^2 \nabla \phi (\phi) \cdot \rho \beta \nabla \phi (\phi) \]

where, \( D_{eff} = nA_1 \beta \sqrt{D_0} [K_1(1 - \phi)^2 + K_2(\phi_0 - \phi)^2] H(\phi_0 - \phi)^2 \beta \phi \) [6]

RESULTS

Parametric study with designated value of dimensionless numbers

<table>
<thead>
<tr>
<th>Dimensionless Number</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{eff} )</td>
<td>0.5, 0.5, 0.04, 0.06</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>20, 30, 40, 50, 60</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.1, 0.2, 0.3, 0.4</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.01, 0.02, 0.03, 0.04</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0, 0.5, 1.0</td>
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<tr>
<td>( A_4 )</td>
<td>0.01, 0.02, 0.03</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>0.01, 0.02, 0.03</td>
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<tr>
<td>( A_6 )</td>
<td>0.01, 0.02, 0.03</td>
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<tr>
<td>( A_7 )</td>
<td>0.01, 0.02, 0.03</td>
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<tr>
<td>( A_8 )</td>
<td>0.01, 0.02, 0.03</td>
</tr>
<tr>
<td>( A_9 )</td>
<td>0.01, 0.02, 0.03</td>
</tr>
</tbody>
</table>

CONCLUSIONS

• The parametric study indicates that maximum packing \( \phi_m \), concentration flux parameters \( \frac{\partial \phi}{\partial t} \), the angle of inclination \( \theta \), pulsating pressure and gravity terms affect the velocity and particle distribution significantly.
• This study is simply a preliminary case and further studies will be performed where the effects of diffusion, heat transfer, such as viscous dissipation and yield stresses will be considered.

References