



### Drag Model for Assemblies of Non-Spherical Particles

Project Award Number DE-FE0031894 **PI: Zhi-Gang Feng, Ph.D. Mechanical Engineering** University of Texas at San Antonio (UTSA)

### 1. Introduction

2. Correlations of drag coefficients for individual non-spherical particles

- 3. Artificial Neural Network (ANN) models for individual nonspherical particles
- 4. Drag models for assemblies of non-spherical particles
- 5. Future Work









Particle hydrodynamic forces exerted on particles play a pivotal role in particulate flow simulation packages

Introduction

Many particles in the real world are non-spherical, with ellipsoids and spherocylinders being among the most common shapes.

Developing a more precise model tailored to non-spherical particles could improve the accuracy of particulate flow simulations.

The applications span a wide array of fields, from biological systems to industrial processes.



Coal Combustion













#### **Technical background**

- Research on non-spherical particle drag is scarce in the literature.
- Existing particulate simulation packages operate under the assumption that particles are spherical, employing spherical drag force models.
- Drag could differ significantly between spherical particles and non-spherical particles



# **UTSA** Descriptions of a Non-spherical Particle



#### The "Size" of non-spherical particles

The volume equivalent diameter:

$$d_V = \frac{3}{\sqrt{\pi}} \frac{6V}{\pi}$$

(5)

The longest dimension of the particle,  $L_L$ .

The shortest dimension of the particle,  $L_s$ .

An intermediate dimension of the particle,  $L_P$ , typically defined as  $(L_L L_S)^{0.5}$ .

The diameter of the smallest sphere that circumscribes the particle,  $d_c$ .

The diameter of the largest sphere that may be enclosed by the particle,  $d_e$ .



#### The "Shape" of a non-spherical particle

The sphericity: defined as the ratio of the surface area of the sphere with equivalent volume to the actual surface area of the particle

Corey shape factor: primarily used for ellipsoids with three semi-axes a>b>c:  $\beta = \frac{c}{\sqrt{ab}}$ 

**Circularity (roundness): the ratio of the area equivalent diameter to the projected perimeter diameter of the particle in the direction of motion** 

Aspect ratio or elongation,  $E_L$ : defined as the ratio of its longest to its shortest dimension,  $L_L/L_S$ 

The *flatness* of a particle,  $F_L$ : defined as the ratio  $L_S/L_I$ . The size of the dimension  $L_I$  is between the longest and the shortest dimensions:  $L_L > L_I > L_S$ .

### **Drag Correlations of A Single Non-Spherical Particle**



### **Spherical particle**

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#### Schiller and Nauman

$$C_D = \frac{24}{Re} \Big( 1 + 0.15 Re^{0.687} \Big)$$

**Clift and Gauvin** 
$$C_D = \frac{2}{R}$$

$$\frac{24}{Re} \left( 1 + 0.15Re^{0.687} \right) + \left( \frac{0.42}{1 + 42500Re^{-1.16}} \right).$$

#### **Non-spherical particle**

Haider & Levenspiel, 1989	$C_D = \frac{24}{Re} (1 + ARe^B) + \frac{C}{1 + \frac{D}{Re}}.$	Loth, 2008	$K_1$ is chosen as $K_1 = \left(\frac{LI}{S^2}\right)^{-0.09}$ in the Ganser (1993) correlation.
	$A = e^{2.3288-6.458\Phi+2.4486\Phi^2}, B = 0.0964 + 0.5565\Phi,$	Hölzer and Sommerfeld, 2008	$C_D = \frac{8}{Re\sqrt{\Phi_{11}}} + \frac{16}{Re\sqrt{\Phi}} + \frac{3}{c^{-1}} + \frac{0.4210^{0.4(-\log\Phi)^{0.2}}}{\Phi_T}.$
	$C = e^{4.905 - 13.8944\Phi + 18.4222\Phi^2 - 10.2599\Phi^3},$		Often time $\Phi_{\parallel}$ is substituted by $\Phi_T$ for convenience.
Swamee & Ojha, 1991	$D = e^{1.4081+12.25640+26.75220} + (\frac{Re}{0.00000000000000000000000000000000000$	Dioguardi, F., and Mele, 2015	$C_D = C_{Ds} \frac{1}{Re^2 \Psi^a} \left(\frac{Re}{1.1833}\right)^{2.0721}$ , $a = Re^{-0.23}$ for $Re < 50$ and $Re^{0.05}$ for $50 < 10^{-0.23}$
	$\beta = \frac{c}{\sqrt{ab}}$ is the Corey shape factor, where a, b, and c = lengths of the three		$Re \le 10^{\circ}$ . The modified sphericity factor $\Psi = \Phi/X$ .
Ganser, 1993	principal axes of the particle in decreasing order of magnitude. $C_D = \frac{24}{ReK_1} \left[ 1 + 0.1118 (ReK_1K_2)^{0.6567} \right] + \frac{0.4305K_2}{1 + \frac{0.3305K_2}{21 + \frac{3305}{ReK_1K_2K_2K_2K_2K_2K_2K_2K_2K_2K_2K_2K_2K_2K$	Bagheri & Bonadonna, 2016	$C_D = \frac{24K_S}{Re} \left[ 1 + 0.125 \left( \frac{ReK_N}{K_S} \right)^{0.667} \right] + \frac{0.46K_N}{1 + \frac{5330}{ReK_N/K_S}}, Re < 3 \times 10^5.$
	$K_1 = \left(\frac{1}{3} + \frac{2}{3\sqrt{\Phi}}\right)^{-1}$ for isometric-like particles or $K_1 = \left(\frac{d_{PA}}{3d_V} + \frac{2}{3\sqrt{\Phi}}\right)^{-1}$ for non- isometric-like particles. $d_{PA}$ is the diameter of the equal projected circular area.		$K_S = \frac{1}{2} \left( F_S^{\frac{1}{3}} + F_S^{-\frac{1}{3}} \right), K_N = 10^{a[-\log(F_N)]^b}$ with $a = 0.45 + \frac{10}{e^{2.5(\log(\rho'))} + 30}$ and
Chien, S. F., 1994.	$K_2 = 10^{1.8148(-\log \Phi)^{0.5743}}.$ $C_D = \frac{30}{Re} + 67.289e^{-5.03\Phi}.$		$b = 1 - \frac{37}{e^{3\log(\rho')} + 100}$ , and $F_S = fe^{1.3} \frac{d_V^3}{L_L L_l L_S}$ and $F_N = f^2 e \frac{d_V^3}{L_L L_l L_S}$ , where flatness
Tran-Cong, et al., 2004	$C_D = \frac{24}{Re\frac{d_{PA}}{d_V}} \left[ 1 + \frac{0.15}{\sqrt{c_P}} \left( Re\frac{d_{PA}}{d_V} \right)^{0.687} \right] + \frac{0.42}{\sqrt{c_P} \left[ 1 + 4.25 \times 10^4 \left( Re\frac{d_{PA}}{L_A} \right)^{-1.16} \right]}.$		$f = \frac{L_S}{L_I}$ and elongation $e = \frac{L_I}{L_L}$ .
	The projected area diameter defined as the diameter of a circle that would have		
	the same area projected in the direction of the motion the particle $d_{PA} = \sqrt{\frac{4A_P}{\pi}}$ .		
	The <i>projected circularity</i> , is the ratio of the perimeter of the projected circle in the direction of motion to the actual perimeter of the particle projected in the		
	same direction: $c_P = \frac{\pi d_{PA}}{2} = \frac{\sqrt{4\pi A_P}}{2}$		

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### **Non-Spherical Particles Implemented in the DNS Code**





Particle Shape	$d_V$	$A_P$	d <sub>A</sub>	$L_L$	$L_I$	$L_S$	Ф
(r is radius, h is height)							
Sphere (r= a)	2a	$4\pi a^2$	2a	2a	2a	2a	1
Cube (side length a)	1.241a	6 <i>a</i> <sup>2</sup>	$\sqrt{6/\pi}a$	$\sqrt{3}a$	$\sqrt{2}a$	а	0.806
Disk (r=a, h=0.2a)	1.063a	$2.4\pi a^{2}$	1.55a	2.01 <i>a</i>	0.2 <i>a</i>	0.2 <i>a</i>	0.471
Cylinder (r=a, h=4a)	2.884a	$10\pi a^2$	3.31a	4.47a	4a	4a	0.832
Spherocylinder (r=a, h=8a)	3.530a	$16\pi a^{2}$	4a	8a	2a	2a	0.779
Prolate (a=b, c=2a)	2.520a	$6.831\pi a^{2}$	2.614a	4a	2a	2a	0.930
Oblate (a=b, c=0.5a)	1.587a	$2.763\pi a^{2}$	1.662a	2a	2a	а	0.912
Cone (r=a, h=2a)	1.587a	$3.236\pi a^2$	1.799a	2.236a	2a	2a	0.778

### **UTSA** Correlation of Drag Coefficients of A Spherocylinder



#### Uniform flow over a Spherocylinder

- Three Dimensionless Parameters Inputs
  - Reynolds Number:  $Re = \frac{\rho U D_e}{\mu}$
  - Aspect ratio:  $\beta = \frac{a+b}{a}$
  - Incident angle:  $\theta$
- Three Coefficients Outputs
  - Drag coefficient: C<sub>D</sub>
  - Lift coefficient:  $C_L$
  - Torque coefficient:  $C_T$







#### Simulation domain size and grid resolutions study



Affect of the domain size to the drag

Re	Grid Resolution $(D/h)$	<b>Domain Size</b> $(L/D)$
<b>0.</b> 1 ≤ <i>Re</i> ≤ 5	10	$18 \times 18 \times 18$
$5 < Re \le 200$	20	$9 \times 9 \times 20$
200 < Re	30	$8 \times 8 \times 24$

Selection of grid resolution and grid size in the simulations

## **UTSA** Direct Numerical Simulation (DNS) Method



#### Validations

#### **Drag Coefficient of a Sphere**



#### Spherocylinder at $\beta = 6$ and $\theta = \pi/3$

Re=10	C <sub>D</sub>	C <sub>L</sub>	C <sub>T</sub>
Zastawny et al.	5.00	0.85	1.2
Ouchene	6.60	1.20	1.50
Present	6.92	1.23	1.57

Re=300	e=300 C <sub>D</sub>		C <sub>T</sub>	
Zastawny et al.	1.25	0.56	0.6	
Ouchene	1.49	0.56	0.84	
Present	1.40	0.53	0.82	

#### **Correlation of Drag Coefficient**

- Aspect Ratio:  $1 \le \beta \le 6$ ,
- Orientation Angle:  $0^0 \le \theta \le 90^0$
- **Reynolds Number:**  $0.1 \le Re \le 300$

 $C_{D,\theta} = C_{D,\theta=0^{\circ}} + (C_{D,\theta=0^{\circ},90^{\circ}} - C_{D,\theta=0^{\circ}})sin^{n}\theta$ 

 $n = 2 - (0.72 - 0.062\beta)(1 - e^{-(0.012 - 0.0034\beta + 0.00038\beta^2)Re}$ 

 $b_0 = 2.107 + 0.00357\sqrt{Re} - 0.00304Re.$  $a_0 = 2.460 + 0.203\sqrt{Re} - 0.00613Re$ .  $C_{D,\theta=0^{\circ}} = (a_0 + a_1\beta^{0.5} + a_2\beta + a_3\beta^{1.5} + 0.15\beta^2)C_{Ds}$  $b_1 = -3.037 - 0.0487\sqrt{Re} + 0.00575Re$ .  $a_1 = -3.461 - 0.324\sqrt{Re} + 0.00912Re$ .  $b_2 = 2.872 + 0.0605\sqrt{Re} - 0.00388Re.$  $a_2 = 2.957 + 0.151\sqrt{Re} - 0.00420Re$ .  $C_{D,\theta=90^{\circ}} = (b_0 + b_1\beta^{0.5} + b_2\beta + b_3\beta^{1.5} + 0.15\beta^2)C_{Ds}$  $b_3 = -1.070 - 0.0106\sqrt{Re} + 0.000641Re.$  $a_2 = -1.084 - 0.0252\sqrt{Re} + 0.000699Re$ 







#### **Results of General Drag Correlation\***

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Comparisons of Correlations (Drag coefficient at  $\theta = 0^o$  and  $90^0$  for  $\beta = 4$ )



Drag coefficients for a spherocylinder in terms of ( $\theta$ ,  $\beta$ , *Re*)



### **Correlation of Drag Coefficients of An Ellipsoid**

#### Drag coefficient of an oblate

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- Aspect Ratio:  $0.1 \le \beta < 1$ ,
- Orientation Angle:  $0^0 \le \theta \le 90^0$
- **Reynolds Number:**  $0.1 \le Re \le 300$

$$C_{D,\theta} = C_{D,\theta=0^{\circ}} + (C_{D,\theta=0^{\circ},90^{\circ}} - C_{D,\theta=0^{\circ}})sin^{n}\theta$$

$$C_{D,\theta=0^{\circ}} = (a_0 + a_1\beta^{0.5} + a_2\beta + a_3\beta^{1.5} + a_4\beta^2)C_{Ds}$$
$$C_{D,\theta=90^{\circ}} = (b_0 + b_1\beta^{0.5} + b_2\beta + b_3\beta^{1.5} + b_4\beta^2)C_{Ds}$$

$$\begin{aligned} a_0 &= 5.049 + 0.219\sqrt{Re} + 0.6841Re - 0.10133Re^{1.5} + 0.00374Re^2 \\ a_1 &= -16.408 + 0.723\sqrt{Re} - 4.2139Re + 0.64318Re^{1.5} - 0.023929Re^2 \\ a_2 &= 29.581 - 6.259\sqrt{Re} + 10.4596Re - 1.59645Re^{1.5} + 0.059298Re^2 \\ a_3 &= -26.511 + 10.481\sqrt{Re} - 11.5757Re + 1.7501Re^{1.5} - 0.064775Re^2 \\ a_4 &= 9.407 - 5.286\sqrt{Re} + 4.7065Re - 0.70377Re^{1.5} + 0.025962Re^2 \end{aligned}$$

$$b_0 = 3.306 + 0.673\sqrt{Re} - 0.1496Re + 0.01182Re^{1.5} - 0.000297Re^2$$
  

$$b_1 = -9.711 - 4.323\sqrt{Re} + 0.9641Re - 0.07901Re^{1.5} + 0.002032Re^2$$
  

$$b_2 = 16.492 + 10.062\sqrt{Re} - 2.2856Re + 0.19106Re^{1.5} - 0.004971Re^2$$
  

$$b_3 = -13.145 - 10.273\sqrt{Re} + 2.3737Re - 0.20098Re^{1.5} + 0.005274Re^2$$
  

$$b_4 = 4.112 + 3.871\sqrt{Re} - 0.9036Re + 0.07712Re^{1.5} - 0.002038Re^2$$





#### Drag coefficient of a prolate

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- Aspect Ratio:  $1 \le \beta \le 3$ ,
- Orientation Angle:  $0^0 \le \theta \le 90^0$
- **Reynolds Number:**  $0.1 \le Re \le 300$

$$C_{D,\theta} = C_{D,\theta=0^{\circ}} + (C_{D,\theta=0^{\circ},90^{\circ}} - C_{D,\theta=0^{\circ}})sin^{n}\theta$$

$$C_{D,\theta=0^{\circ}} = (a_0 + a_1\beta^{0.5} + a_2\beta + a_3\beta^{1.5} + 0.35\beta^2)C_{Ds}$$
$$C_{D,\theta=90^{\circ}} = (b_0 + b_1\beta^{0.5} + b_2\beta + b_3\beta^{1.5} + 0.5\beta^2)C_{Ds}$$

$$a_0 = 2.993 + 0.2841\sqrt{Re} - 0.00961Re$$
  

$$a_1 = -5.026 - 0.5279\sqrt{Re} + 0.01813Re$$
  

$$a_2 = 4.799 + 0.3082\sqrt{Re} - 0.01113Re$$
  

$$a_3 = -2.087 - 0.063\sqrt{Re} + 0.00235Re$$

 $b_0 = 3.020 - 0.0196\sqrt{Re} + 0.00159Re$ 

 $b_1 = -5.862 - 0.0026\sqrt{Re} - 0.00423Re$ 

 $b_2 = 6.239 + 0.0241 \sqrt{Re} + 0.00355 Re$ 

 $b_3 = -2.868 - 0.0005 \sqrt{Re} - 0.00118 Re$ 









#### **Limitations of Correlations**

1. These methods are primarily designed for analyzing relationships between two variables, making it challenging to extend them to handle three or more variables effectively.

2.Correlations are highly sensitive to outliers in the data, often resulting in skewed and inaccurate results.

**3.**Risk of overfitting, where the model fits the training data too closely and performs poorly when applied to new, unseen data.

4. Unable to develop accurate correlations for lift and torque coefficients due to the intricate nature of the particle shapes a



#### **Completed Master Thesis:**

1. Joshua Conner (2022), "Prediction of the flow dynamics of a sphere translating near a plane wall using a muti-output deep learning model."

2. Sergio Molina (2023), "Implementing artificial neural networks to estimate coefficients of drag, lift, and torques of spherocylinder particles."

3. Daniel Hinojosa (2024), "Using an artificial neural network to predict the drag, lift and torque coefficients of an ellipsoid in a viscous fluid."

**Ongoing Doctoral Dissertation:** 

Jack Smith (expected completion in 2025), "Developing artificial neural network models for assemblies of non-spherical particles."

### Neural Network Model for An Spherocylinder



### **Collect Data**

- 1200 data points generated via Direct Numerical Simulations (DNS)
- Input features:
  - -Aspect Ratio,  $\beta$ : [1.0 6.0]
  - -Reynolds Number, *Re*: [0.1 300]
  - -Angle of Incident,  $\theta$  :  $[0^{\circ} 90^{\circ}]$
- Output features:
  - -Coefficient of Drag,  $C_D$ : [0 400]
  - -Coefficient of Lift,  $C_L$ : [0 60]
  - -Coefficient of Torque,  $C_T$ : [0 6]



# UTSA Neural Network Model for An Spherocylinder

### **Distributions in Data**

- The output label data is right-skewed, exponentially distributed
- Skewed distributions lead to model learning bias due to overrepresentation
- The range of values is large which can also result learning bias towards larger values
  - -Coefficient of Drag,  $C_D$ : [0 400]
  - -Coefficient of Lift,  $C_L$ : [0 60]
  - -Coefficient of Torque,  $C_T$ : [0 6]

#### UTSA **Designing Multi-Layered Neural Network (MLNN)**

300

250

200

150

100

50

0.0

0.2

0.4



#### **Data Preprocessing**

Data transformation via Box-Cox transformation

$$- x_{trans} = \begin{cases} \frac{x^{\lambda}}{\lambda}, \ \lambda \neq 0\\ \ln(x+1), \ \lambda = 0 \end{cases}$$
$$- y_{trans} = \begin{cases} \frac{y^{\lambda}}{\lambda}, \ \lambda \neq 0\\ \ln(y+1), \ \lambda = 0 \end{cases}$$

-  $\lambda$  set to 0.25



### **Neural Network Model for An Spherocylinder**



#### Model: "sequential\_60"

Layer (type)	Output	Shape	Param #				
dense_420 (Dense)	(None,	3)	12				
dense_421 (Dense)	(None,	50)	200				
dense_422 (Dense)	(None,	150)	7650				
dense_423 (Dense)	(None,	350)	52850				
dense_424 (Dense)	(None,	500)	175500				
dense_425 (Dense)	(None,	350)	175350				
dense_426 (Dense)	(None,	50)	17550				
dense_427 (Dense)	(None,	3)	153				
Total params: 429,265 Trainable params: 429,265							

Trainable params: 429,265 Non-trainable params: 0

Hyper	
Parameters	Values
	0.00005
Learning Rate	0.0001
	0.001
	1
Layers	5
	10
	32
Batch Size	225
	512

#### **Selected Neural Network Model**

- Learning Rate 0.001
- Batch Size 32
- Epoch 1000

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- Cost Function Mean Squared Logarithmic Error
- Activation Function (hidden layers) ReLU

Learning Rate	Batch Size	Layers	Train Time	RMSE	R² Cd	R² Cf	R² Cm
0.001	32	5	12m 57s	5.5	0.98	0.98	0.98

#### Table 6. MLNN Best Performance Results on Unobserved Data

Learning Rate	Batch Size	Layers	RMSE UD	R <sup>2</sup> UD Cd	R <sup>2</sup> UD Cf	R <sup>2</sup> UD Cm
0.001	32	5	2.1	0.99	0.88	0.94

### UTSA Neural Network Model for An Spherocylinder



#### **Model Performance**







#### **Comparison with the correlation of drag correlations**

- 1000 random cases for aspect ratio, incident angle, and Reynolds number
- MLNN achieved a correlation coefficient of 99.9%
- MLNN coefficient of drag estimates fit the correlation extremely well





![](_page_22_Figure_8.jpeg)

## UTSA Artificial Neural Network Model for An Ellipsoid

![](_page_23_Picture_1.jpeg)

![](_page_23_Figure_2.jpeg)

Drag, Lift and Torge Prediction

# UTSA Artificial Neural Network Model for An Ellipsoid

![](_page_24_Picture_1.jpeg)

![](_page_24_Figure_2.jpeg)

Drag, Lift, & Torque Prediction Error Distribution

## UTSA Artificial Neural Network Model for An Ellipsoid

![](_page_25_Picture_1.jpeg)

#### DNS, ANN predictions, and present correlation comparisons

![](_page_25_Figure_3.jpeg)

### Drag model of an assembly of non-spherical particles

### **Additional input parameters**

• Solid fractions

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- Orientations at high solid fractions
- Mixture ratio of different shapes of particles

![](_page_26_Picture_5.jpeg)

![](_page_26_Picture_6.jpeg)

![](_page_26_Picture_7.jpeg)

![](_page_26_Picture_8.jpeg)

![](_page_26_Picture_9.jpeg)

![](_page_27_Picture_0.jpeg)

![](_page_27_Picture_1.jpeg)

Stokes flow over an assembly of face-centered arrays of spheres.

Analytical solutions of Hashimoto (1959) for  $\phi < 0.2$ 

Numerical solution of Sangani and Acrivos (1982)

![](_page_27_Figure_5.jpeg)

![](_page_27_Picture_6.jpeg)

Dimensionless drag:

$$F = \frac{F_d}{3\pi\mu dU}$$

![](_page_28_Picture_0.jpeg)

![](_page_28_Picture_1.jpeg)

### Influence of different configurations at the same solid fraction and the same $\frac{dp}{dz}$

![](_page_28_Figure_3.jpeg)

#### Influence of number of particles used in a simulation box for $\phi = 0.2$ and Re = 56

Number of Particles	50	150	200
Dimensionless Drag F	9.32	9.50	9.44

# **UTSA** Drag model of an assembly of spheres

![](_page_29_Picture_1.jpeg)

Ergun: 
$$F^*(\phi, Re) = \frac{150\phi}{(18(1-\phi)^2 + 1.75/(18(1-\phi)^2 Re.))}$$
  
Wen and Yu:  $F^*(\phi, Re) = \frac{(1+0.15Re^{0.687})(1-\phi)^{-3.7}}{(1-\phi)^{-3.7}}$ .  
Tang et al. (2015):  
•  $F^*(\phi, Re) = \frac{10\phi}{(1-\phi)^2} + (1-\phi)^2 (1+1.5\sqrt{\phi}) + \left[ 0.11\phi(1+\phi) - \frac{0.00456}{(1-\phi)^4} + \left( 0.169(1-\phi) + \frac{0.0644}{(1-\phi)^4} \right) Re^{-0.343} \right] Re$ .

![](_page_29_Figure_3.jpeg)

![](_page_29_Figure_4.jpeg)

Random distributions of 100 spheres in a cube for  $\phi = 0.1$  and  $\phi = 0.4$ .

![](_page_29_Figure_6.jpeg)

![](_page_30_Picture_0.jpeg)

![](_page_30_Picture_1.jpeg)

#### Individual drag force of 100 spherocylinders in an assembly

1. Individual drag forces fluctuate within approximately 10% of the average drag force.

2. The lift forces exerted on the particles are relatively insignificant when compared to the drag forces

![](_page_30_Figure_5.jpeg)

![](_page_30_Figure_6.jpeg)

*Dimensionless fluid-sphere forces of 100 particle at Re=18.3 and*  $\phi = 0.1$ 

![](_page_31_Picture_0.jpeg)

![](_page_31_Picture_1.jpeg)

#### **Effect of different configurations**

![](_page_31_Figure_3.jpeg)

#### **Comparison with sphere drag models**

![](_page_31_Figure_5.jpeg)

![](_page_32_Picture_0.jpeg)

# Average dimensionless drag of an assembly (80 particles) of mixed spheres and ellipsoids at solid fraction $\phi = 0.1$ at different mixed ratio

![](_page_32_Figure_3.jpeg)

![](_page_32_Picture_4.jpeg)

![](_page_33_Picture_1.jpeg)

#### Average dimensionless drag for spheres and ellipsoids:

Percentage of ellipsoids	Re	Average <i>F</i> for all particles	Average F for spheres	Average F for ellipsoids
0%	179	11.83	11.83	N/A
25%	175	12.13	11.45	14.16
75%	163	12.96	11.42	13.47
100%	153	13.84	N/A	13.84

1. At the same pressure gradient, Reynolds number decreases as the percentages of nonspherical ellipsoids increases.

2. The average drag for spheres is less than that for ellipsoids, i.e., no-spherical particles would have higher drag coefficients in comparison with spheres

![](_page_34_Picture_0.jpeg)

![](_page_34_Picture_1.jpeg)

Neural Network Model for the drag coefficients of assemblies of particles Non-spherical particles:

spherocylinders, ellipsoids, and mixed bi-disperse assembly Input parameters:

Reynolds number, aspect ratio, solid fraction, orientation, bi-disperse mixed ratio

![](_page_35_Picture_0.jpeg)

![](_page_35_Picture_1.jpeg)

#### **Student contributors to this project**

- Graduate students: Joshua Corner, Daniel Hinojosa, Sergio Molina, Miguel Cortina, Jack Smith, and James Standard.
- Undergraduate student: Andres Leon Islas, Mahnoor Bokhari, James Moseley, and Joshua Beltran

#### **Journal Publications:**

"Wall Effects on the Flow Dynamics of a Rigid Sphere in Motion." Journal of Fluids Engineering 143, no. 8 (2021): 081106.

"Review—Drag Coefficients of Non-Spherical and Irregularly Shaped Particles." ASME. J. Fluids Eng. June 2023; 145(6): 060801.

"A general and accurate correlation for the drag on spherocylinders." International Journal of Multiphase Flow 168 (2023): 104579.

![](_page_36_Picture_0.jpeg)

![](_page_36_Picture_1.jpeg)

# Thank you for your time and attention!

![](_page_36_Picture_3.jpeg)