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Computational Analysis of Advanced Cooling Strategies for High-Efficiency and Long-Service-Life Gas Turbines

Tom I-P. Shih
School of Aeronautics and Astronautics, Purdue University

Mark Bryden
Ames National Laboratory, U.S. Dept. of Energy

Richard Dalton and Rich Dennis
National Energy Technology Laboratory, U.S. Dept. of Energy

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Concerns on climate change is now a dominate force driving R&D in the electric-power-generation sector. Want technologies that produce near-zero to zero CO_2 emissions!

For gas turbines,

- this means transitioning the fuel from natural gas to blends of NG and H_2 or ammonia to 100% H_2 or ammonia fuels.

What are the challenges?

- Combustors to handle flashback, thermoacoustics,
- Geometries for blades and vanes (e.g., angles of attack) to match the required flow rates with the new fuels.
- Thermal management to sustain and enhance efficiency and service life with the new flow rates and fuels.
- Materials that can handle the caustic hydrogen fuels.
- Infrastructure to support hydrogen as a fuel.

Our focus is on advancing thermal management for the turbine component.

Basically,
the material properties change so behaviors and dominating mechanisms change.

BUT,
the fundamental remain the same.

On designs to meet the challenges, low hanging fruits are gone.

Thus, must understand things in much greater depth than before.

How to obtain the understanding needed?

- experiments
- theory and computations

"A theory is something nobody believes, except the person who made it. An experiment is something everybody believes, except the person who made it."

— Albert Einstein

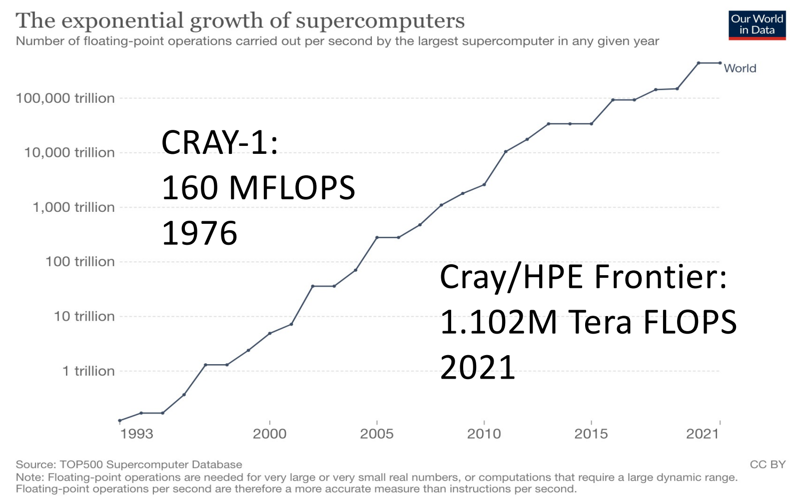
Computations has advanced tremendously!

- Hardware

  Cray1 → Frontier →.... → quantum computing

- Software

  CFD, CSM, ... → MDO → Machine Learning → AI



It is now possible to perform 1st principle simulations of hard problems – like those in gas turbines – to provide the depth of understanding needed.

The objective of this study to address simulations of turbulent flows that pervade throughout the gas turbine!

Development of Hybrid LES-RANS Methods for Turbine Cooling Simulations

Wanjia Zhang and Tom I-P. Shih

School of Aeronautics and Astronautics
Purdue University

- Zhang, W. and Shih, T.I-P., "Adaptive Downstream Tensorial Eddy Viscosity for Hybrid LES-RANS Simulations," *International Journal for Numerical Methods in Fluids*, Vol. 93, No. 6, June 2021, pp. 1825-1842 (DOI: 10.1002/fld.4954).
- Zhang, W. and Shih, T.I-P., "Hybrid LES Method with Adaptive Downstream Anisotropic Eddy Viscosity Model," *International Journal for Numerical Methods in Fluids*, 2022; 1-20. doi: 10.1002/fld.5124.

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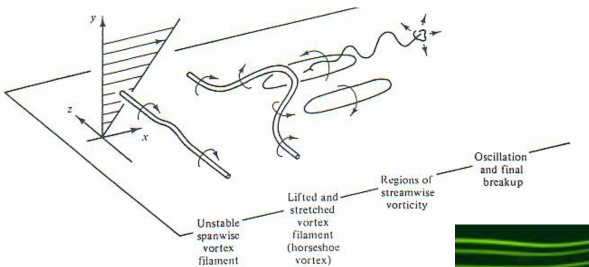
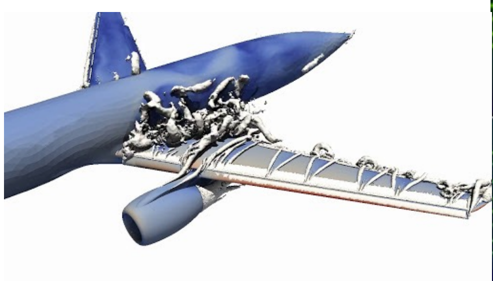
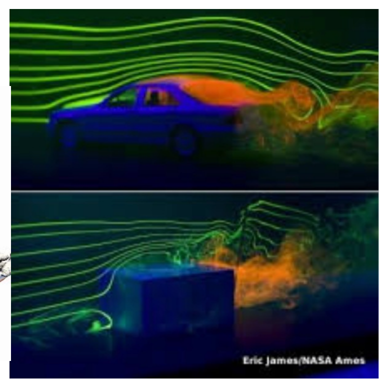
Physics of Turbulence

Turbulent flow involves many scales created by

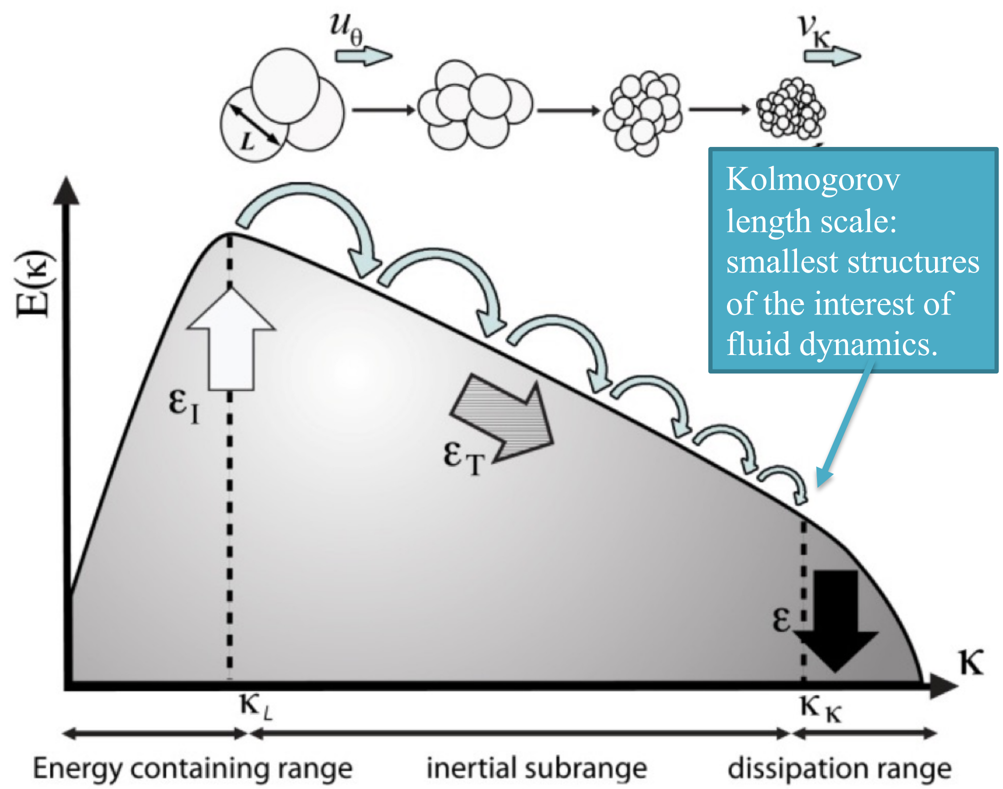
- instability and its sustenance
- vortex stretching and tilting
- dissipation to thermal energy



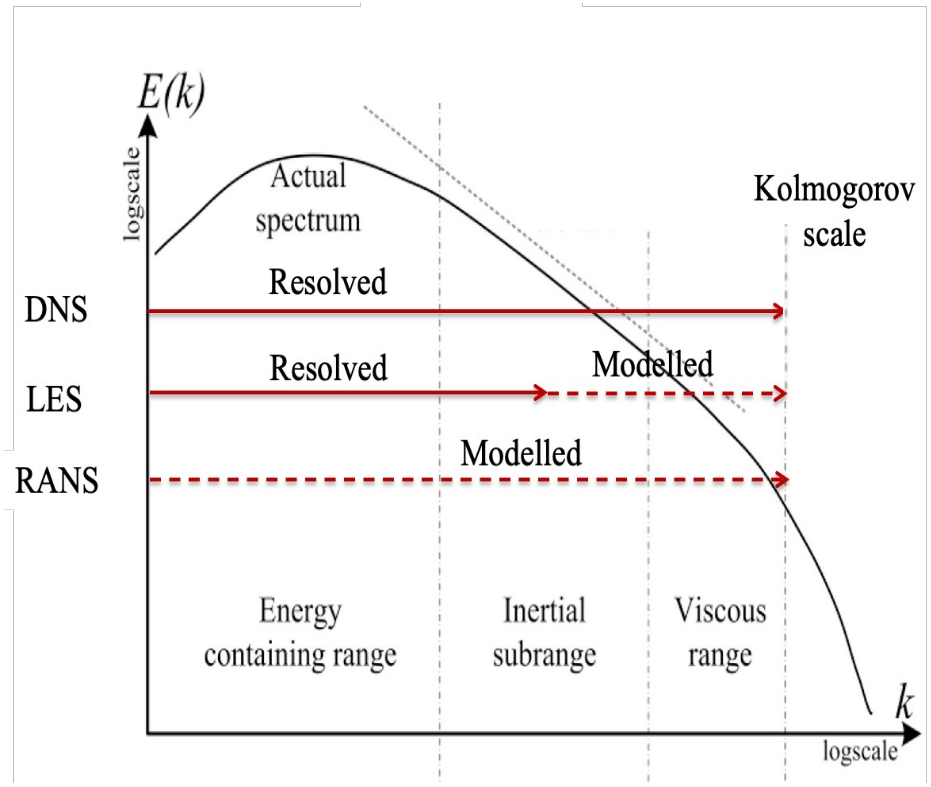




Energy Spectrum of Turbulent Kinetic Energy



Modeling and Simulation of Turbulence



RANS: $\bar{\phi}(x) = \lim_{t_0 \to \infty} \int_0^{t_0} \phi(x,t)\, dt$

LES: $\hat{\phi}(x,t) = \int_\Omega \mathcal{G}(x-x')\phi(x',t)\, dx'$

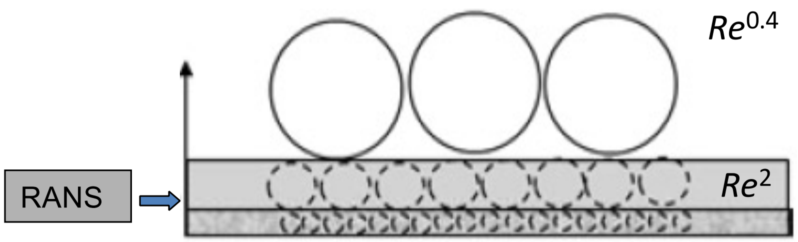
- Direct Numerical Simulation (DNS): Resolves all turbulent scales. ⇒ accurate, but still prohibitively expensive
- Large Eddy Simulation (LES): only small, isotropic turbulent scales, which are less affected by geometry and boundary conditions, are modeled ⇒ accurate, but still expensive.
- Reynolds-Averaged Navier-Stokes (RANS): all turbulent scales are modeled ⇒ so less accurate except for simple flows (e.g., unseparated boundary layers), but affordable.

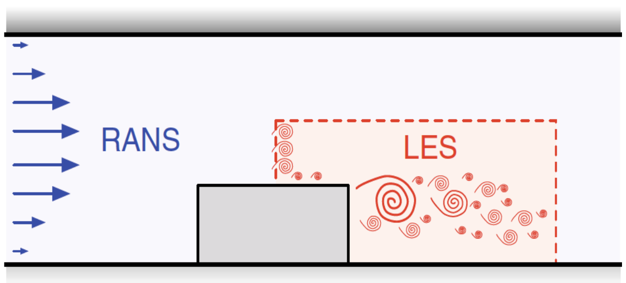
Name	DNS	LES	RANS
Empiricism	No	Low	High
Unsteady	Yes	Yes	No (can be)
# of points (Boeing wing)	10^{20}	10^{11}	10^7
In Service (Boeing)	2080*	2045*	1995

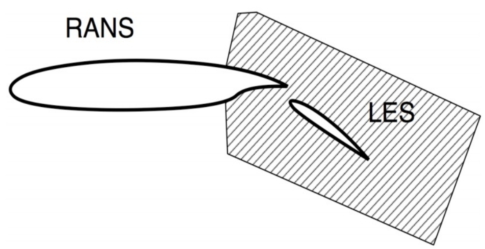
* Spalart, Boeing

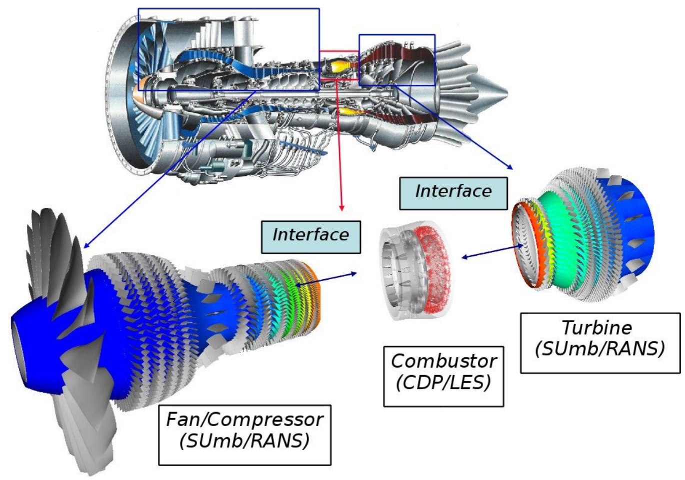
Hybrid RANS-LES Methods

Hybrid RANS- LES combines the best of LES and RANS to get accurate solutions more efficiently. Apply LES where needed and RANS where applicable.









For hybrid methods to be useful, the LES region should be as small as possible in the flow domain to reduce computational cost.
What are the challenges?

Classification of Hybrid Methods

Previous classifications:

- *Nonzonal* vs *Zonal* (Shur et al., 1999; Spalart, 2009)
- *Global* vs *Zonal* (Sagaut et al., 2006)
- *Unified* vs *Segregated modeling* (Fröhlich et al., 2008)
- *Seamless*, *Blended* and *Zonal* (K. Hanjalić et al., 2015)

Our Classification

Continuous Flow

Variable: solution from LES & RANS are **continuous** at LES-RANS interfaces

Pros	Cons
• Easy to implement • Widely used	• Physical inconsistency • Grey area

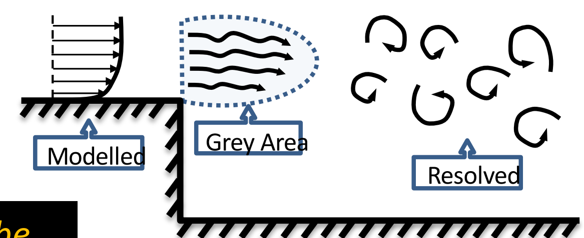
Discontinuous Flow

Variable: solution from LES & RANS are **discontinuous** at LES-RANS interfaces, but statistics (e.g., mean, Reynolds stresses) are continuous

Pros	Cons
• Sound physical meaning	• Complexity • Interface conditions

RANS: $\bar{\phi}(x) = \lim_{t_0 \to \infty} \int_0^{t_0} \phi(x,t)\,dt.$

LES: $\hat{\phi}(x,t) = \int_{\Omega} \mathcal{G}(x-x')\phi(x',t)\,dx'$



In the gray area, the solution is neither LES or RANS so physical meaning is unclear. Maybe OK for external flows, where RANS is next to the wall, but problematic for internal flows.

Previous Work Organized per Our Classification

Continuous Flow Variables (Unified)
Instantaneous solution of RANS and LES are continuous at where RANS and LES regions meet.

- **Sharp Interface**
  Sharp interface to distinguish RANS and LES regions (Reynolds stresses, … are mismatched.)
  - **Detached Eddy Simulation - DES97** (Spalart et al., 1997)
  - **Hybrid Two-layer URANS-LES** (Temmerman et al., 2005)
  - **Zonal DES** (Deck, 2005)
- **Diffused Interface**
  No clear interface but a transition area between RANS and LES regions with blending function (ambiguous in this area)
  - **Mixed Transport Equations**
    - **DDES & IDDES** (Spalart et al., 2006);(Shur et al., 2008)
    - **Limited Numerical Scales -LNS** (Batten et al., 2000)
  - **Mixed Transport Properties**
    - **Weighted Sum Function** (Baurle et al., 2003)
    - **Flow Simulation Methodology - FSM** (Speziale et al., 1998)

Discontinuous Flow Variables

- Zonal Multi-domain Method (Quéméré et al., 2002).
- Gas Turbine Coupling (Schlüter et al., 2005).
- Coupling RANS downstream (Von Terzi et al., 2007).
- Embedded LES-RANS solver (Anupindi et al., 2017).

Interface Conditions at the LES-RANS Interface

RANS → LES

Same as LES inflow boundary condition, where the mean statistics are known, but not its distribution across the length scales.

RANS outflow BC from LES data:

$p_{RANS} = \overline{p_{LES}}$
other flow variables extrapolated from the RANS region

LES inflow BC from RANS data:

$\phi_{LES} = \phi_{RANS} + \phi', \phi = \rho u_i, (\rho, \rho e)$

ϕ' is the artificial fluctuation.

RANS provide pressure to upstream LES

LES → RANS

LES provide RANS mean flow variables **+ turbulence quantities**

LES outflow BC from RANS data:

$\phi_{LES} = \phi_{RANS} + \phi'$

$\frac{\partial \phi'}{\partial t} + \mathrm{U_c} \frac{\partial \phi'}{\partial n} = 0, \phi' = \phi_{LES} - \phi_{RANS}$

$\phi = \rho, \rho u_i, \rho e$

RANS inflow BC from LES data:

$\phi_{\mathrm{RANS}} = \overline{\phi_{\mathrm{LES}}}, \qquad \phi = \rho u_i, T$

$\phi_{RANS} = \mathrm{k}, \varepsilon(\omega)\ or\ \mu_t$

How to get turbulent Viscosity from upstream LES solution?

Since most hybrid LES-RANS methods use scalar eddy-viscosity methods in the RANS region, need to get μ_t. How?

- **Average LES data to get mean quantities (e.g., k, ω) at the interface and then calculate μ_t:** Medic et al. (2006), Schlüter et al.(2005), König et al.(2010), Roidl et al. (2012) and Anupindi et al. (2015).
- **Solve turbulent transport equations for the entire domain with averaged LES data:** Quéméré et al. (2002) and Von Terzi et al. (2010)
- **Solve Reynolds Averaged Navier-Stokes (RANS) equation for turbulent viscosity:** Nolin et al. (2005)

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}_{I,j}}{\partial x_j} = \frac{\partial \mathbf{F}_{V,j}}{\partial x_j}$$

where $\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho e \end{bmatrix}$ $\mathbf{F}_{I,j} = \begin{bmatrix} \rho u_j \\ \rho u_1 u_j + \delta_{1j} p \\ \rho u_2 u_j + \delta_{1j} p \\ \rho u_3 u_j + \delta_{1j} p \\ (\rho e + p) u_j \end{bmatrix}$ and $\mathbf{F}_{V,j} = \begin{bmatrix} 0 \\ \sigma_{1j} \\ \sigma_{2j} \\ \sigma_{3j} \\ \sigma_{jk} u_k + q_j \end{bmatrix}$

$$\sigma_{ij} = (\mu + \mu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) \quad \text{and} \quad q_j = \frac{\mu + \mu_t}{Pr}\frac{\partial T}{\partial x_j}$$

Bars and tildas above the flow variables omitted.

- **Over-determined:**

  1 unknown and 3 equations

- Choose the least stiff equation (usually the streamwise momentum equation).

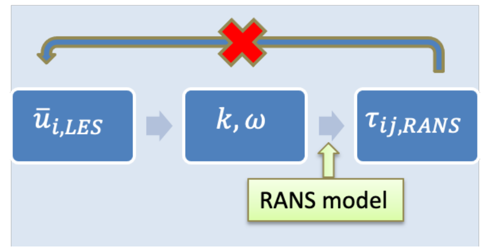
How to get turbulent Viscosity from upstream LES solution?

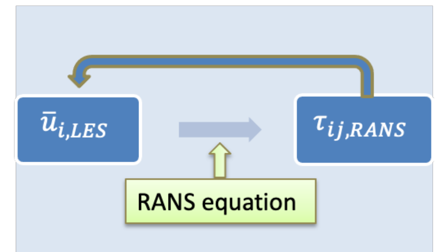
So, what's the problem?

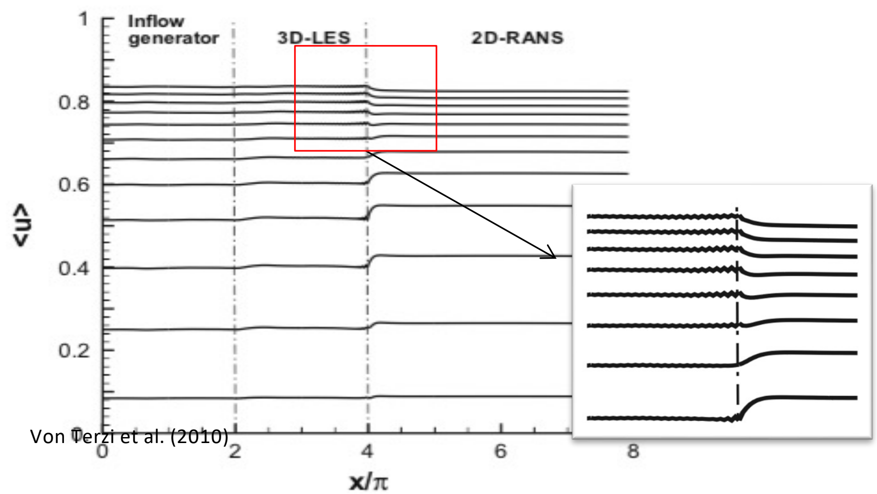
For the 1st two methods:
The μ_t obtained (e.g., $\mu_t = c_\mu \rho k^2/\epsilon$) is not the μ_t in the LES solution.

For the 3rd method based on the mom. eq:
μ_t in the LES region is anisotropic (tensorial), but μ_t in the RANS region is isotropic (scalar).

Thus, RANS could not sustain the LES solution even though the correct mean flow variables ($\bar{\rho}$,) are transferred to the RANS region at the LES-to-RANS interface.







Von Terzi et al. (2010)

LES-to-RANS Interface Condition: What's the problem?

- The "downstream" RANS model must be able to continue the turbulence predicted by the "upstream" LES. Otherwise, there will be a mismatch generated at the LES-to-RANS interface.

- Previous studies only focused on how to transfer data from LES to RANS at the LES-to-RANS interface. BUT, that is not enough because the "downstream" RANS model could not sustain the LES data so RANS regions are quite small (typically only a small region next to walls).

Objective

- Develop an anisotropic RANS model based on the Reynolds stresses from the upstream LES solution at the LES-to-RANS interface by using the “three-momentum equation approach.
- Develop a method to modify the downstream RANS model from isotropic (scalar) to anisotropic (tensorial) so upstream LES solution can be sustained in the downstream RANS region.
- Assess the models and methods developed.

I. Background

II. Method 1: Downstream Tensorial Eddy Viscosity Model

i. Tensorial Eddy Viscosity at LES-to-RANS Interface

ii. Downstream Adaptive Model

iii. Numerical Methods

iv. Test cases:

1. Channel Flow
2. Periodic Hill Flow

III. Method 2: Downstream Nonlinear Eddy Viscosity Model

IV. Summary

Anisotropic Eddy Viscosity at LES-to-RANS Interface from LES

- The momentum equations for downstream RANS region:

$$\frac{\partial\left(\rho_{RANS}u_{i,RANS}u_{j,RANS}\right)}{\partial x_j}+\frac{\partial p_{RANS}}{\partial x_j}-\frac{\partial}{\partial x_j}\left(2\mu S_{ij}-\frac{2}{3}\mu S_{kk}\delta_{ij}\right)-\frac{\partial \tau_{ij,RANS}}{\partial x_j}=0$$

$$\text{where } S_{ij}=\frac{1}{2}\left(\frac{\partial u_{i,RANS}}{\partial x_j}+\frac{\partial u_{j,RANS}}{\partial x_i}\right)$$

- The RANS model that can sustain the LES profile downstream of the LES-to-RANS interface, $\tau_{ij,RANS}$, must satisfy:

$$\boldsymbol{\mathcal{N}}\left(\tau_{ij,RANS}\right)=\frac{\partial\left(\overline{\rho_{LES}}\tilde{u}_{i,LES}\tilde{u}_{j,LES}\right)}{\partial x_j}+\frac{\partial \overline{p_{LES}}}{\partial x_j}-\frac{\partial}{\partial x_j}\left(2\mu S_{ij}-\frac{2}{3}\mu S_{kk}\delta_{ij}\right)-\frac{\partial \tau_{ij,RANS}}{\partial x_j}=0$$

$$\text{where } S_{ij}=\frac{1}{2}\left(\frac{\partial \tilde{u}_{i,LES}}{\partial x_j}+\frac{\partial \tilde{u}_{j,LES}}{\partial x_i}\right)$$

Tensorial Eddy Viscosity at LES-to-RANS Interface from LES Solution

- Use the "3-momentum equations" approach to get the tensorial eddy viscosity:

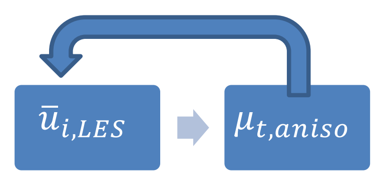
$$\frac{\partial(\bar{\rho}\widetilde{u_1}\widetilde{u_1}+\bar{p})}{\partial x_1}+\frac{\partial(\bar{\rho}\widetilde{u_1}\widetilde{u_2})}{\partial x_2}+\frac{\partial(\bar{\rho}\widetilde{u_1}\widetilde{u_3})}{\partial x_3}=\frac{\partial}{\partial x_1}\left[2(\mu+\boldsymbol{\mu_{t,n}})S_{11}-\frac{2}{3}\bar{\rho}k\right]+\frac{\partial}{\partial x_2}\left[2(\mu+\boldsymbol{\mu_{t,12}})S_{12}\right]+\frac{\partial}{\partial x_3}\left[2(\mu+\boldsymbol{\mu_{t,13}})S_{13}\right]$$

$$\frac{\partial(\bar{\rho}\widetilde{u_1}\widetilde{u_2})}{\partial x_1}+\frac{\partial(\bar{\rho}\widetilde{u_2}\widetilde{u_2}+\bar{p})}{\partial x_2}+\frac{\partial(\bar{\rho}\widetilde{u_2}\widetilde{u_3})}{\partial x_3}=\frac{\partial}{\partial x_1}\left[2(\mu+\boldsymbol{\mu_{t,12}})S_{12}\right]+\frac{\partial}{\partial x_2}\left[2(\mu+\boldsymbol{\mu_{t,n}})S_{22}-\frac{2}{3}\bar{\rho}k\right]+\frac{\partial}{\partial x_3}\left[2(\mu+\boldsymbol{\mu_{t,23}})S_{23}\right]$$

$$\frac{\partial(\bar{\rho}\widetilde{u_1}\widetilde{u_3})}{\partial x_1}+\frac{\partial(\bar{\rho}\widetilde{u_2}\widetilde{u_3})}{\partial x_2}+\frac{\partial(\bar{\rho}\widetilde{u_3}\widetilde{u_3}+\bar{p})}{\partial x_3}=\frac{\partial}{\partial x_1}\left[2(\mu+\boldsymbol{\mu_{t,13}})S_{13}\right]+\frac{\partial}{\partial x_2}\left[2(\mu+\boldsymbol{\mu_{t,23}})S_{23}\right]+\frac{\partial}{\partial x_3}\left[2(\mu+\boldsymbol{\mu_{t,n}})S_{33}-\frac{2}{3}\bar{\rho}k\right]$$

- Since the nonlinear terms in the above eqs are already known from LES solution, the above equations are linear in the unknowns: $\boldsymbol{\mu_{t,n}}$, $\boldsymbol{\mu_{t,12}}$, $\boldsymbol{\mu_{t,13}}$, $\boldsymbol{\mu_{t,23}}$.

- However, the problem is now under-determined with 4 unknowns & 3 eqs. **Thus, a constraint is needed.**



Tensorial Eddy Viscosity at LES-to-RANS Interface from LES Solution

<u>Constraint to reduce the unknowns from 4 to 3:</u>

- Want a criterion to identify the most important component of the tensorial eddy viscosity. The production rate of turbulent kinetic energy could be one because its magnitude is significantly affected by the Reynolds stresses.

$$P = -\overline{\rho u_i' u_j'} \frac{\partial \widetilde{u}_i}{\partial x_j}$$

- To develop the criterion, the production rate is rewritten to be connected to the Reynolds stresses:

$$P = -\overline{\rho u_i' u_j'} \frac{\partial \widetilde{u}_i}{\partial x_j} = -\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} S_{ij} = \sum_{i=1}^{3}\sum_{j=1}^{3} 2\mu_{t,ij} \left(S_{ij}\right)^2 = \sum_{i=1}^{3}\sum_{j=1}^{3} \theta_{ij}$$

- By comparing the values of θ_{12}, θ_{23} and θ_{13} (result can be written in $\bar{\theta}_{i_a j_a} > \bar{\theta}_{i_b j_b} > \bar{\theta}_{i_c j_c}$), the dominant viscosity could be found.

- Let the least dominant one to be equal to the most important one. The viscosity components could be reduced from 4 to 3.

$$\mu_{t,\alpha} = \mu_{t,i_a j_a} = \mu_{t,i_c j_c}, \quad \mu_{t,\beta} = \mu_{t,i_b j_b} \text{ and } \quad \mu_{t,\gamma} = \mu_{t,n}$$

I. Background

II. Method 1: Downstream Tensorial Eddy Viscosity Model

  i. Tensorial Eddy Viscosity Model at the LES-to-RANS Interface

  ii. Downstream Adaptive Model

  iii. Numerical Methods

  iv. Test cases:

    1. Channel Flow

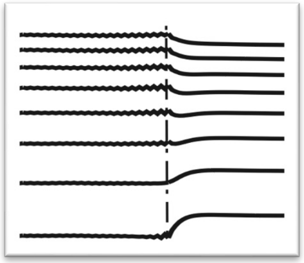
    2. Periodic Hill Flow

III. Method 2: Downstream Nonlinear Eddy Viscosity Model

IV. Summary

Downstream Adaptive Model

Problem: Though the mean flow variables and μ_t match those at the LES-to-RANS interface, these values change quickly in the RANS region because the downstream RANS model – if based on a scalar eddy viscosity model (e.g., $k - \epsilon$ or SST) – do not have the capability to sustain the LES solution described by a tensorial eddy viscosity.



- Could the tensorial eddy viscosity obtained at the LES-to-RANS interface be propagated to the downstream RANS region? One way is to invoke Rodi's weak-equilibrium assumption (1972), which states that the anisotropy is approximately constant following a fluid particle; i.e.,

$$\frac{D}{Dt}\left(\frac{\tau_{ij}}{\bar{\rho}k}\right) = 0\,, \qquad \text{for i, j} = 1, 2, \text{and } 3$$

- Dividing the above equation by the same equation with another Reynolds stress to eliminate the turbulent kinetic energy and apply the anisotropic eddy viscosity

$$\Rightarrow \frac{D}{Dt}\left(\frac{\tau_{ij}}{\tau_{lm}}\right) = 0\,, \qquad \text{for i, j, l, m} = 1,2,3$$

- Thus, based on the weak-equilibrium assumption, the tensorial eddy viscosity obtained at the LES-to-RANS interface can be transferred from the interface to the downstream by scaling and maintaining the same anisotropy.

Downstream Adaptive Model

- **Scaling**: assuming the ratio of the deviation of the predominant component of the Reynolds stress τ_α, i.e., τ_α^* based on the $\mu_{t,RANS}$ from the downstream RANS model, to the turbulent kinetic energy k also follows the weak equilibrium assumption:

$$\frac{D}{Dt}\left(\frac{\tau_\alpha}{k}\cdot\frac{k}{\tau_\alpha^*}\right) = \frac{D}{Dt}\left(\frac{\mu_{t,\alpha}}{\mu_{t,RANS}}\right) = 0$$

- **Anisotropy**: Since $\mu_{t,\alpha}$ is already known (see slide 20), the other components of the tensorial eddy viscosity will be computed with respect to that component, τ_α.

$$\frac{D}{Dt}\left(\frac{\tau_\alpha}{\tau_{i_c j_c}}\right) = 0$$

where $\tau_\alpha = \mu_{t,\alpha} S_{i_a j_a}$ and $\tau_{i_c j_c} = \mu_{t,\gamma} S_{i_c j_c}$

The above two equations + the anisotropic $\boldsymbol{\mu_t}$ from the LES constitute the downstream adaptive RANS model.

The method developed is adaptive because the anisotropic turbulent viscosities in the downstream RANS model is adapted by the upstream LES solution.

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III. Method 2: Downstream Nonlinear Eddy Viscosity Model

IV. Summary

Numerical Methods

Code: HiFiLES (High Fidelity Large Eddy Simulation)

- Governing equation:
  compressible Navier-Stokes equation with thermally perfect gas
  LES: WALE (Wall-Adapting Local Eddy-viscosity)
  RANS: one equation Spalart-Allmaras model
- Numerical scheme:
  - High-order Energy-Stable Flux Reconstruction scheme
  - Time scheme: explicit time-stepping with 4^{th} order Runge-Kutta method

NOTE: To assess the accuracy of the methods developed to transfer LES info to the downstream RANS region and to modify the downstream RANS model, only one-way coupling was considered. This is to ensure that the downstream RANS models do not disrupt the upstream LES solution while assessing the performance of the downstream RANSD model developed.

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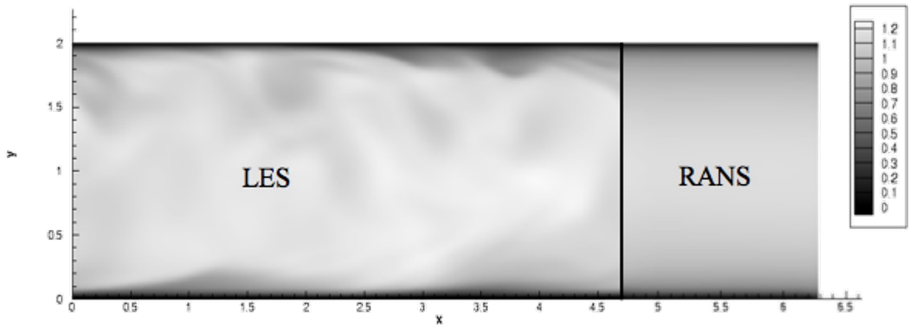
III. Method 2: Downstream Nonlinear Eddy Viscosity Model

IV. Summary

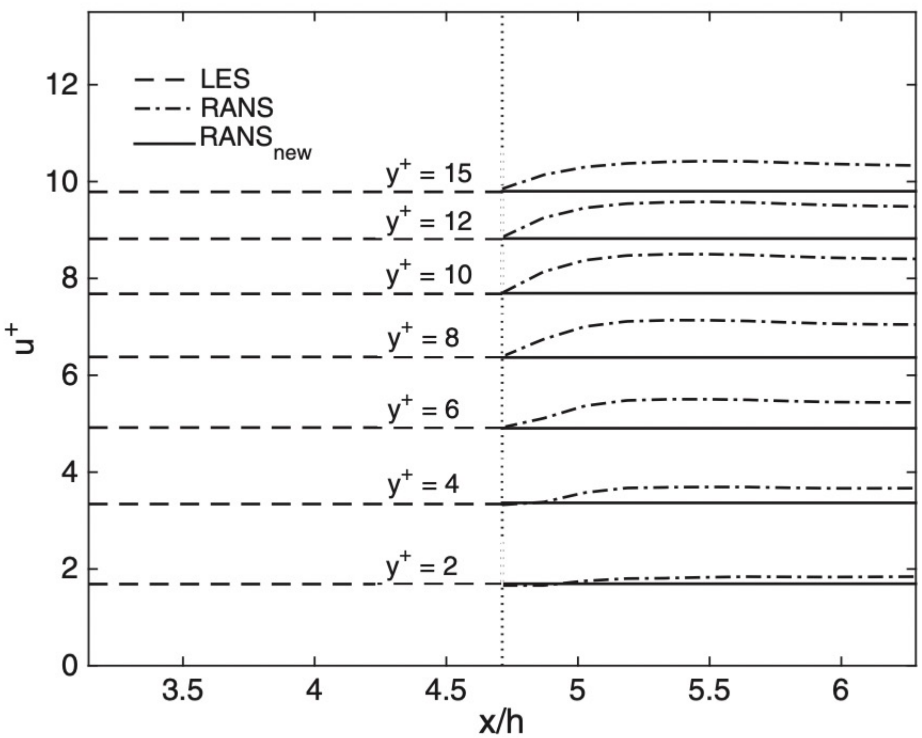
Channel flow

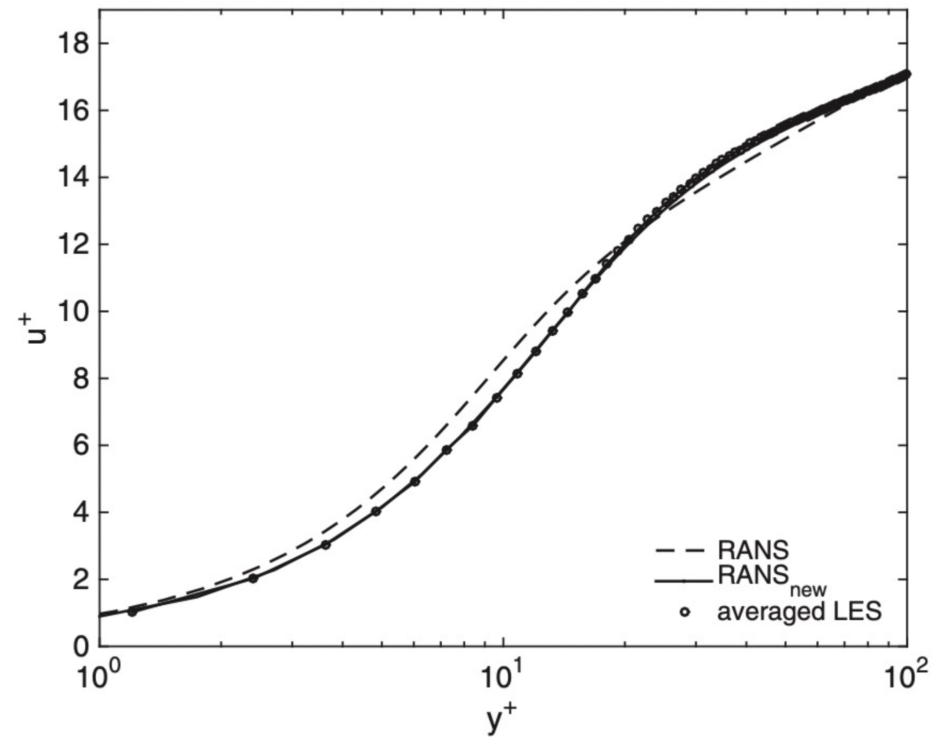
- A fully developed channel flow with **one-way coupling** is tested as a validation case for turbulent viscosity reconstruction procedure.
  - Dimension: $2\pi h(x) \times 2h(y) \times \pi h(z)$, where h is the half channel width
  - Reynolds number: $\mathrm{Re}_\tau = \rho u_\tau h/\mu = 180$, where u_τ is the friction velocity.
  - Mach number: $M = 0.2$
  - Boundary conditions: No-slip walls; periodic BC in streamwise and spanwise direction with constant body force for LES region. Characteristic outflow condition for RANS outlet.
  - Mesh:
    $12(x) \times 24(y) \times 18(z)$ for LES
    $3(x) \times 24(y)$ for RANS



- Spatial scheme: 3rd-order Flux Reconstruction
- DOFs: $36(x) \times 72(y) \times 46(z)$ for LES (coarse) and $9(x) \times 72(y)$ for RANS

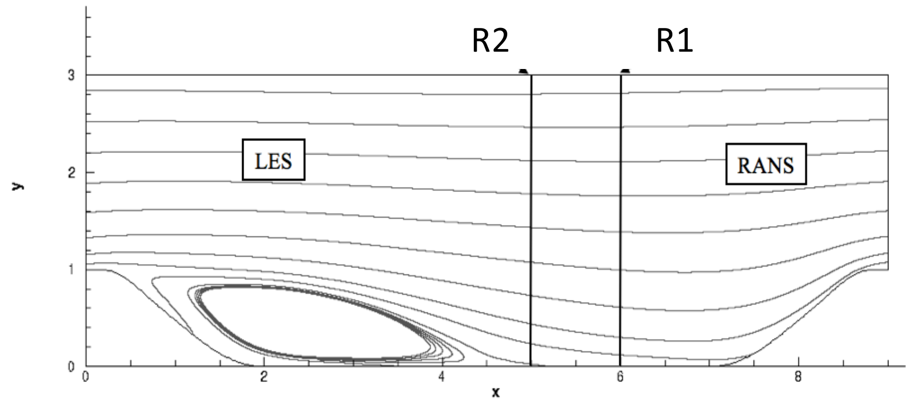
Channel flow





Periodic Hill Flow

- Size: $9h(x)\times3h(y)\times4h(z)$,
  where h is the hill height
- $Re = \rho u_b h/\mu = 10595$
- Mach number: $M = 0.2$



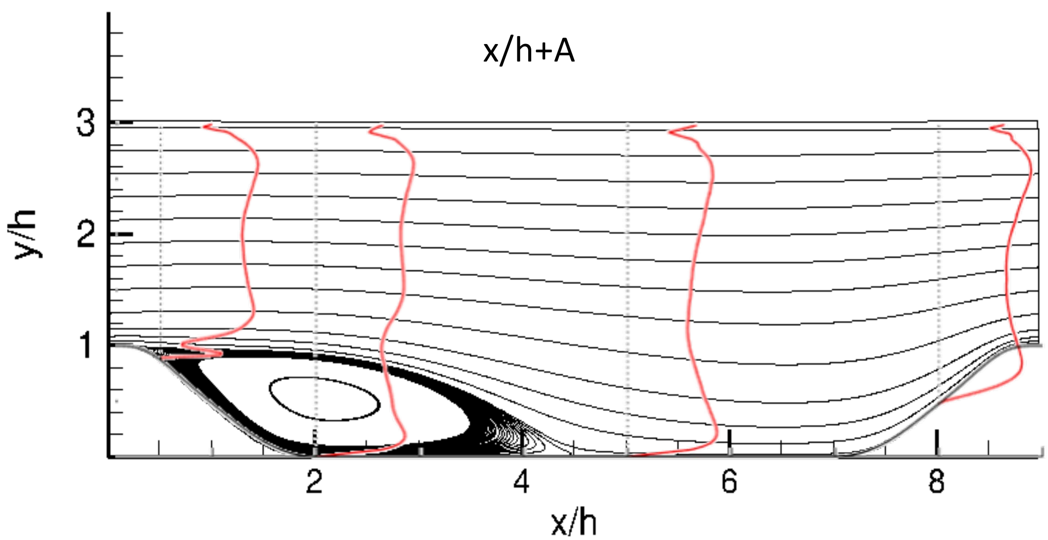
- BCs: (One-way coupling)
  No-slip walls;
  periodic BC in streamwise and spanwise direction for LES region.
  characteristic outflow condition for RANS outlet.

	Streamwise length	Mesh: $N_x\times N_y\times N_z$	DOF
pure LES	9h	64 × 32 × 40	256 × 128 × 160
pure RANS	9h	48 × 32 × 1	192 × 128 × 1
RANS part: R1	3h	16 × 32 × 1	64 × 128 × 1
RANS part: R2	4h	20 × 32 × 1	80 × 128 × 1

Periodic Hill Flow

Why this case is chosen?

- The presence of streamline curvature at the post-attachment zone cannot be accurately predicted by scalar eddy viscosity models.
- The change of the anisotropy of Reynolds stresses is moderate (weak-equilibrium assumption is valid).



flatness parameter:

$$A = 1 + 9(II + III)$$

isotropic: $A = 0$

where

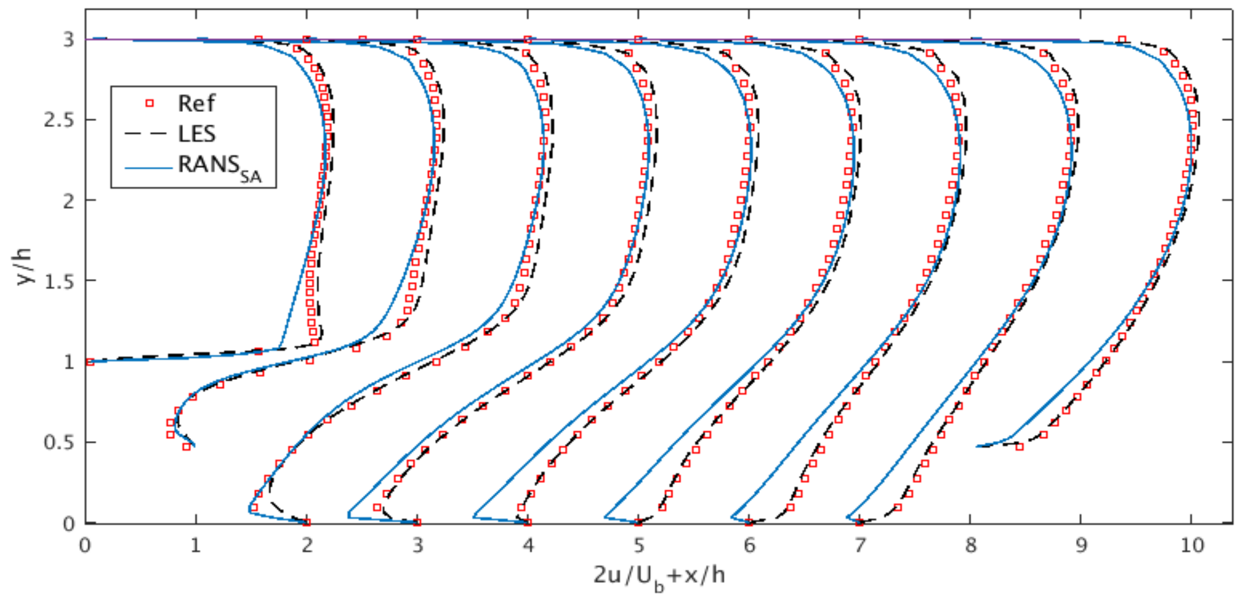
$$II = -\frac{b_{ij}b_{ij}}{2}$$

$$III = \frac{b_{ij}b_{jk}b_{ki}}{3}$$

$$b_{ij} = \frac{\overline{\rho u_i' u_j'}}{2\bar{\rho}\mathrm{k}} - \frac{1}{3}\delta_{ij}$$

Periodic Hill Flow

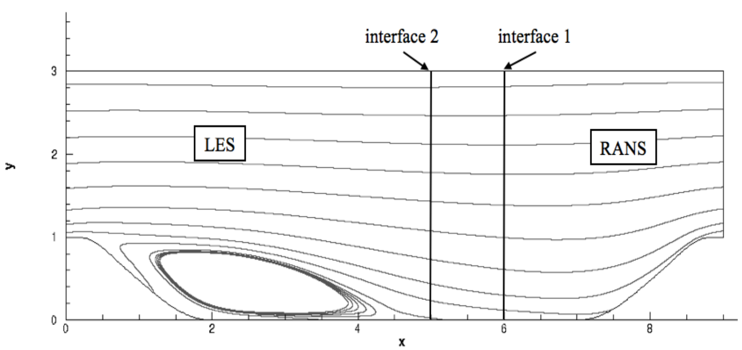
The LES results are first compared with the reference to check the quality of the upstream LES information, where the pure RANS data are also presented to show the bad prediction of Spalart-Allamaras model for this case.



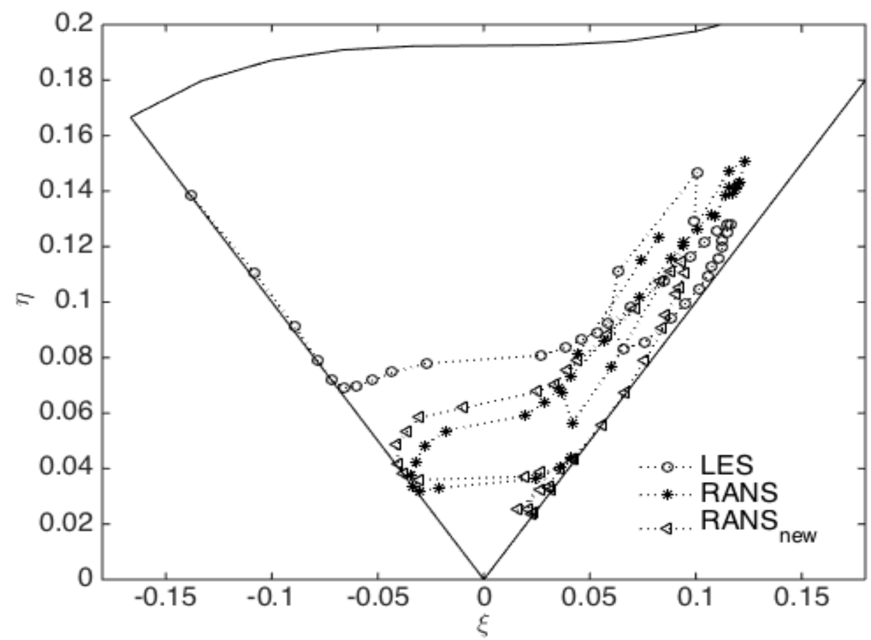
Reattachment point	RANS	LES	Ref. (J. Fröhlich et al.)
x/L	7.66	4.53	4.56-4.72

Periodic Hill Flow

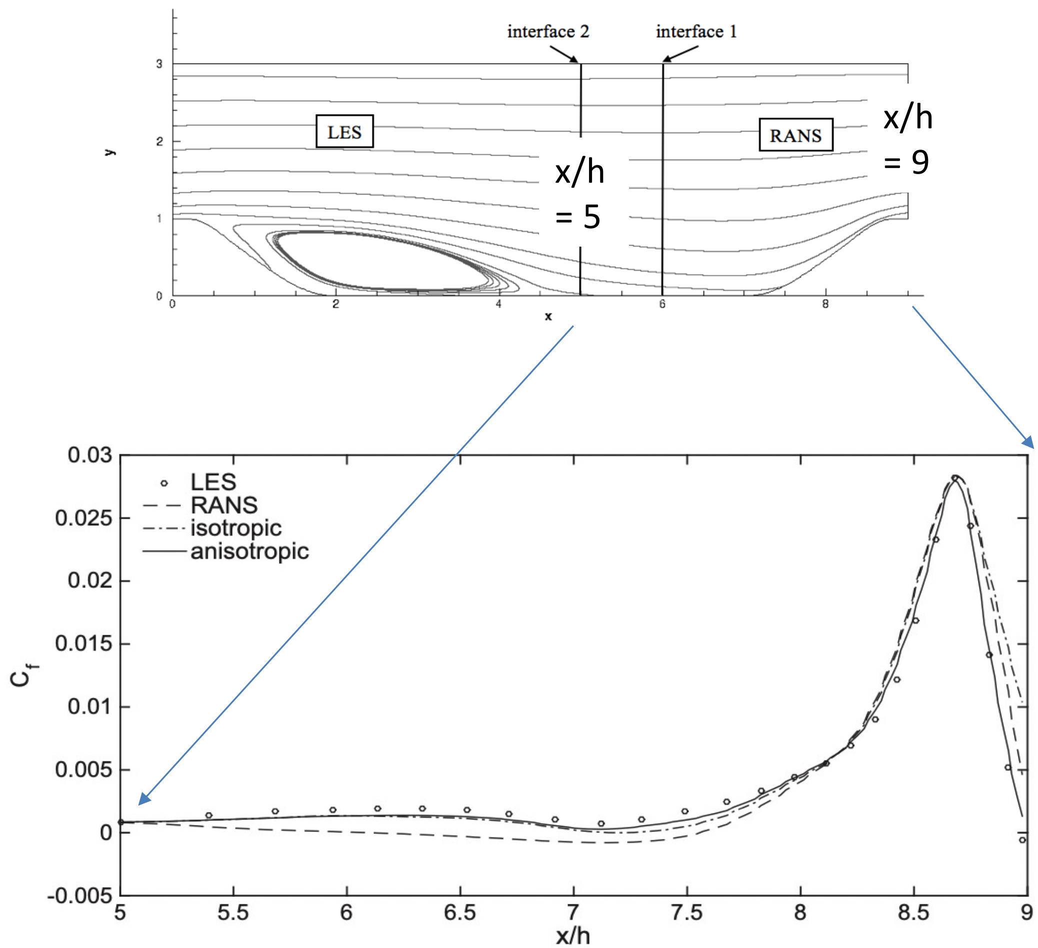
The physical realizability of Reynolds stresses is first assessed by examining the Lumley triangle along the interface.

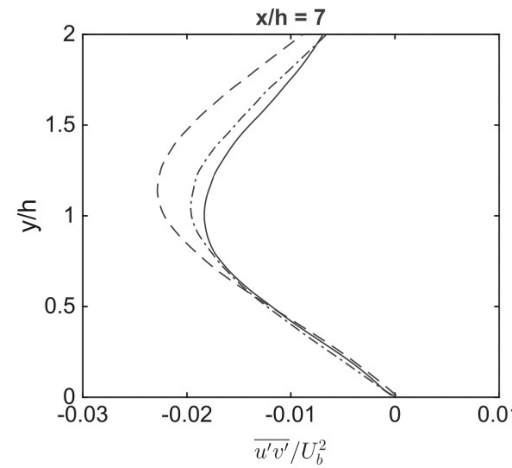


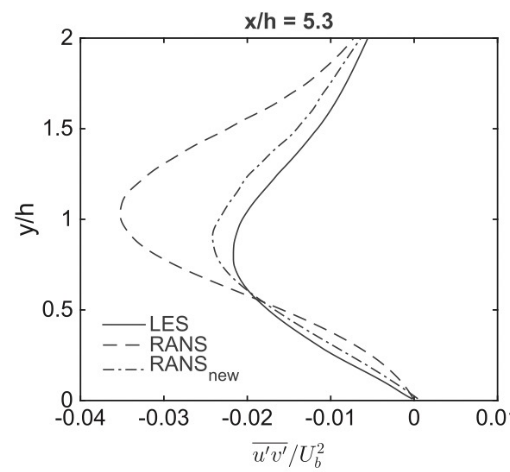
Invariant map along the interface.



Periodic Hill Flow







I. Background

II. Methd 1: Downstream Tensorial Eddy Viscosity Model

III. Method 2: Downstream Nonlinear Eddy Viscosity Model

i. Nonlinear Eddy Viscosity Model

ii. Downstream Adaptive Model

iii. Numerical Methods

iv. Test case - Film Cooling

IV. Summary

Nonlinear Eddy Viscosity Model

One way to allow the Reynolds stresses $\tau_{ij,RANS}$ to satisfy RANS momentum equations without relying on the **choice of the coordinate system** is that they should be modelled as a function of powers of the mean flow gradients, i.e., constitutive relations (see Pope (2004)):

$$
\begin{aligned}
-\tau_{ij} = {} & \frac{2}{3}k\delta_{ij} + 2a_2\mathfrak{J}k\left(U_{i,j} + U_{j,i} - \frac{2}{3}U_{k,k}\delta_{ij}\right) + 2a_4\mathfrak{J}^2k\left(U_{i,j}^2 + U_{j,i}^2 - \frac{2}{3}\Pi_1\delta_{ij}\right) \\
& +2a_6\mathfrak{J}^2k\left(U_{i,k}U_{j,k} - \tfrac{1}{3}\Pi_2\delta_{ij}\right) + 2a_7\mathfrak{J}^2k \\
& +2a_8\mathfrak{J}^3k\left(U_{i,k}U_{j,k}^2 + U_{i,k}^2U_{j,k} - \tfrac{2}{3}\Pi_3\delta_{ij}\right) \\
& +2a_{10}\mathfrak{J}^3k\left(U_{k,i}U_{k,j}^2 + U_{k,i}^2U_{k,j} - \frac{2}{3}\Pi_3\delta_{ij}\right) \\
& +2a_{12}\mathfrak{J}^4k\left(U_{i,k}^2U_{j,k}^2 - \frac{1}{3}\Pi_4\delta_{ij}\right) + 2a_{13}\mathfrak{J}^4k\left(U_{k,i}^2U_{k,j}^2 - \frac{1}{3}\Pi_4\delta_{ij}\right) \\
& +2a_{14}\mathfrak{J}^4k\left(U_{i,k}U_{l,k}U_{l,j}^2 + U_{j,k}U_{l,k}U_{l,i}^2 - \frac{2}{3}\Pi_5\delta_{ij}\right) \\
& +2a_{16}\mathfrak{J}^5k\left(U_{i,k}U_{l,k}^2U_{l,j}^2 + U_{j,k}U_{l,k}^2U_{l,i}^2 - \frac{2}{3}\Pi_6\delta_{ij}\right) \\
& +2a_{18}\mathfrak{J}^6k\left(U_{i,k}U_{l,k}U_{l,m}^2U_{j,m}^2 + U_{j,k}U_{l,k}U_{l,m}^2U_{i,m}^2 - \frac{2}{3}\Pi_7\delta_{ij}\right)
\end{aligned}
$$

where $U_{i,j} = \frac{\partial \tilde{u}_i}{\partial x_j}$, $\mathfrak{J} = k/\varepsilon$, $\Pi_1 = U_{i,k}U_{k,i}$, $\Pi_2 = U_{i,k}U_{i,k}$, $\Pi_3 = U_{i,k}U_{i,k}^2$, $\Pi_4 = U_{i,k}^2U_{i,k}^2$,
$\Pi_5 = U_{i,k}U_{l,k}U_{l,i}^2$, $\Pi_6 = U_{i,k}U_{l,k}^2U_{l,i}^2$ and $\Pi_7 = U_{i,k}U_{l,k}U_{l,m}^2U_{i,m}^2$

Nonlinear Eddy Viscosity Model

Since there are only 3 momentum equations in the RANS region, the relation is first **truncated to quadratic tensorial forms**:

$$-\tau_{ij} = \frac{2}{3}k\delta_{ij} - 2C_\mu \mathfrak{I} k S^*_{ij} + 2\alpha_1 \mathfrak{I}^2 k \left(S^{(2*)}_{ij} + \Omega^2_{ij}\right)$$
$$+\alpha_2 \mathfrak{I}^2 k \left(S^{(2*)}_{ij} - \Omega^2_{ij} - S^*_{ik}\Omega_{kj} + \Omega_{ik}S^*_{kj}\right)$$
$$+\alpha_3 \mathfrak{I}^2 k \left(S^{(2*)}_{ij} - \Omega^2_{ij} + S^*_{ik}\Omega_{kj} - \Omega_{ik}S^*_{kj}\right)$$

where $S_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$, $S^*_{ij} = S_{ij} - \frac{1}{3}\frac{\partial \tilde{u}_k}{\partial x_k}\delta_{ij}$, $S^{(2*)}_{ij} = S^2_{ij} - \frac{1}{3}S^2_{kk}\delta_{ij}$,

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i}\right), \quad \Omega^2_{ij} = \Omega_{ik}\Omega_{kj}$$

Then, **the rapid distortion constraint** (Shih et al., (1994)) is applied. With the rapid distortion theory, an isotropic turbulence should remain isotropic under a rapid rotating flow with $S_{ij} = 0$, so:

$$b_{ij} = -\frac{\tau_{ij}}{2k} - \frac{1}{3}\delta_{ij} = \frac{1}{2}\mathfrak{I}^2\Omega^2_{ij}(2\alpha_1 - \alpha_2 - \alpha_3) = 0$$

Nonlinear Eddy Viscosity Model

- Applying the rapid distortion constraint to the quadratic form of the constitutive relations, we have a nonlinear eddy viscosity model with 3 undetermined coefficients:

$$\tau_{ij,RANS} = 2C_\mu \mathfrak{I} k S_{ij}^* - C_1 \mathfrak{I}^2 k \left(2S_{ij}^{(2*)} - S_{ik}^*\Omega_{kj} + \Omega_{ik}S_{kj}^*\right)$$
$$-C_2 \mathfrak{I}^2 k \left(2S_{ij}^{(2*)} + S_{ik}^*\Omega_{kj} - \Omega_{ik}S_{kj}^*\right) - \frac{2}{3}k\delta_{ij}$$

- To get the value of time scale $\mathfrak{I}$, an approximation based on the Bradshaw's hypothesis is derived with the mean velocity gradients available in LES region:

$$\mathfrak{I} = \frac{1}{\sqrt{\beta_c S_{ij} S_{ij}}} \qquad with\ \beta_c = 0.09$$

- By solving the resulting linear system of equations derived from RANS momentum equations, the values of C_μ, C_1 and C_2 can be obtained, which are adaptive by case and variant in space.

Downstream Adaptive Model

- By solving the 3-RANS mom eqs with LES data and the constraint imposed, the anisotropic eddy viscosity from the LES solution at the LES-to-RANS interface is obtained.
- The anisotropic eddy viscosity from the LES solution is transferred to the existing downstream RANS model by scaling its computed turbulent viscosity and enabling anisotropy through C_μ, C_1 and C_2.
- These values are transferred from the LES-to-RANS to the downstream RANS region by invoking Rodi's weak equilibrium assumption, where C_μ, C_1 and C_2 are taken to be constant along the streamline, i.e.,

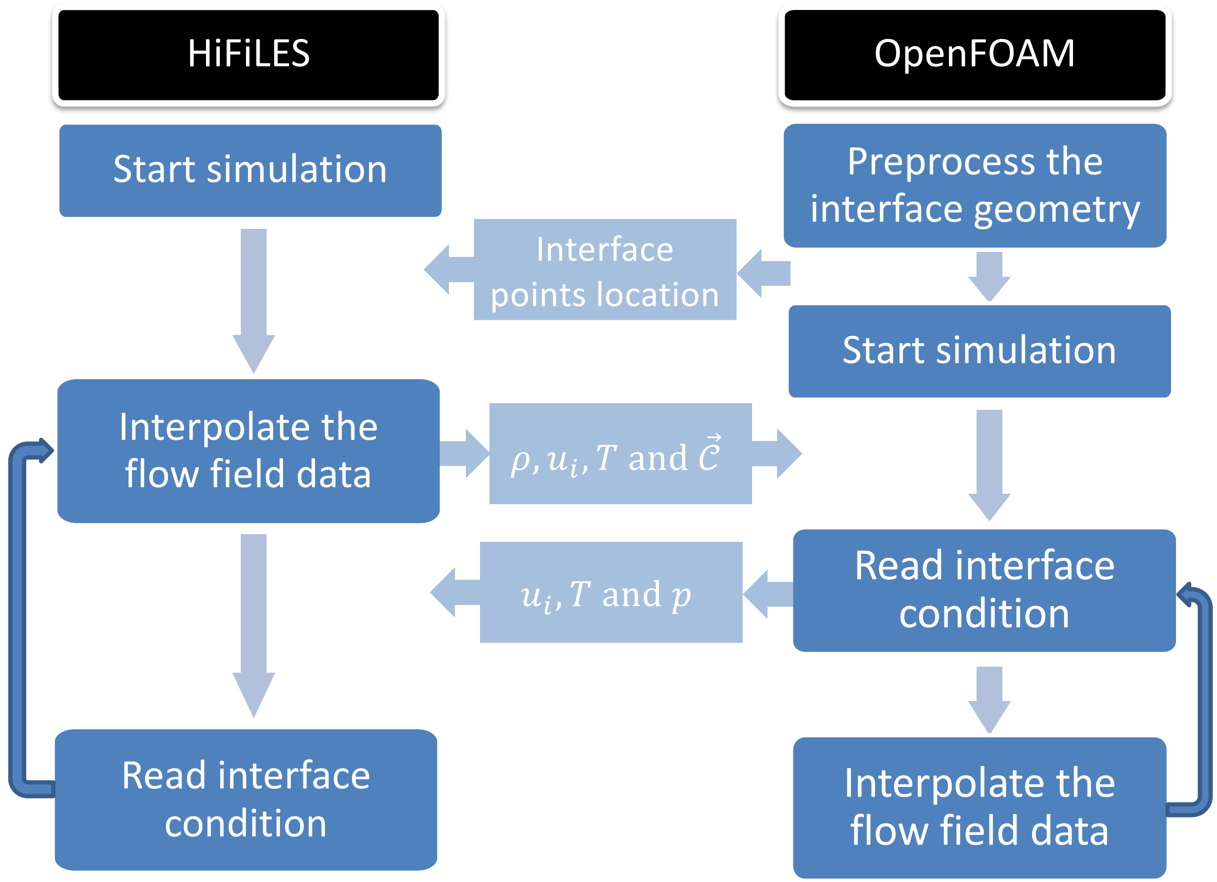
$$\frac{D\vec{C}}{Dt} = 0, \text{for } \vec{C} = C_\mu, C_1 \text{ and } C_2$$

The method described above to obtain the values of C_μ, C_1 and C_2 in the downstream RANS region constitute the new anisotropic eddy viscosity model, referred to as NLEV_Adaptive, to distinguish it from the original NLEV model, denoted as NLEV_Original.

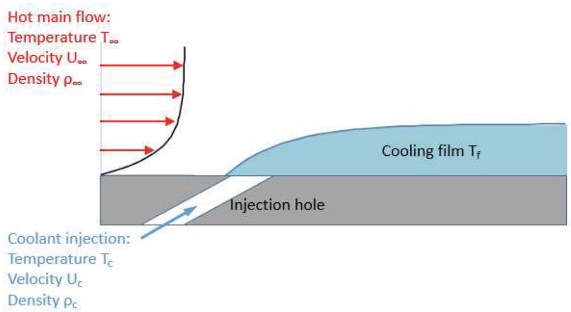
Numerical Methods

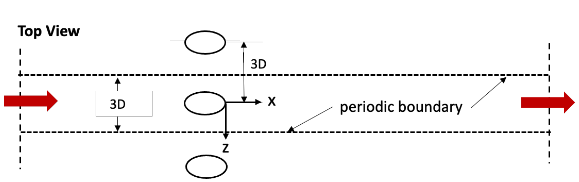
HiFiLES	OpenFOAM
LES	RANS
Compressible	Compressible
Density-based	Pressure-based
Explicit Runge-Kutta	Implicit
Discontinuous Galerkin	Finite Volume
Unstructured mesh	Unstructured mesh
Non-dimensional	Dimensional

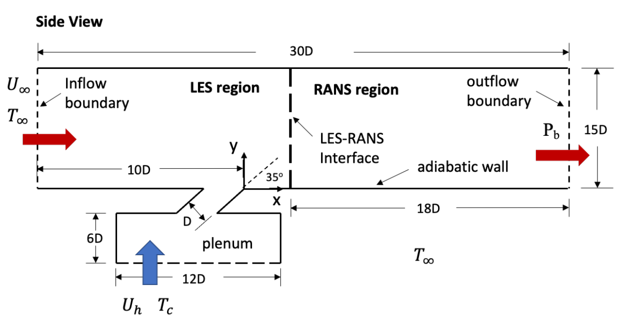
Numerical Methods with Two-Way Coupling

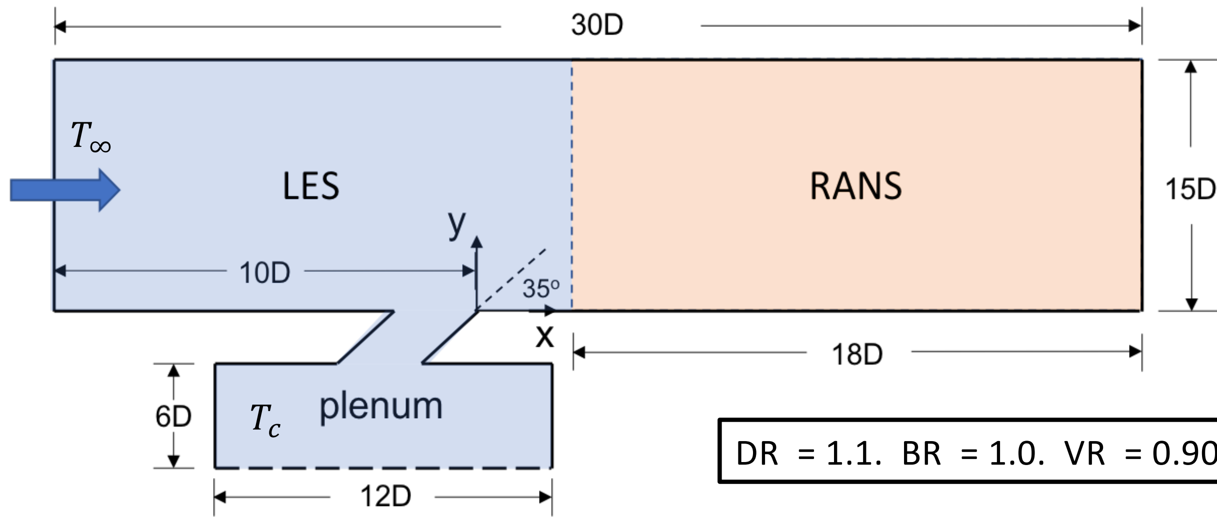


Test Case: Film Cooling









DR = 1.1. BR = 1.0. VR = 0.909

- D = 2.6 mm
- U_∞ = 36.35 m/s
- $T_\infty = 297.3K$
- $T_c = 270.8K$
- M = 0.1
- $Re = \rho U_\infty D/\mu$ = 6160
- B.C.s:
  Synthetic turbulent inlet B.C.;
  Pressure Outlet B.C.;
  Adiabatic no-slip walls;
  Periodic B.C. in spanwise direction;
  Mass flow rate inlet B.C. for plenum

	LES	RANS
Code	HiFiLES	OpenFOAM
Model	WALE	NLEV-Adaptive
DoF/Mesh	14M	1M

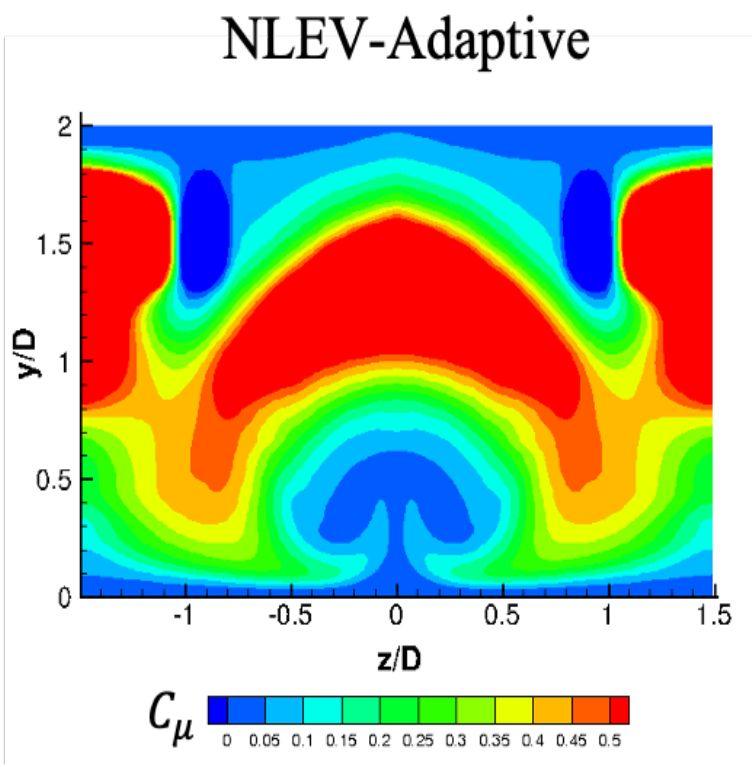
Mesh size: pure LES ~1.6 times of the hybrid
CPU-hours: pure LES ~2.3 times of the hybrid.

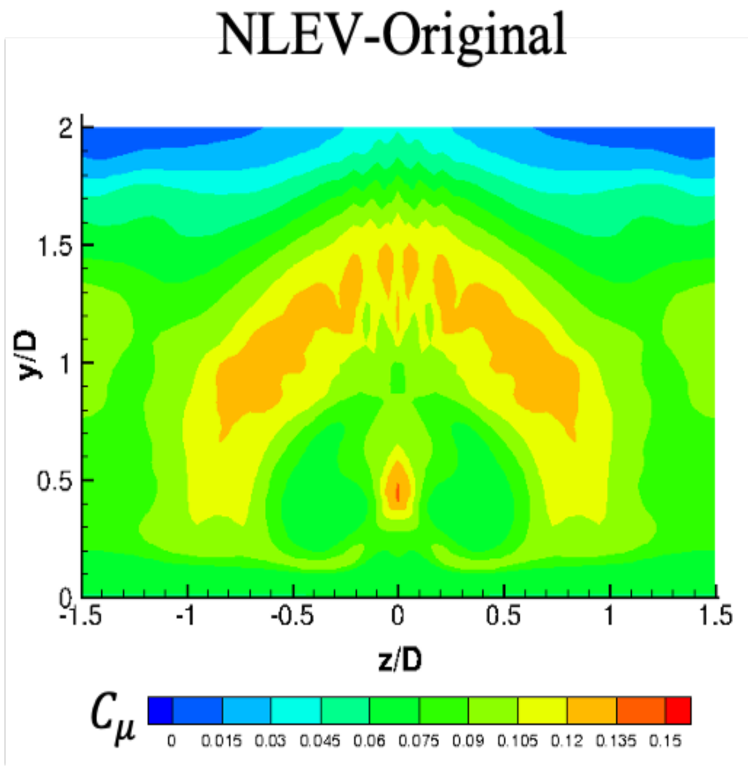
Summary of Parameters Used in Downstream RANS Models*

Downstream RANS model	**Parameters C_μ, C_1 and C_2**
NLEV-Adaptive (Two-way coupling)	Calculated from upstream LES solutions.
Realizable $k-\varepsilon$ (One-way coupling)	$C_\mu = \frac{1}{A_0 + A_s^* U^* k/\varepsilon}$, $C_1 = C_2 = 0$
NLEV-Original (One-way coupling)	$C_\mu = \frac{1}{A_0 + A_s^* U^* k/\varepsilon}$, $C_1 = -C_2 = \frac{\sqrt{1-9C_\mu^2\left(\frac{S^*k}{\varepsilon}\right)^2}}{C_0 + 6\frac{S^*k}{\varepsilon}\frac{\Omega^*k}{\varepsilon}}$

* $A_0 = 6.5$, $C_0 = 1.0$, $A_s^* = \sqrt{6}cos\phi$, $\phi = \frac{1}{3}\arccos\left(\sqrt{6}W^*\right)$, $W^* = \frac{S_{ij}^* S_{jk}^* S_{ki}^*}{(S^*)^3}$ *and* $U^* = \sqrt{S_{ij}^* S_{ij}^* + \Omega_{ij}\Omega_{ij}}$

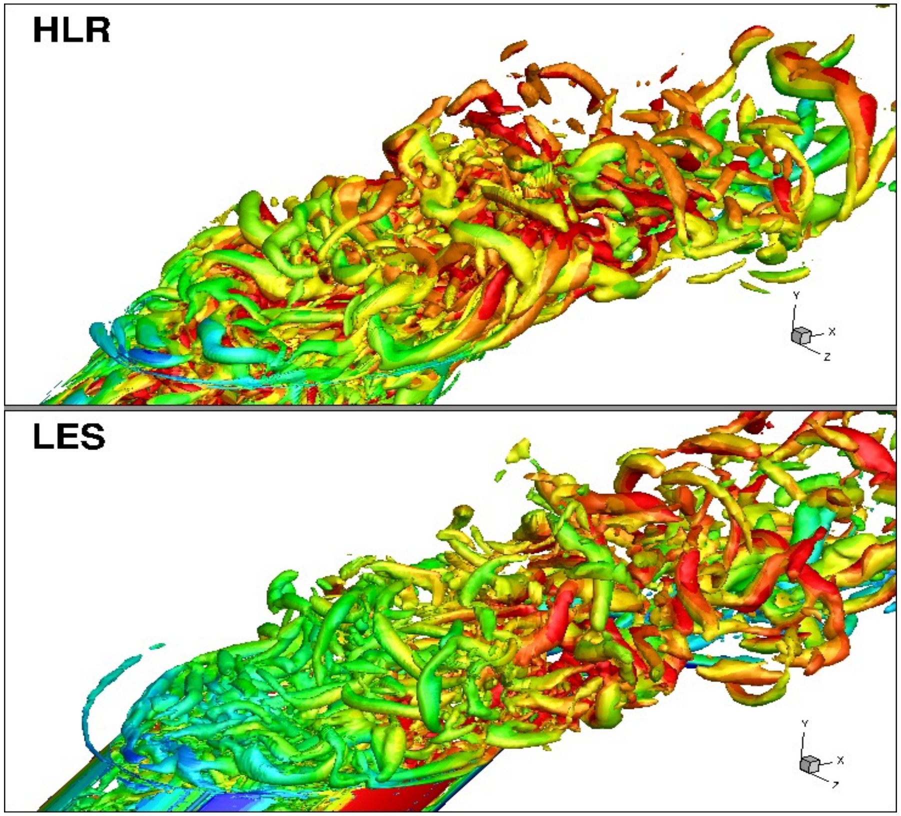
Parameter Value in Downstream Adaptive RANS Model at LES-to-RANS Interface





Film Cooling: LES Region

Iso-surface of Q-criterion colored by instantaneous streamwise velocity (Q value set to 10)



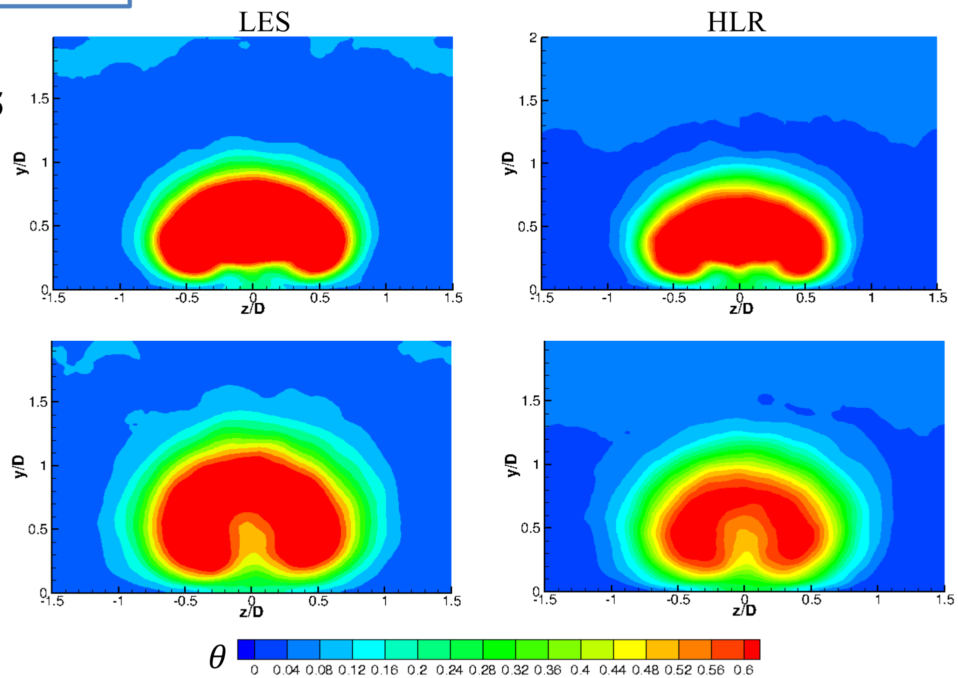
Film Cooling: LES Region

$$\theta = \frac{T - T_\infty}{T_c - T_\infty}$$

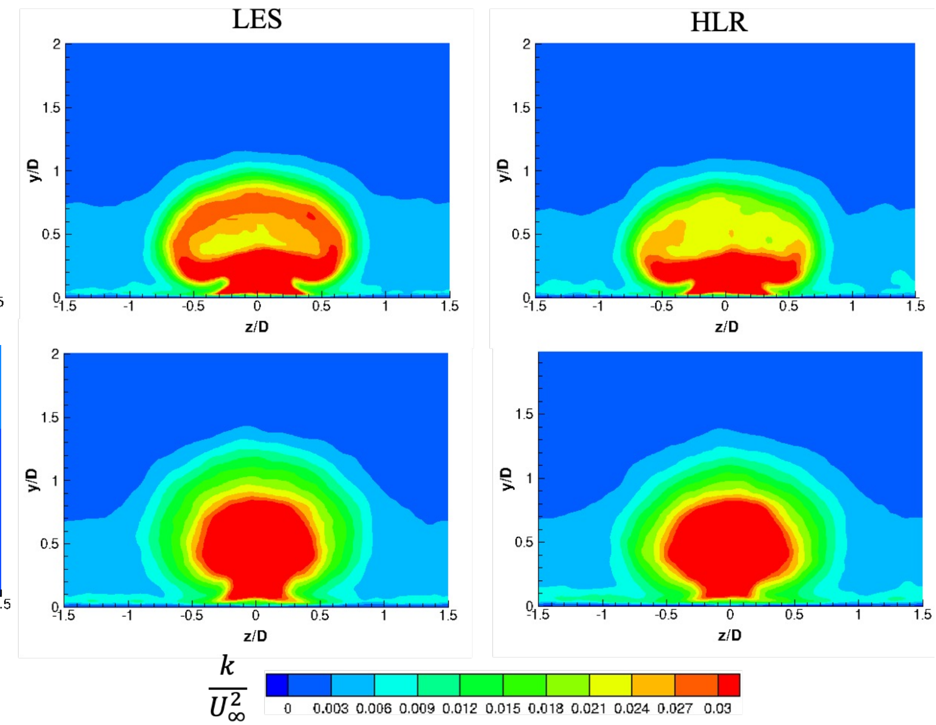
Non-Dimensional Temperature θ

x/D = 0.5

x/D = 2

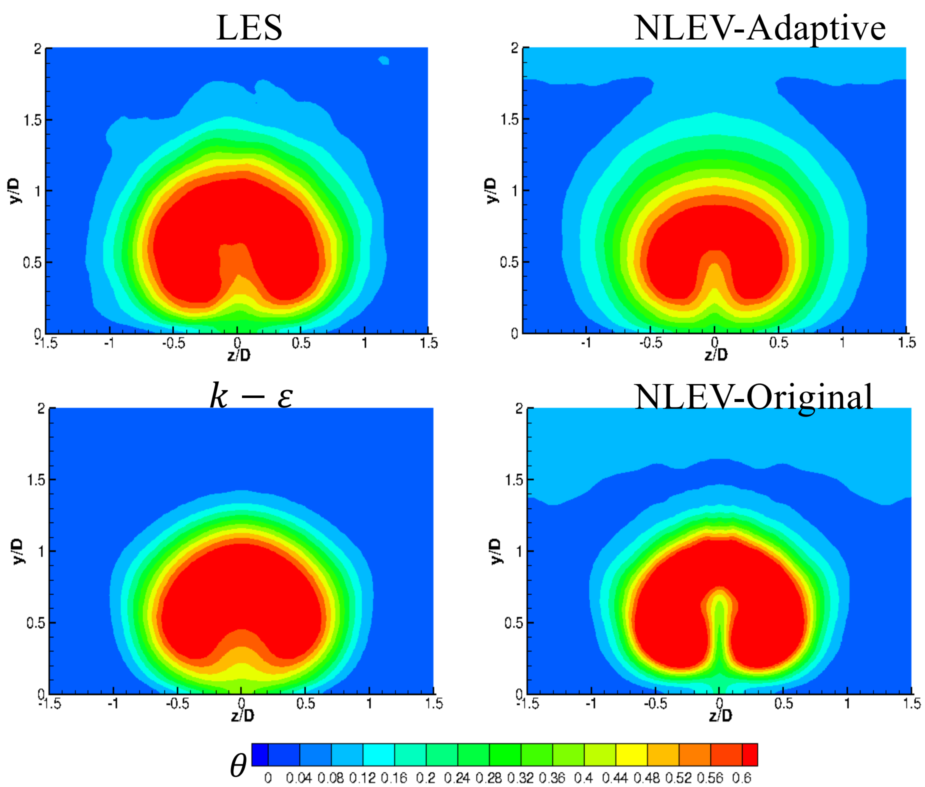


Turbulent Kinetic Energy

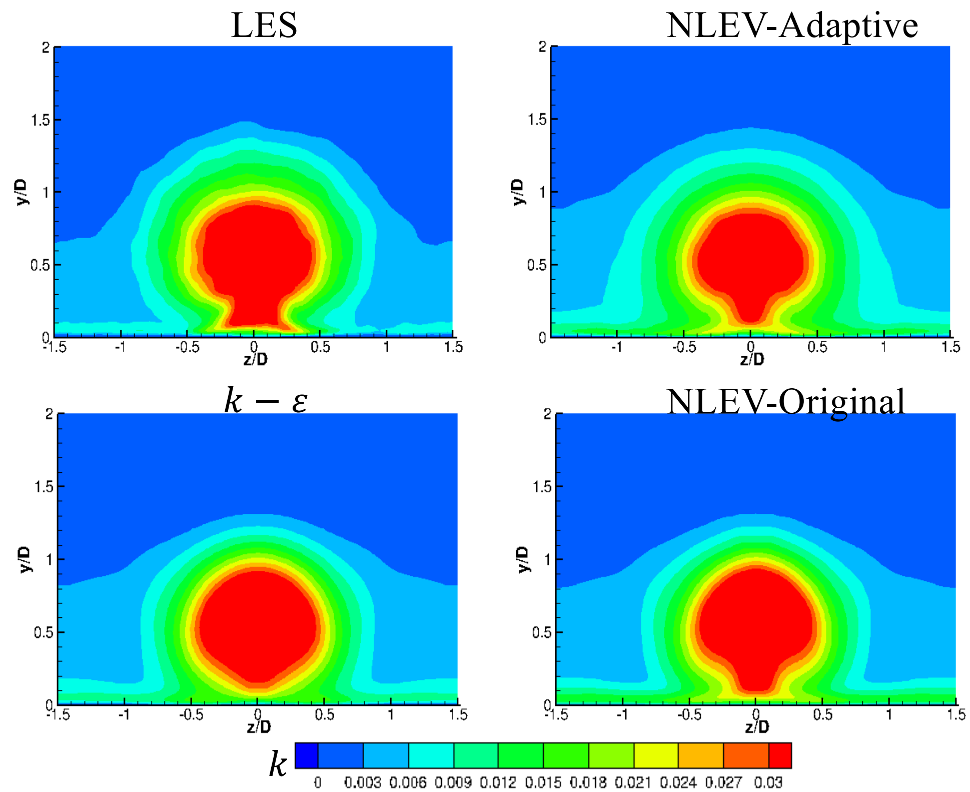


Film Cooling: RANS Region at x/D = 2.5

Non-Dimensional Temperature

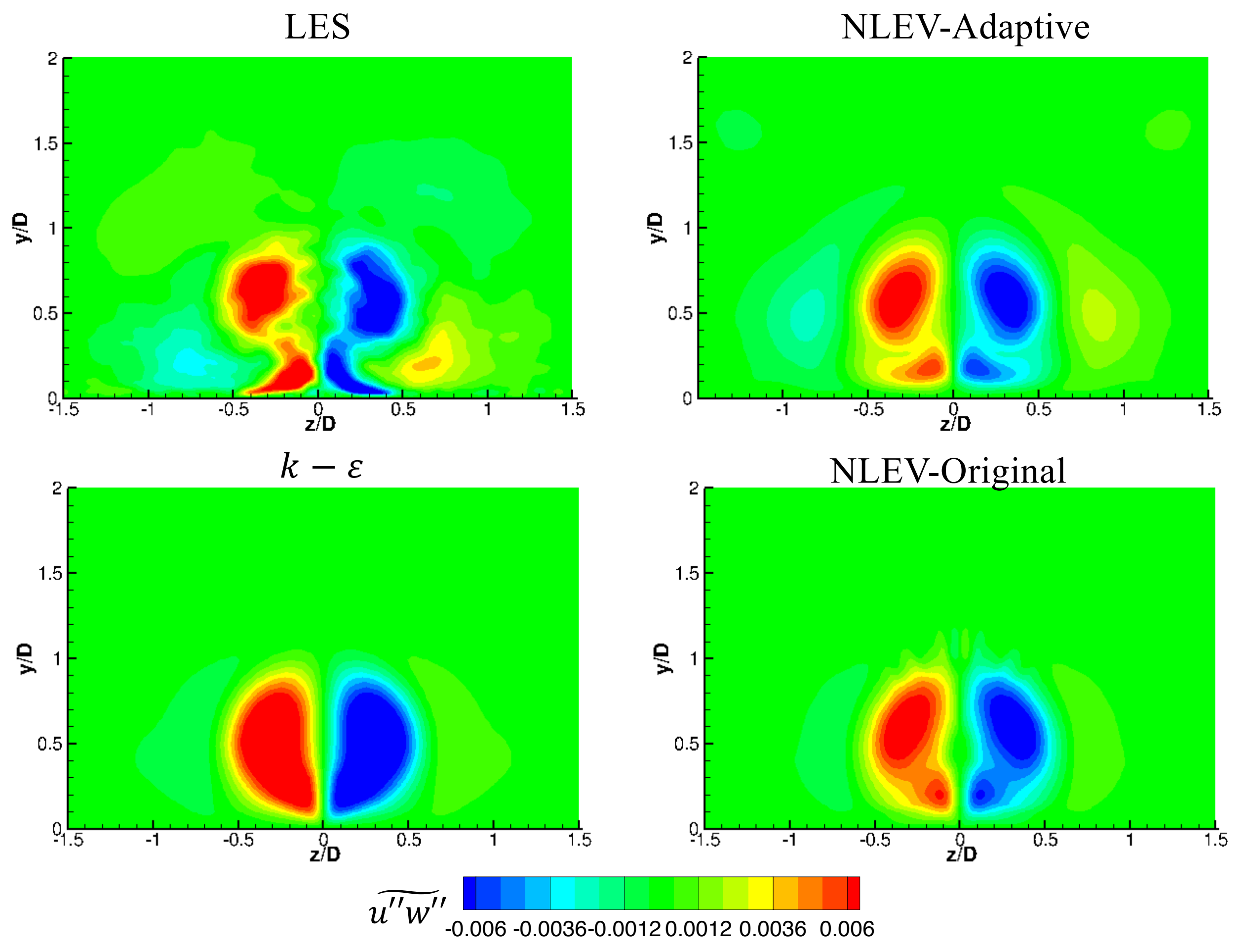


Turbulent Kinetic Energy



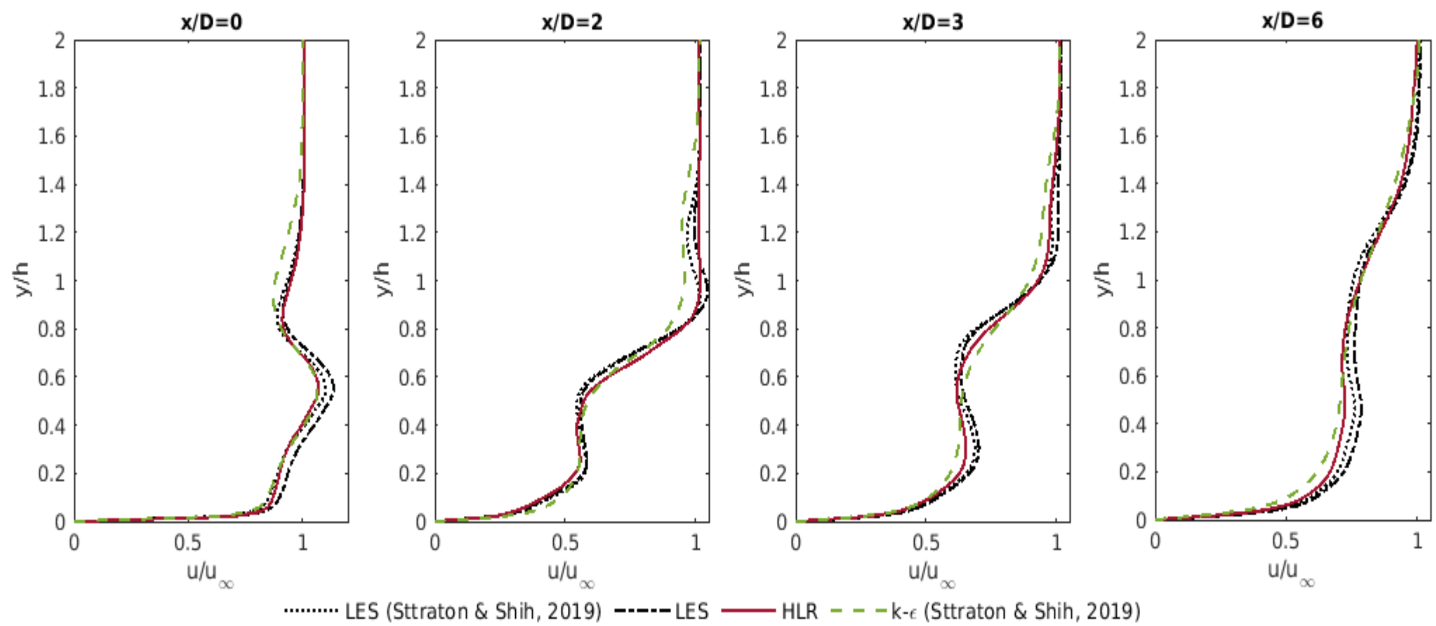
Film Cooling: RANS Region

Reynolds stress <u'w'> at x/D=2.5



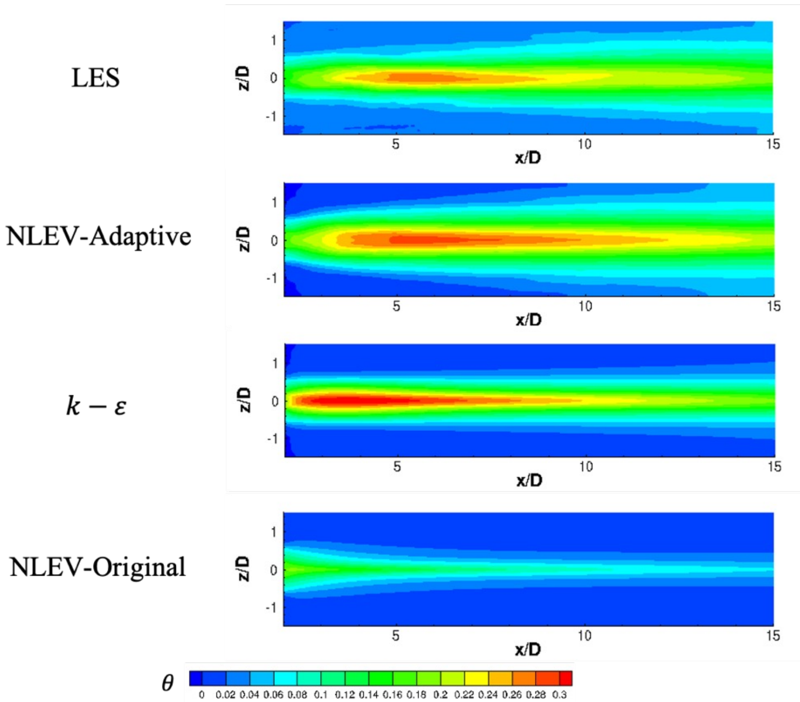
Film Cooling

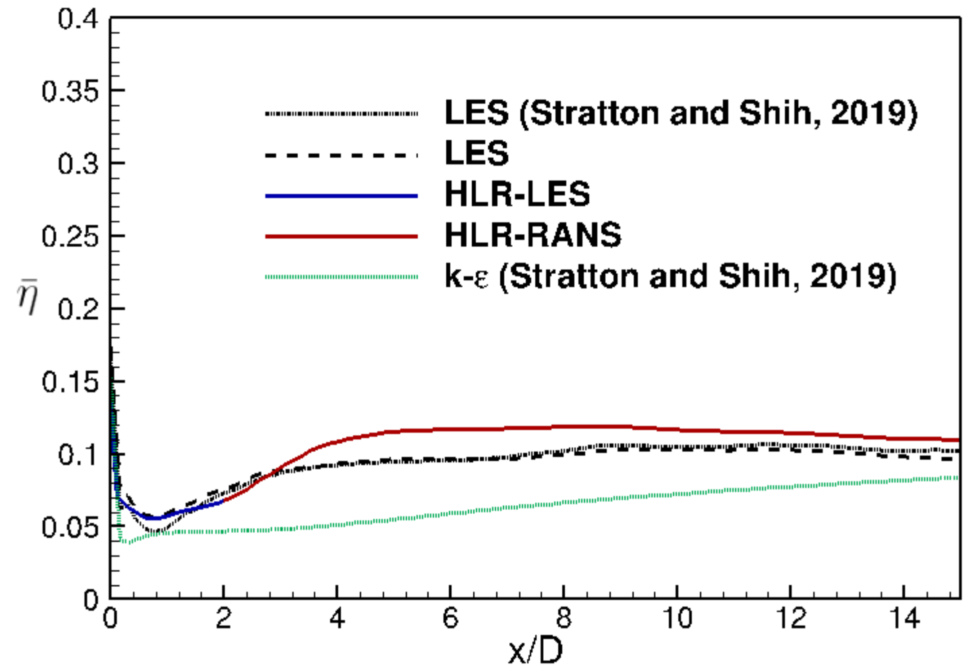
Streamwise Velocity Profiles at centerline



Hybrid simulation (HLR) matches the LES data better compared to the $k - \varepsilon$ model which fail to predict the curvature of the profile inside the jet.

Film Cooling Adiabatic Effectiveness





When compared to LES, the max. relative error is 10% for HLR-LES and 22% for HLR-RANS for $\bar{\eta}$.

The total number of cells reduced by 35% when compared to the LE. Cost was reduced by 57%.

I. Background

II. Method 1: Downstream Tensorial Eddy Viscosity Model

III. Method 2: Downstream Nonlinear Eddy Viscosity Model

IV. Summary

Key Contributions on Hybrid LES-RANS Methods

- Removed the “gray” area in LES-RANS solutions via the concept of discontinuous flow variables (DVF) approach.
- Removed instabilities and increased efficiency by developing 2 downstream RANS models that can sustain the upstream LES solution so LES region can be significantly reduced in size.
- Implemented DFV method in 2 open-source codes: HiFILES and OpenFOAM.
- Assessed models on turbulent flows in straight duct, periodic hill, and film-cooling of a flat plate and obtained good results.

Comments? Questions?