

Bridging the Gap: Coupled Poromechanical and Earthquake Simulation to Model Induced Seismicity

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Budget & research team

Project timeline: October 2023 – September 2025

Budget: 300k (Y1) - **500k (Y2)** - 500k (Y3)

LLNL Research team:

- Matteo Cusini
- Kayla Kroll
- Nicola Castelletto
- Randy Settgast
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Collaborators (unfunded):

- Vidar Stiernström (Stanford)
- Matteo Frigo (Stanford)
- Eric Dunham (Stanford)

“ *High-resolution storage estimates that more critically assess the integrity of individual storage complexes in terms of their ability to sequester CO₂ without significant leakage [...] and **avoid triggering of injection-induced seismicity** will eventually be required.* ”

Getting to Neutral. Chapter 6, page 87, 2020.

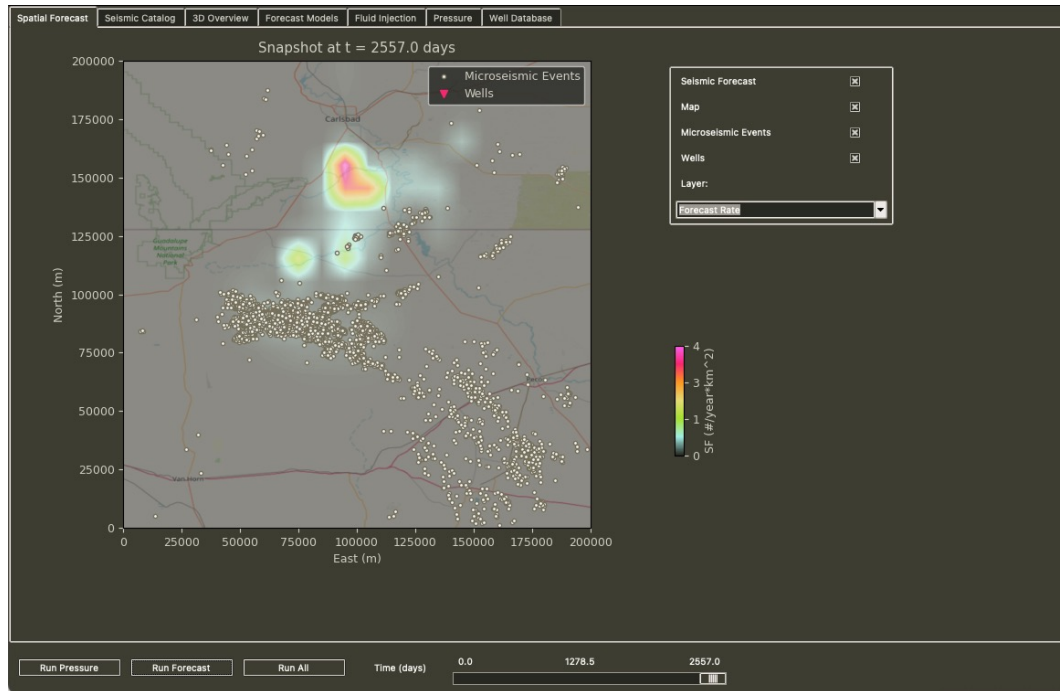
We need models to understand **induced seismicity!**



- How can we **quantify** hazards at new site?
- What **data** are useful to reduce uncertainty?
- Which type of data is most important to quantify induced seismicity hazards?
- Are there **effective management strategies**?

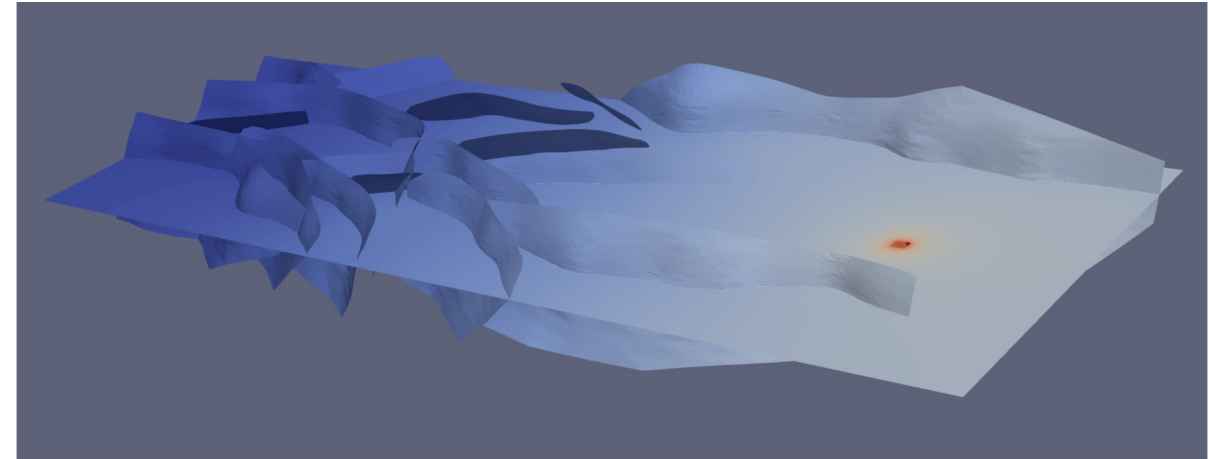
Models of induced seismicity

Reduced order models (Orion)



- Provide information about the seismicity rate
- Low computational cost
- Can be used by non-experts

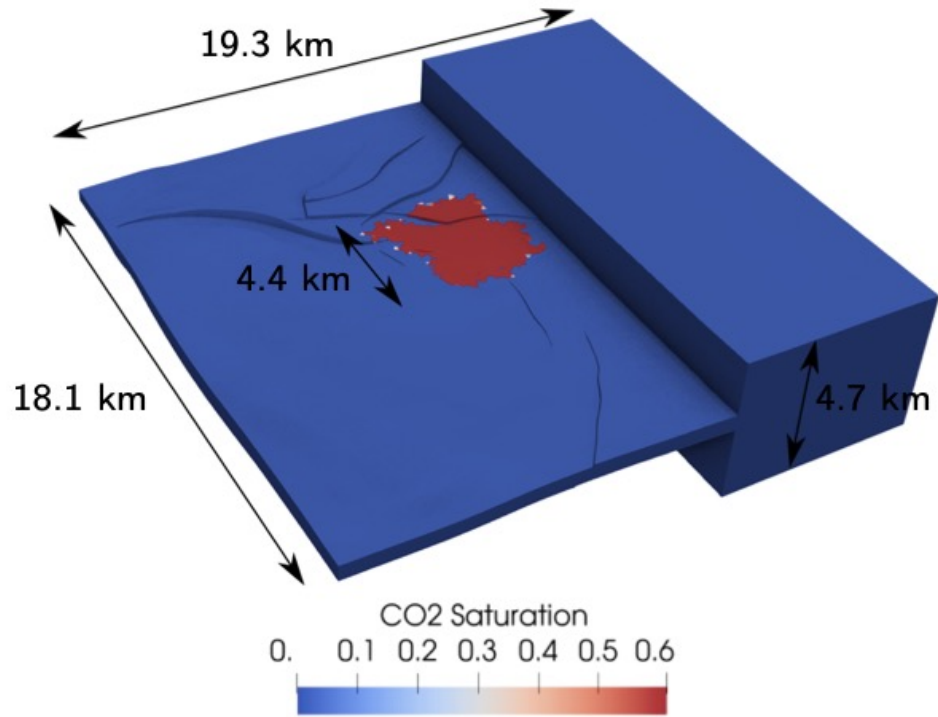
High fidelity models (this project)



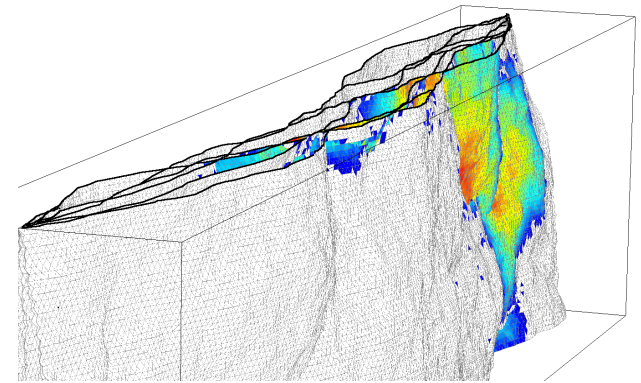
- Provide event locations and magnitudes
- Computationally expensive
- Necessary to train/validate ROM

Key ingredients

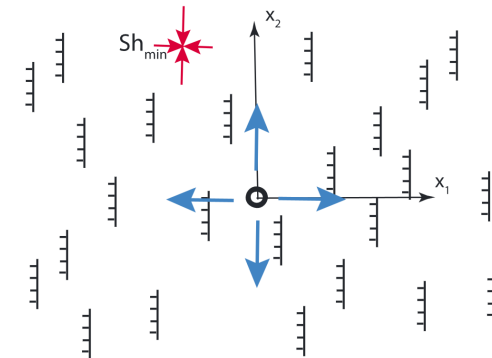
A poromechanics simulation module



An earthquake model



Two-way Coupling



[Seagall and Lu (2015)]

Objectives and subtasks

Objective: develop, within the open-source **GEOS** simulation framework, a high-fidelity coupled poromechanical and earthquake rupture simulator.

Subtask 1 – Quasi-static fault stability analysis capability

- **Deliverable:** quasi-static fault modeling capability in the open-source GEOS framework.

Subtask 2 – Quasi-dynamic fault modeling capability

- **Deliverable:** a coupled poromechanics-earthquake (HM+E) simulation capability in the open-source GEOS framework.

Subtask 3 – Demonstration of the applicability of the developed framework

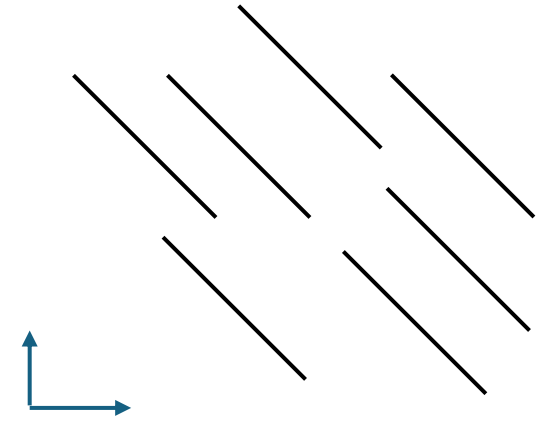
- **Deliverable:** a demonstration of the applicability of the developed capabilities through the modeling of induced-seismicity at a real GCS site.

High-fidelity poromechanics & ROM for seismicity

- Earthquake rate equations are derived from rate-state friction
- We assume we know faults orientation

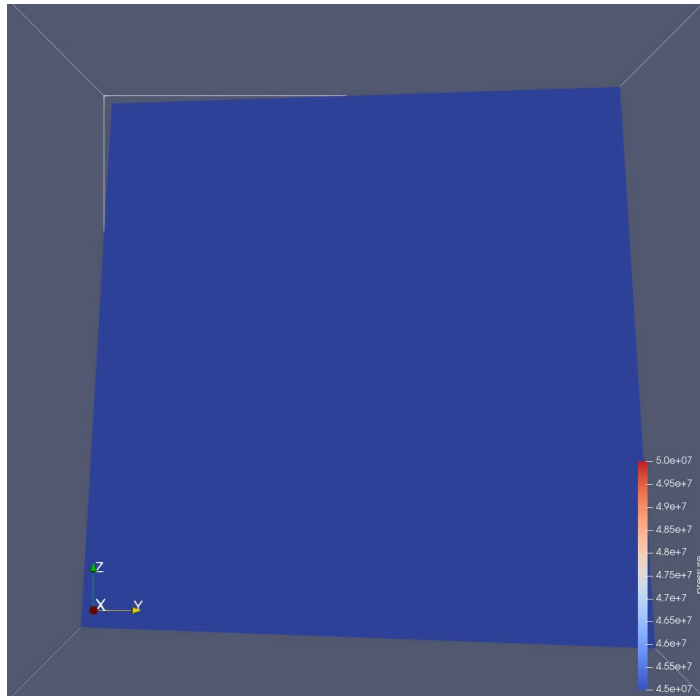
$$\dot{R} = \frac{1}{t_a} \hat{R}(t_a \dot{g}(t) - \hat{R}) \quad t_a = \frac{a\sigma_0}{\dot{\tau}_r}$$
$$\dot{g}(t) = \frac{\dot{\tau}(t)\sigma(t) - \tau(t)\dot{\sigma}(t)}{a\sigma(t)}$$

- σ and τ extracted from a poromechanics simulation.



- Provides a seismicity rate with no information about location and magnitude of the events

Otaniemi Geothermal field, Finland



$$\sigma_0 = 155\text{MPa}$$

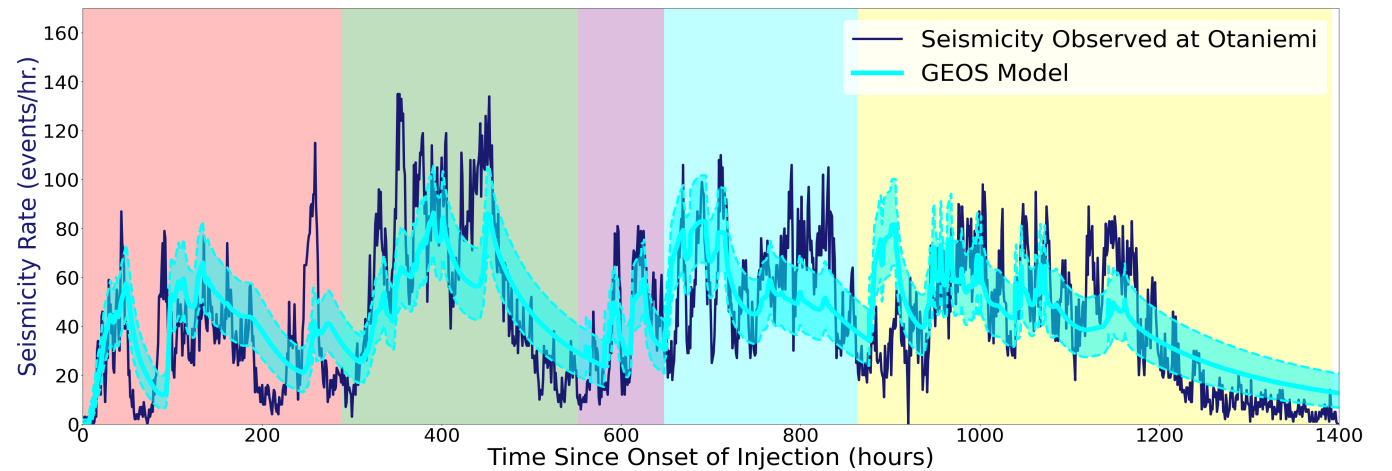
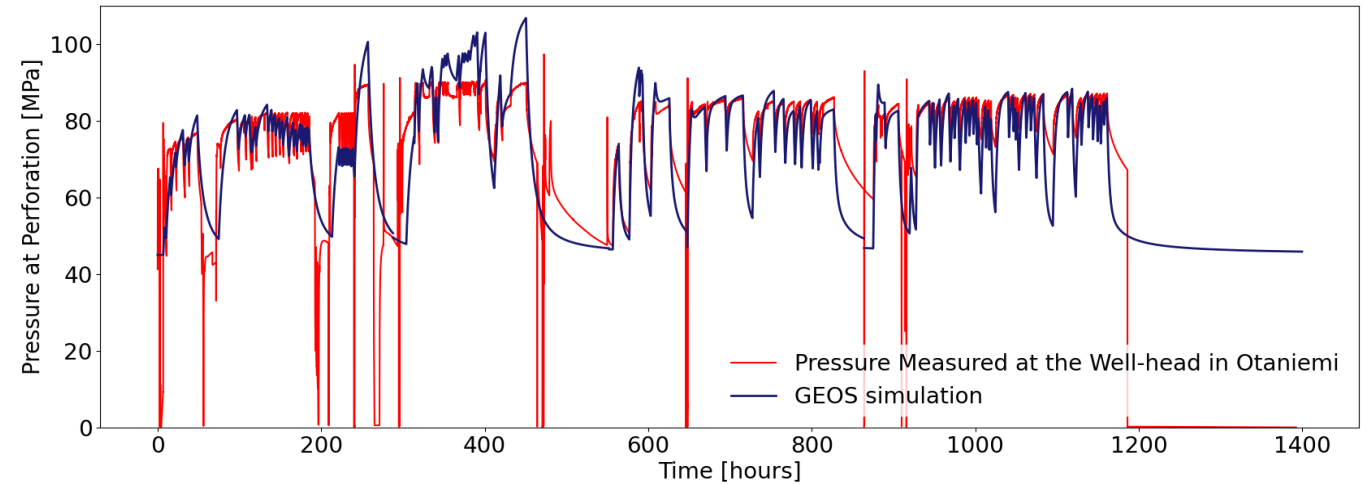
$$k = 8 \cdot 10^{-16}\text{m}^2$$

$$p_0 = 45\text{MPa}$$

$$a = 6 \cdot 10^{-5}$$

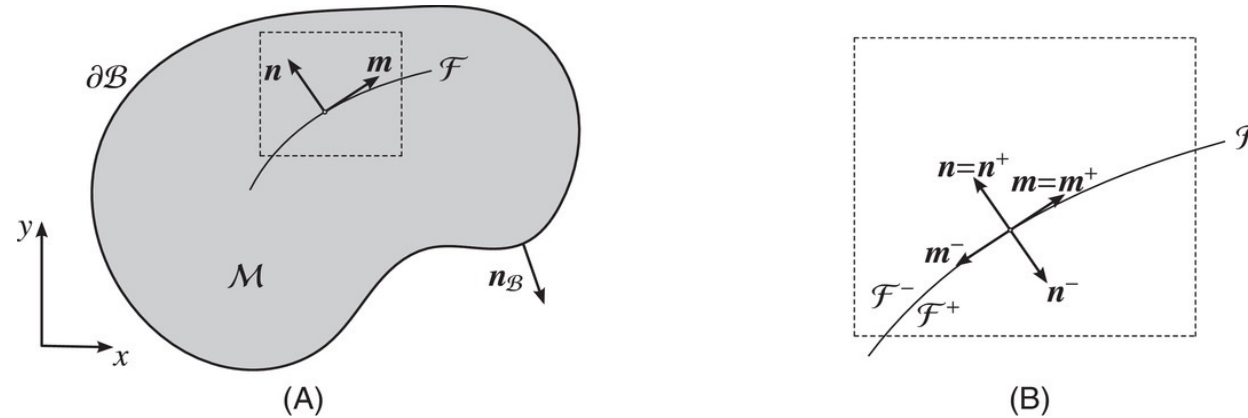
$$\tau_0 = 0.6\bar{\sigma}_0 = 66\text{MPa}$$

$$\dot{\tau}_r = 1\text{kPa/yr}$$



Two-way coupled poromechanics & earthquake model

Step 1: explicitly represent faults in the poromechanical model



$$-\nabla \cdot (\boldsymbol{\sigma}' - bp\mathbf{1}) - \rho\mathbf{g} = 0 \quad \text{on } \mathcal{M} \times (\mathbf{0}, \mathbf{T}] \quad \text{Linear momentum balance}$$

$$\dot{m}_{\pi}^m + \nabla \cdot (\rho_{\pi} \mathbf{v}_{\pi}^m) - q_{\pi}^{fm} - q_{\pi}^m = 0 \quad \text{on } \mathcal{M} \times (\mathbf{0}, \mathbf{T}] \quad \text{Matrix mass balance}$$

$$[[\boldsymbol{\sigma}]] \cdot \mathbf{n} = 0 \quad \text{on } \mathcal{F} \times (\mathbf{0}, \mathbf{T}] \quad \text{Stress continuity across the fracture}$$

$$\dot{m}_{\pi}^f + \nabla \cdot (\rho_{\pi} \mathbf{v}_{\pi}^f) - q_{\pi}^{mf} - q_{\pi}^f = 0 \quad \text{on } \mathcal{F} \times (\mathbf{0}, \mathbf{T}] \quad \text{Fault mass balance}$$

Contact constraints & friction law

Normal contact conditions

$$\lambda_n = \boldsymbol{\lambda} \cdot \mathbf{n} \leq 0 \quad \text{on } \mathfrak{F} \times (0, T]$$

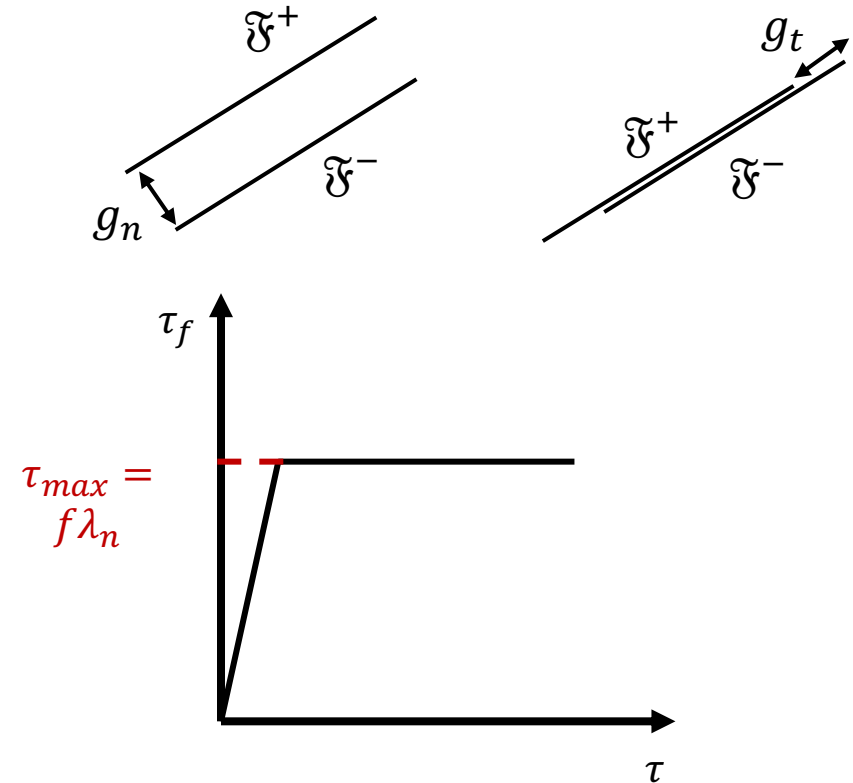
$$g_n = \llbracket \mathbf{u} \rrbracket \cdot \mathbf{n} \geq 0 \quad \text{on } \mathfrak{F} \times (0, T]$$

$$\lambda_n g_n = 0 \quad \text{on } \mathfrak{F} \times (0, T]$$

Coulomb friction law

$$\|\boldsymbol{\lambda}_t\|_2 - \tau_{max} = 0 \quad \text{on } \mathfrak{F} \times (0, T]$$

$$\dot{g}_t \dot{t}_t - \tau_{max} \|\dot{\mathbf{g}}_t\|_2 = 0 \quad \text{on } \mathfrak{F} \times (0, T]$$



Not suited to model seismicity!

Rate- and state-dependent friction

Friction is a function of slip velocity (V) and state variable (θ):

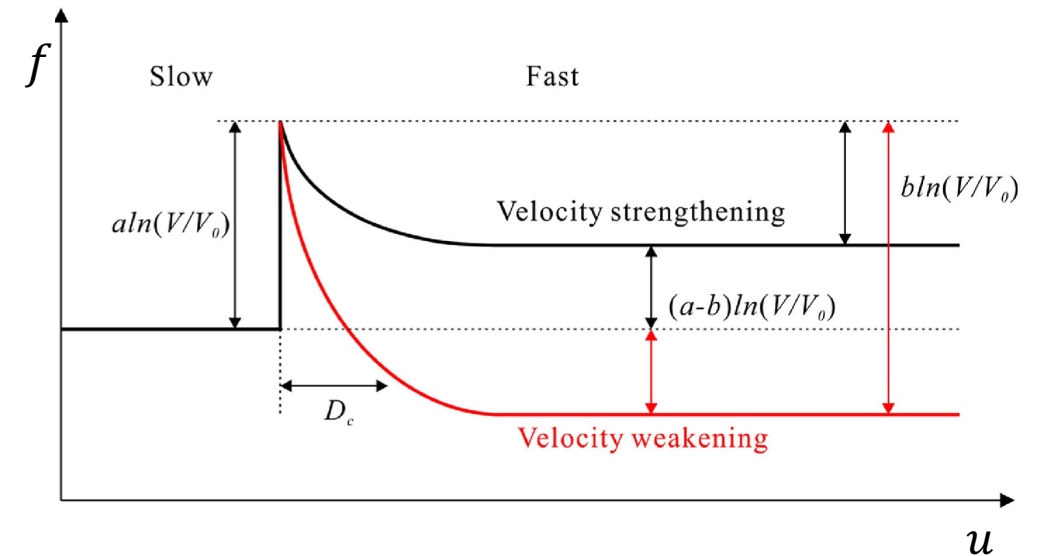
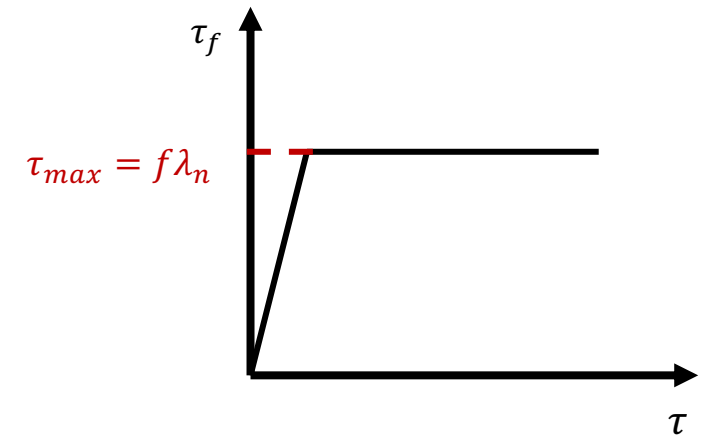
$$f = f_0 + a \ln\left(\frac{V}{V_0}\right) + b \ln\left(\frac{V_0\theta}{D_c}\right)$$

$$\frac{\partial\theta}{\partial t} = -\frac{V\theta}{D_c} \left[\ln\left(\frac{V\theta}{D_c}\right) \right] \quad \text{slip law [Ruina, 1983]}$$

At Steady-state ($\frac{\partial\theta}{\partial t} = 0$):

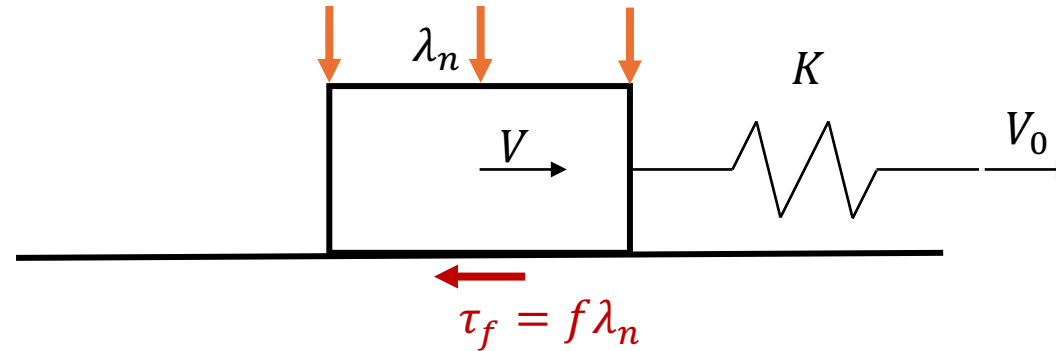
$$f_{ss} = f_0 + (a - b) \ln\left(\frac{V}{V_0}\right)$$

$\begin{cases} a - b < 0 & \text{steady-state velocity weakening} \\ a - b > 0 & \text{steady-state velocity strengthening} \end{cases}$



[modified after Y. Huang et al, Earthq. Research Adv. (2023)]

0D earthquake model: spring-slider system [1/3]



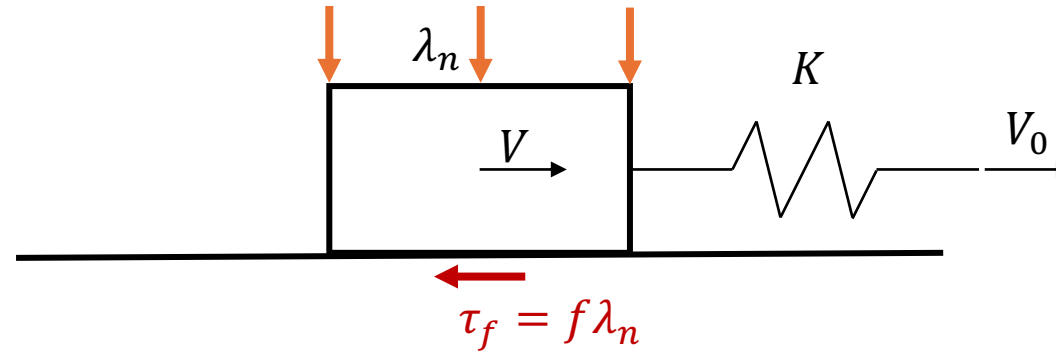
Quasi-dynamic approximation

$$\lambda_t(t) = \tau_0 + \hat{t} \cdot t - K\delta - \boxed{\eta V}$$

ηV is the radiation-damping term where η is the shear impedance

$\tau_0 + \hat{t} \cdot t - K\delta - \eta V - f(\theta, V)\lambda_n = 0$	Force balance
$\frac{\partial \delta}{\partial t} - V = 0$	Slip evolution
$\frac{\partial \theta}{\partial t} + G(V, \theta) = 0$	Slip/Aging law

0D earthquake model: spring-slider system [2/3]



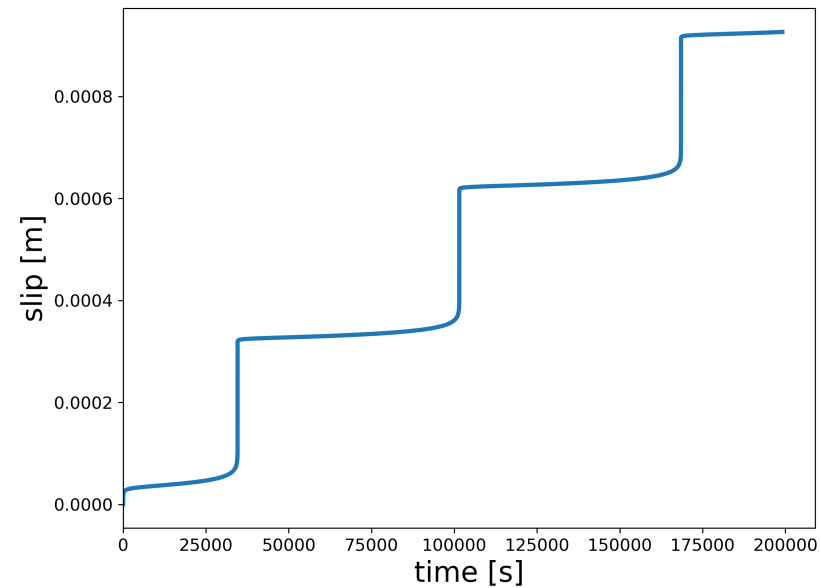
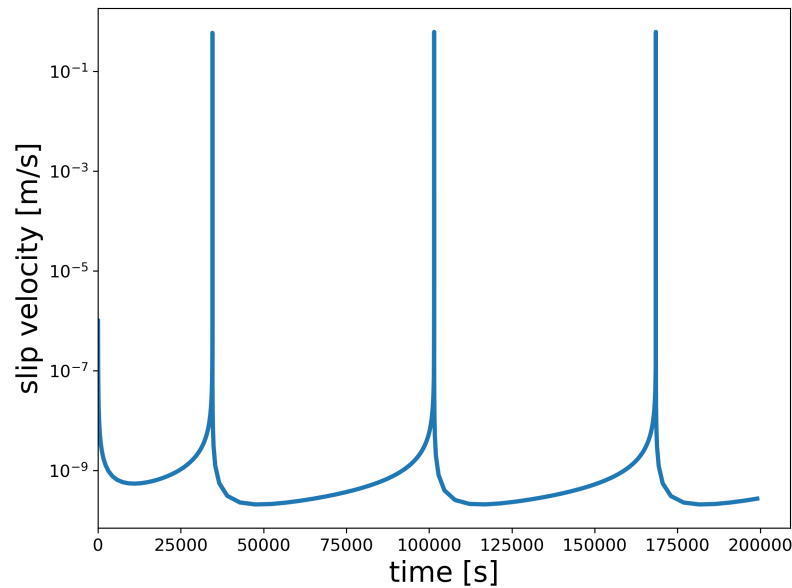
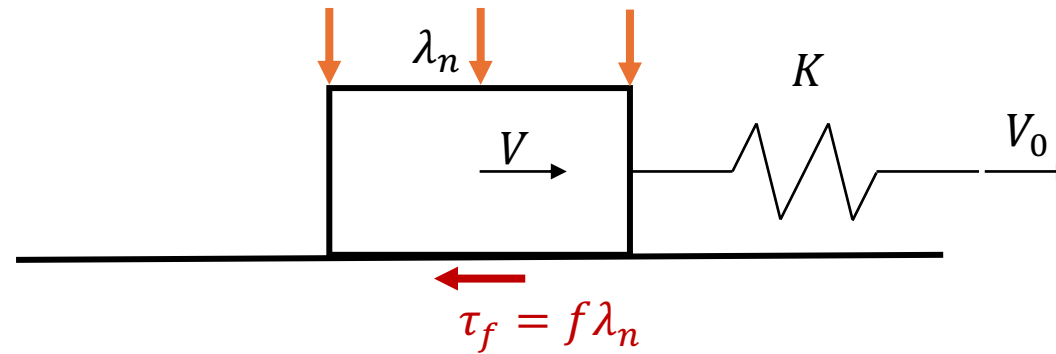
We can discretize with Euler-backward*...

$$\begin{aligned} r_1(\theta, V) &= \tau_n + \hat{\tau} \cdot \Delta t - K(\delta_n + V\Delta t) - \eta V - f(\theta, V)\lambda_n = 0 \\ r_2(\theta, V) &= \frac{\theta - \theta_n}{\Delta t} + G(\theta, V) = 0 \end{aligned}$$

...and solve with the Newton-Raphson method

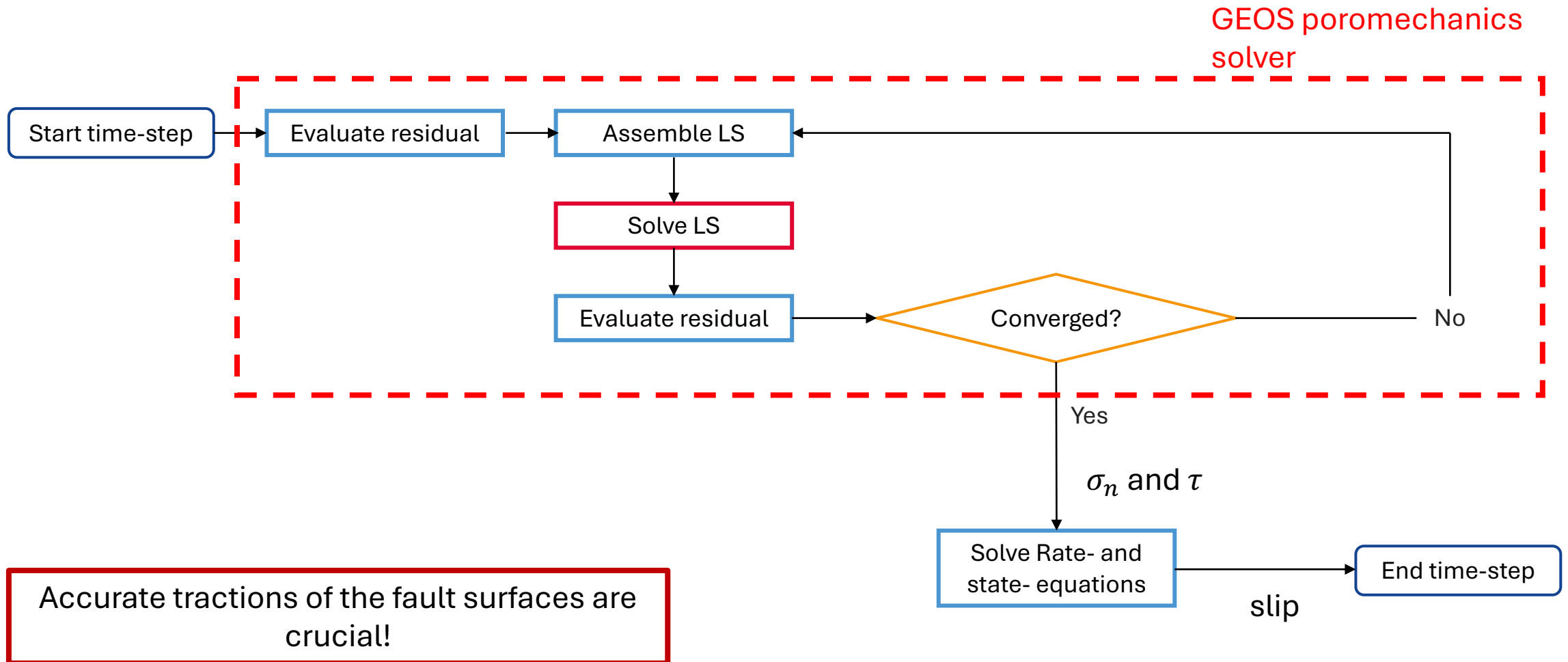
*we have also explored other time-integrators

0D earthquake model: spring-slider system [3/3]

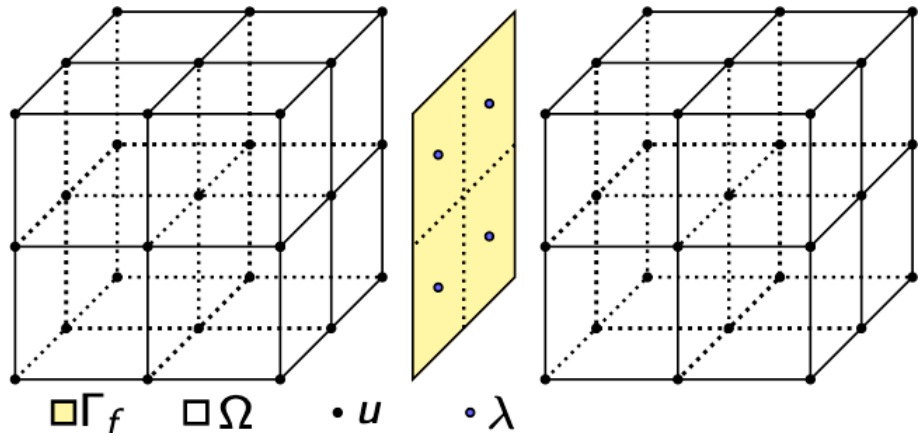


The peaks are characteristic events (i.e., quakes)

Coupled (poro)mechanics & quasi-dynamic earthquake model



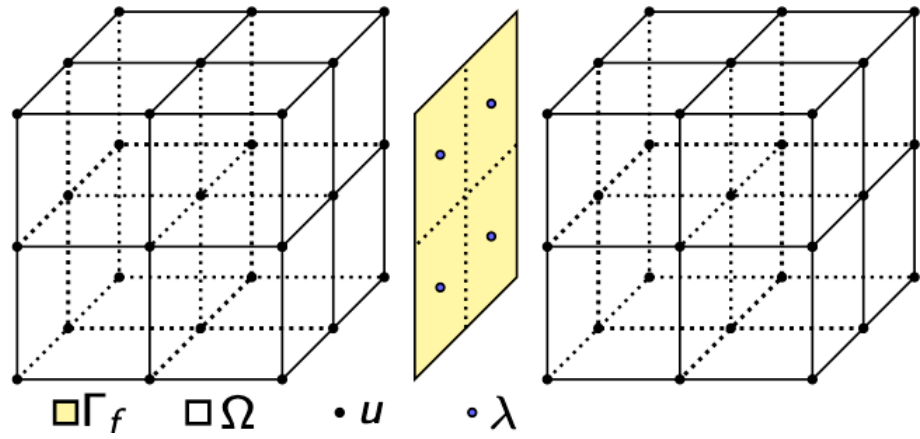
Discretization of explicitly represented faults



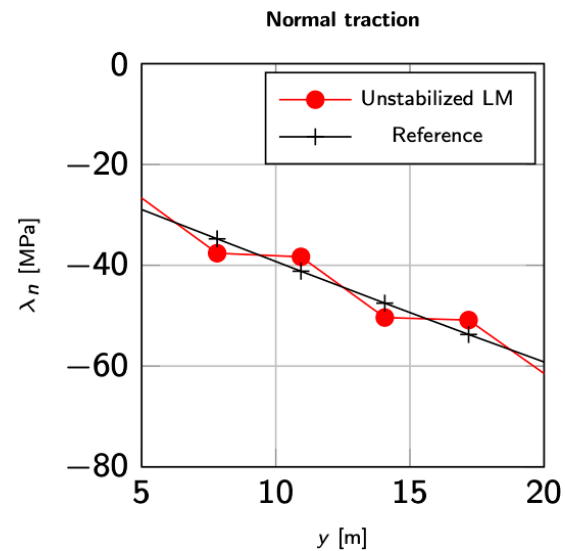
Results in the following saddle-point problem

$$\begin{bmatrix} K_{uu} & C_{u\lambda} \\ C_{\lambda u} & 0 \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = - \begin{bmatrix} r_u \\ r_\lambda \end{bmatrix}$$

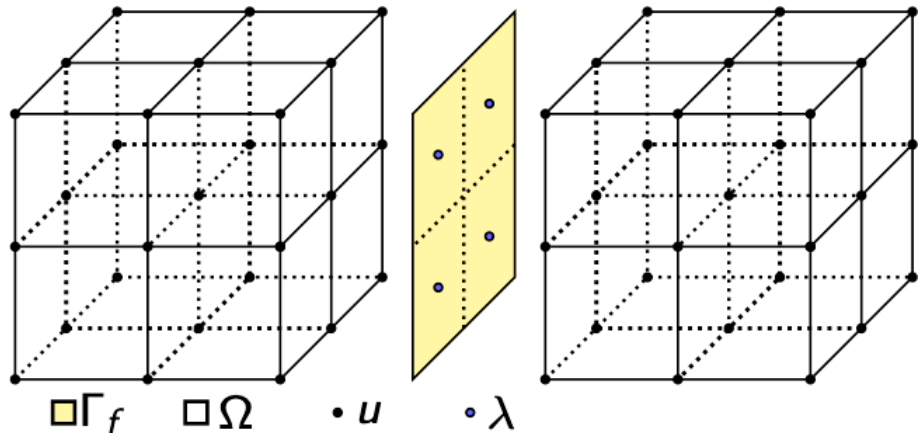
Discretization of explicitly represented faults



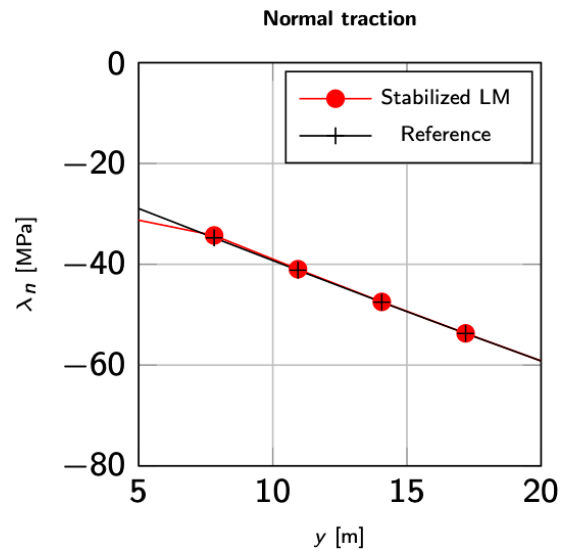
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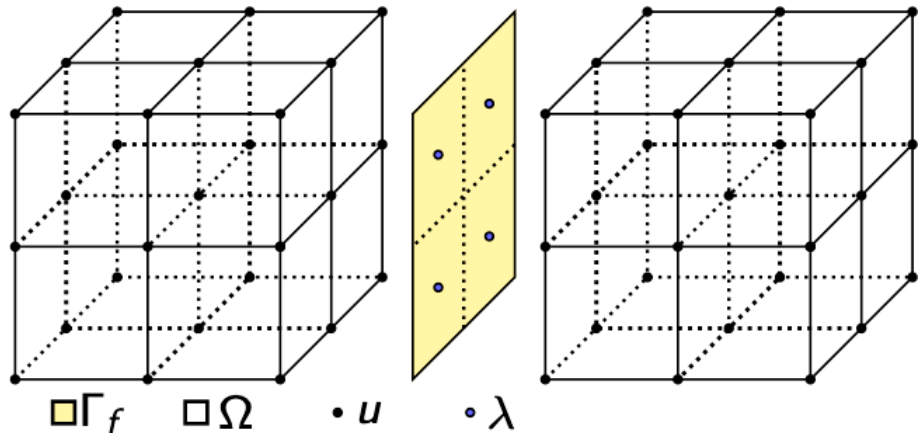
Discretization of explicitly represented faults



Results in the following saddle-point problem



Discretization of explicitly represented faults

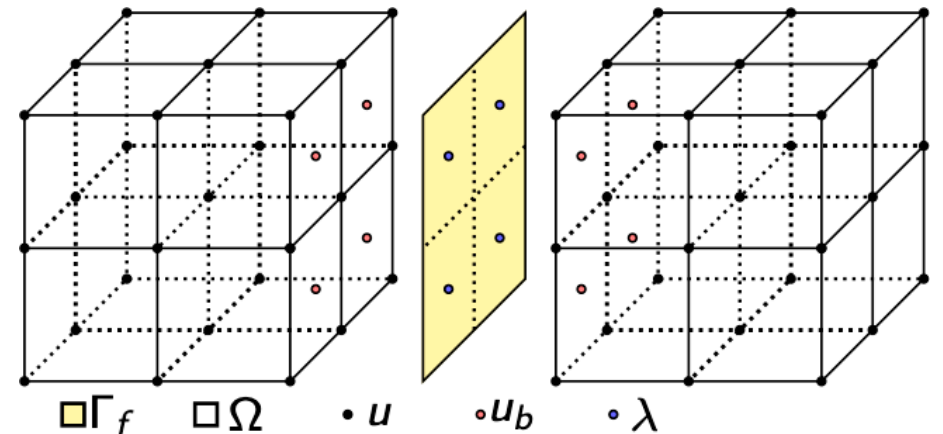
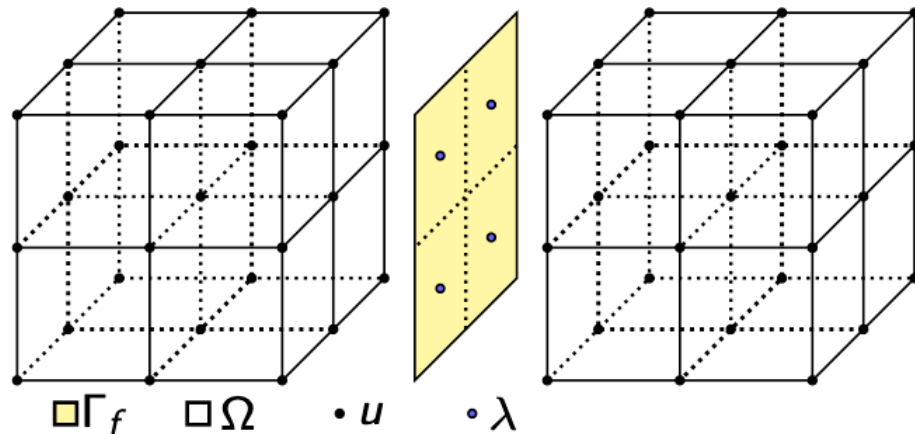


Results in the following saddle-point problem

$$\begin{bmatrix} K_{uu} & C_{u\lambda} \\ C_{\lambda u} & A_{stab} \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = - \begin{bmatrix} r_u \\ r_\lambda \end{bmatrix}$$

The stabilization matrix affects the solution and it is only exact for hexahedral elements.

Discretization of explicitly represented faults



Results in the following saddle-point problem

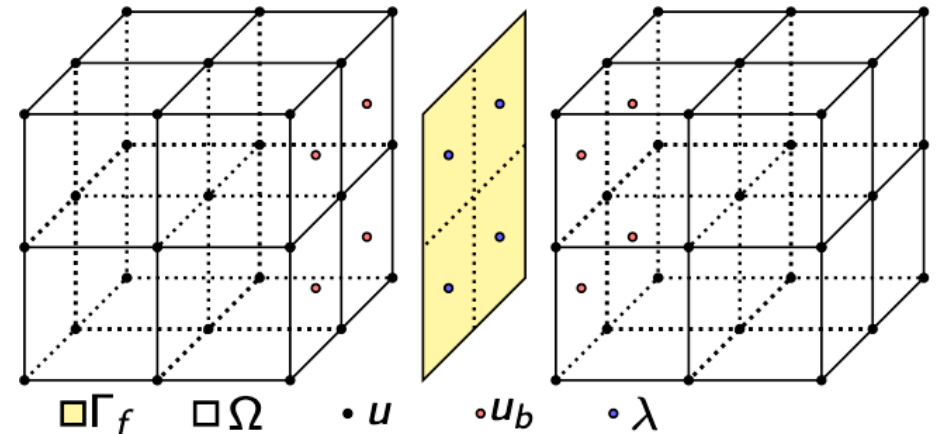
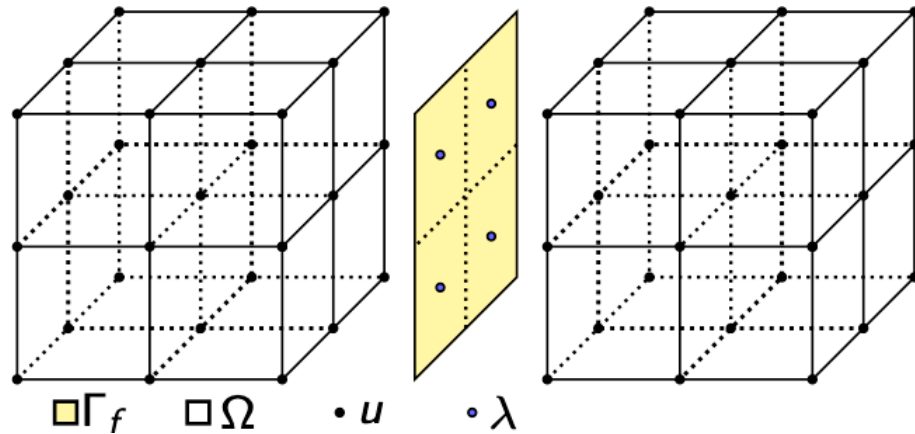
$$\begin{bmatrix} K_{uu} & C_{u\lambda} \\ C_{\lambda u} & A_{stab} \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = - \begin{bmatrix} r_u \\ r_\lambda \end{bmatrix}$$

$$\begin{bmatrix} A_{bb} & A_{bu} & A_{bt} \\ A_{ub} & A_{uu} & A_{ut} \\ A_{tb} & A_{tu} & 0 \end{bmatrix} \begin{bmatrix} u_b \\ u \\ \lambda \end{bmatrix} = - \begin{bmatrix} r_b \\ r_u \\ r_\lambda \end{bmatrix}$$

The stabilization matrix affects the solution and it is only exact for hexahedral elements.

- Does not affect the solution
- It is generic for all element types (as long as we can write the bubble)
- Can be statically condensed

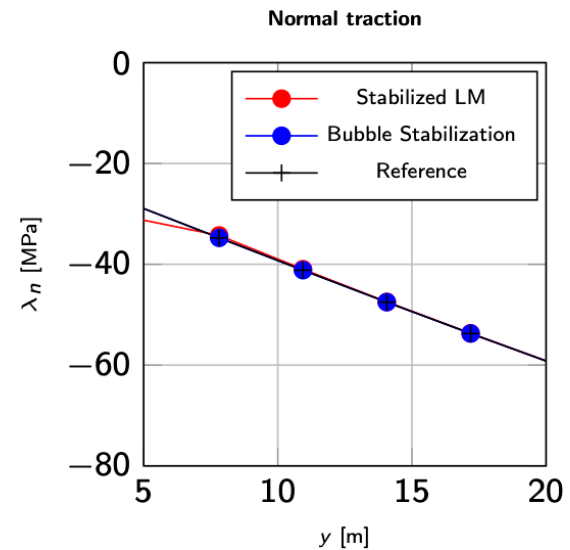
Discretization of explicitly represented faults



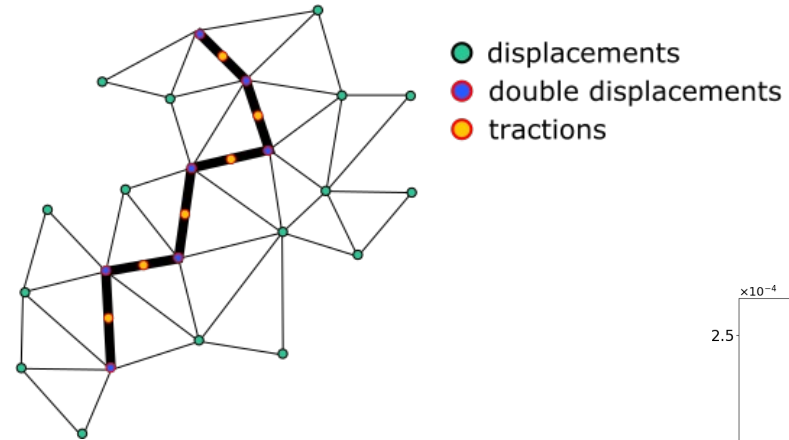
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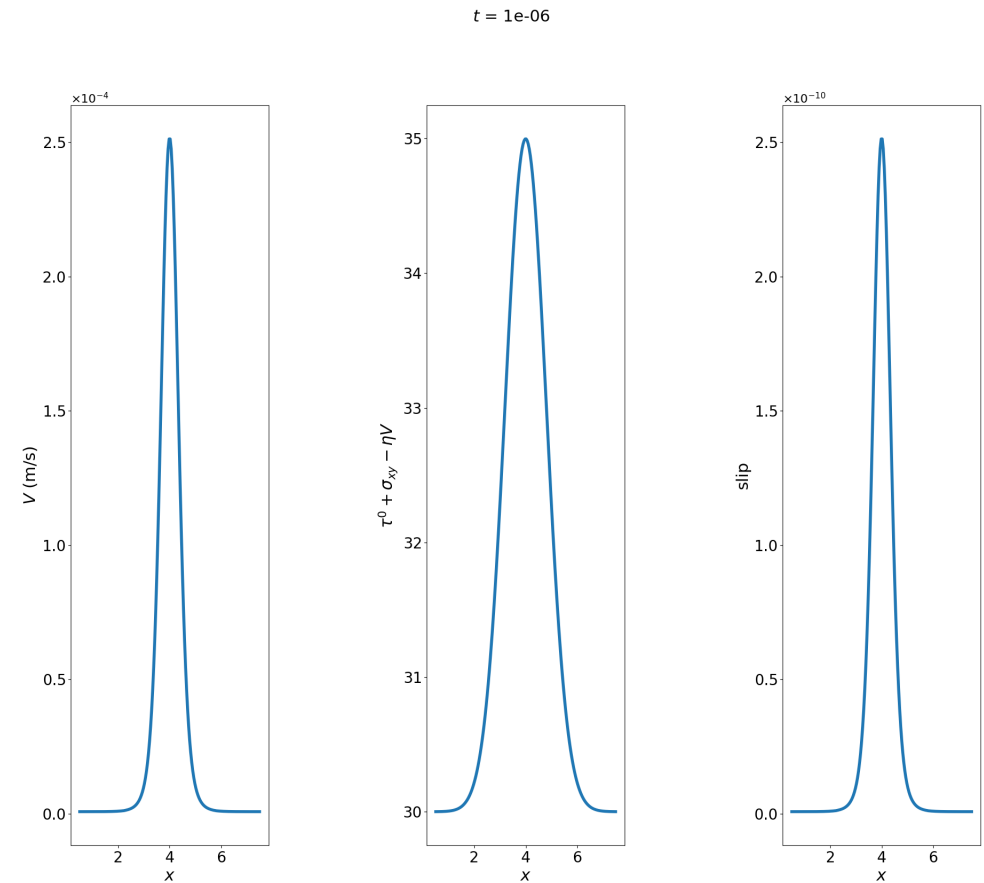
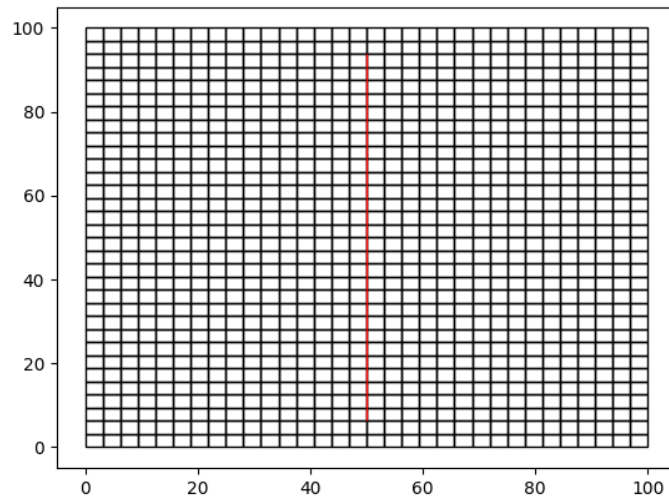
The stabilization matrix affects the solution and it is only exact for hexahedral elements.



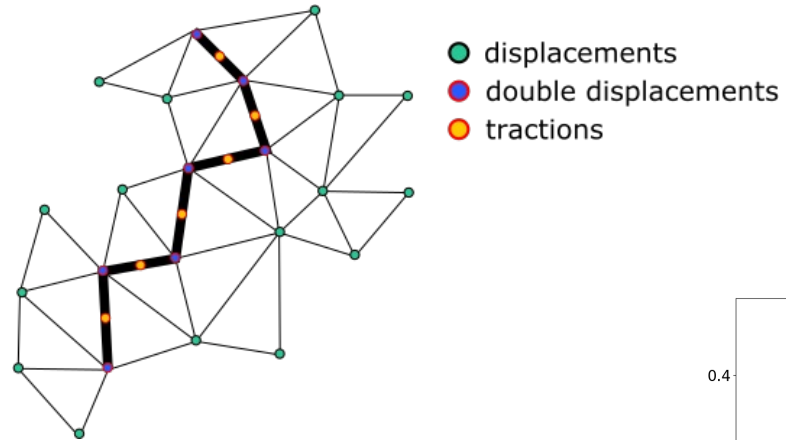
Coupled (poro)mechanics and earthquake model



Slip is prescribed as a b.c. to the mechanics

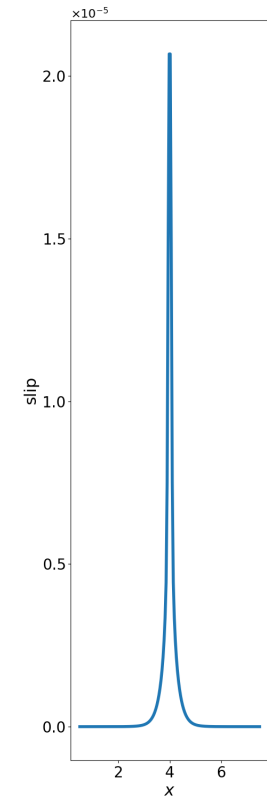
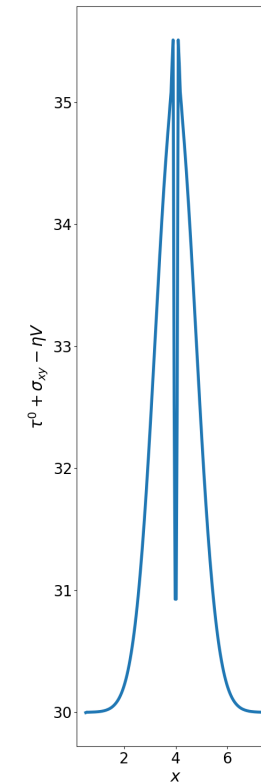
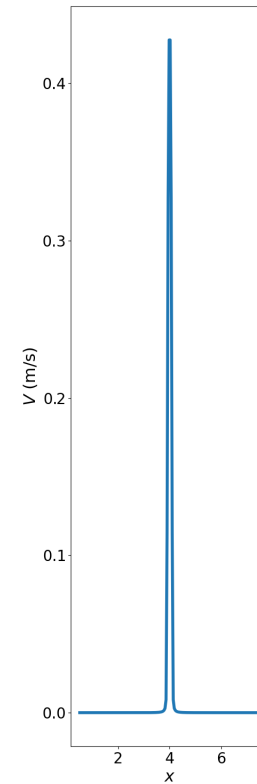
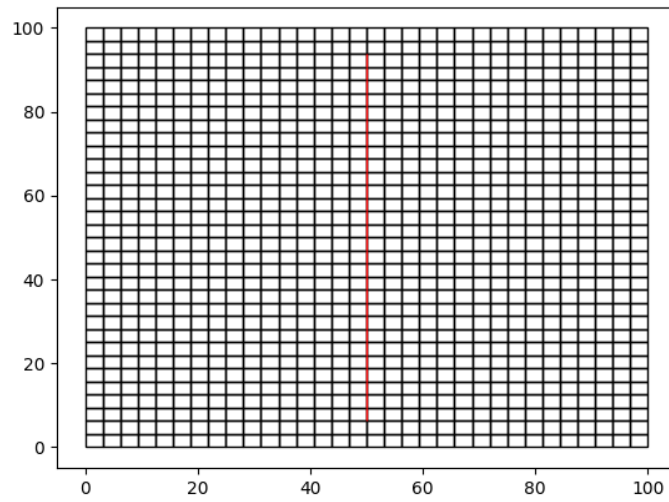


Coupled (poro)mechanics and earthquake model

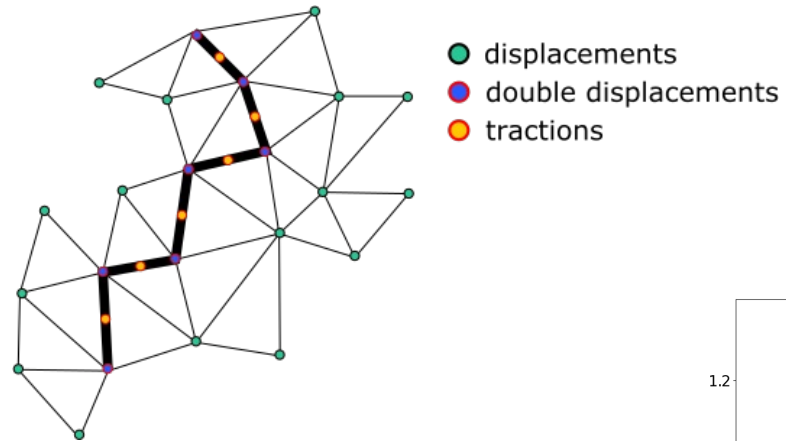


Slip is prescribed as a b.c. to the mechanics

$t = 0.006174535166632781$

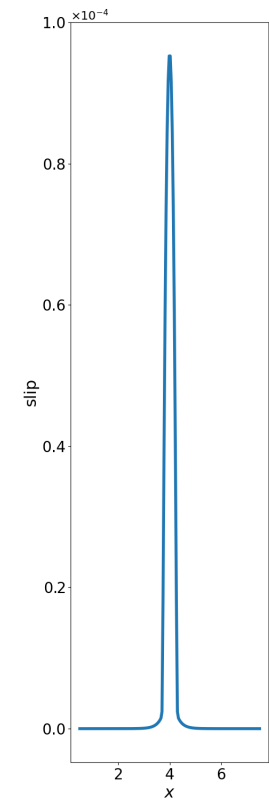
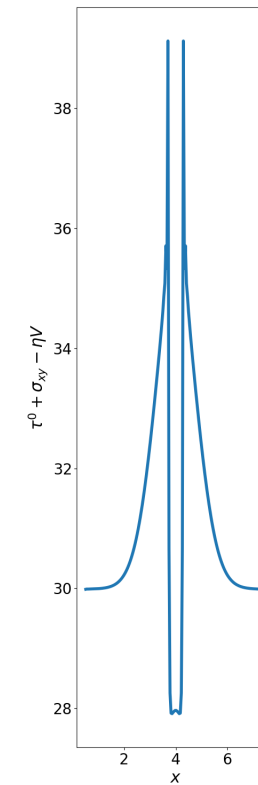
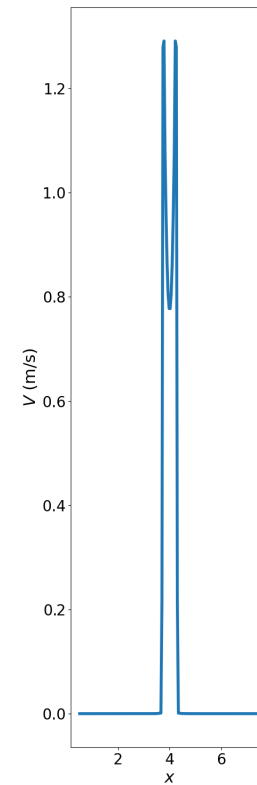
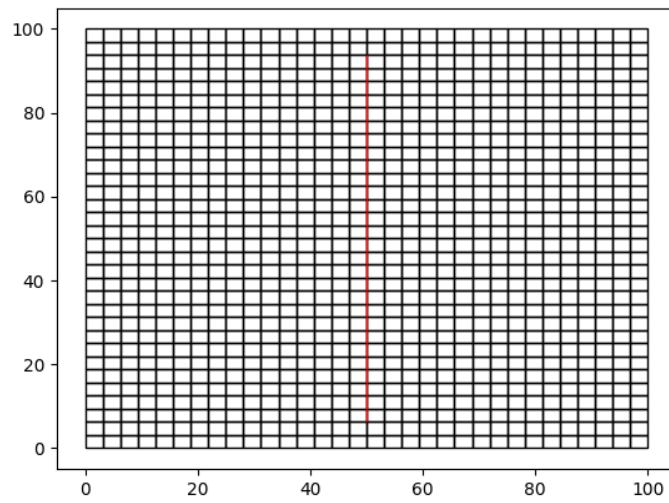


Coupled (poro)mechanics and earthquake model

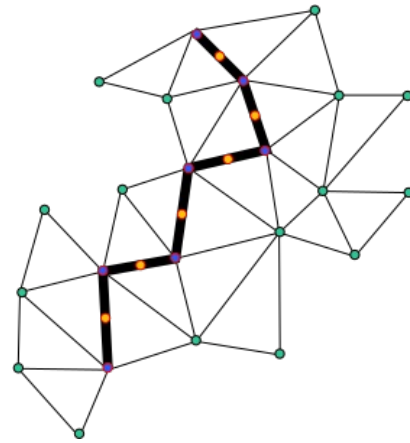


Slip is prescribed as a b.c. to the mechanics

$t = 0.006277664624308697$



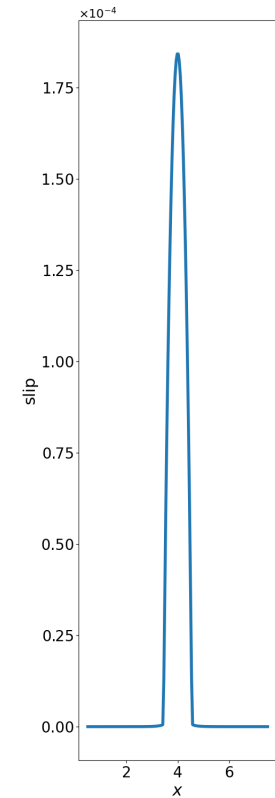
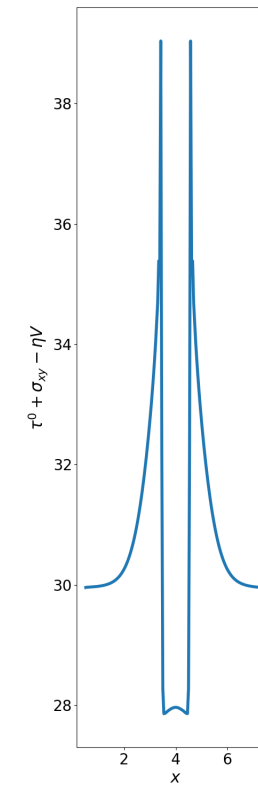
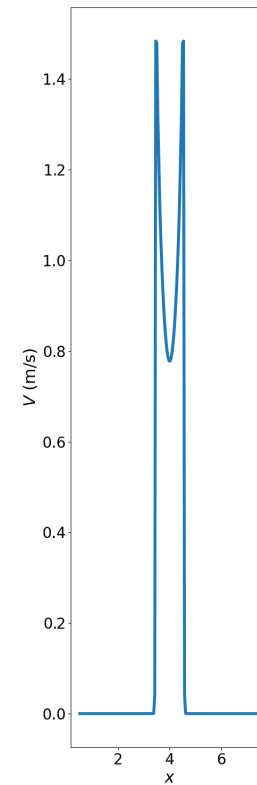
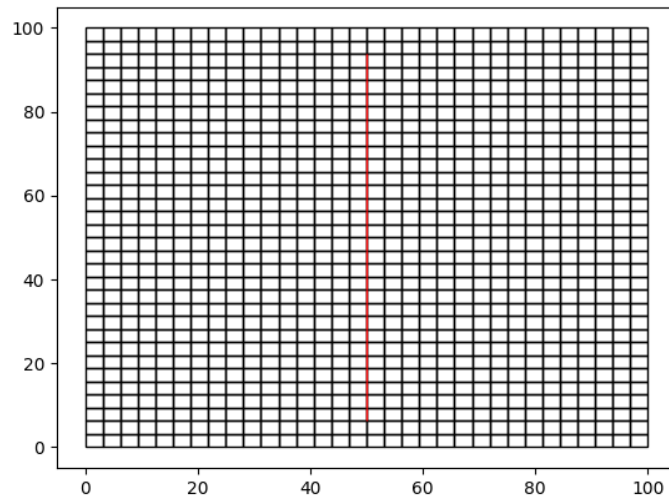
Coupled (poro)mechanics and earthquake model



- displacements
- double displacements
- tractions

Slip is prescribed as a b.c. to the mechanics

$t = 0.006391951728227186$



Year 1: subtasks & milestones overview

Subtask 1 – Quasi-static fault stability analysis capability

- 1.1 – Implementation of a conforming discretization approach to model faults in a poroelastic medium.
- 1.2 – Implementation of constitutive laws that account for the dependency of fault permeability on stressing conditions.

Milestone 1.1: Poromechanical solver with Lagrange multiplier-based contact enforcement implemented in GEOS and validated with numerical examples (Completed).

Year 2: subtasks & milestones overview

Subtask 2 – Quasi-dynamic fault modeling capability

- 2.1 – *Implementation of a rate- and state-dependent friction model.* We enrich the framework devised in subtask 1.1 with a rate- and state-dependent friction model.
- 2.2 – *Development of a prototype quasi-dynamic earthquake rupture modeling capability.* We will develop a prototype earthquake rupture simulator and implement it in the GEOS framework.
- 2.3: *Development of a strategy to couple poromechanics with a quasi-dynamic earthquake rupture physics.*

Milestone 2.1: Rate- and state- friction model implemented and validated. [Fully prototyped & GEOS implementation ongoing]

Milestone 2.2: Prototype quasi-dynamic earthquake rupture modeling capability completed. [80%]

Milestone 2.3: Prototype coupled poromechanics and quasi-dynamic earthquake rupture modeling capability completed. [50%]

Thank you

