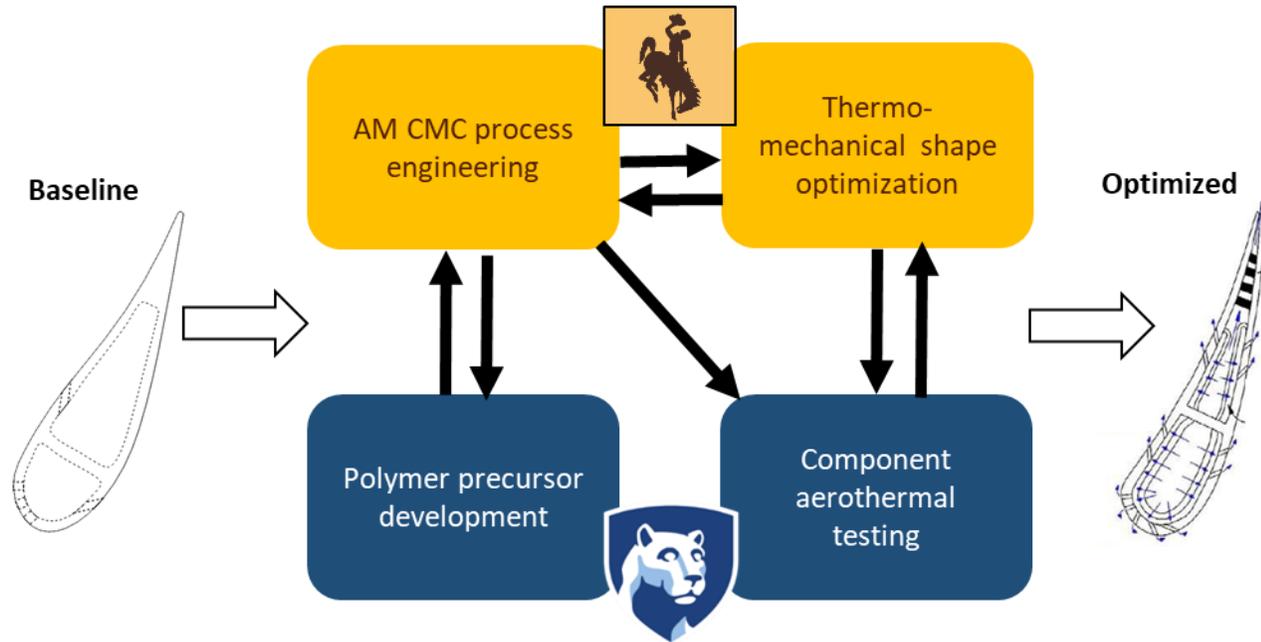


# Development of Additive Manufacturing for Ceramic Matrix Composite Vanes



**Dr. Stephen Lynch**  
**Dr. Michael Hickner**

Students: Andrew Fox, Shruti Gupta



**Dr. Carl Frick**  
**Dr. Ray Fertig**

Students: Jason Young, Jennifer Hankins,  
Kyle Kuhn

**Project: DE-FE0031758**

# This review covers the motivation, project plan, and Year 3 progress on our UTSR-supported project



**Motivation: enable superalloy-like cooling features in CMC-like materials to enable higher temperature operation**



**Approach: push polymer-derived ceramics development through additive manufacturing engineering, polymer science, part design, and performance verification**

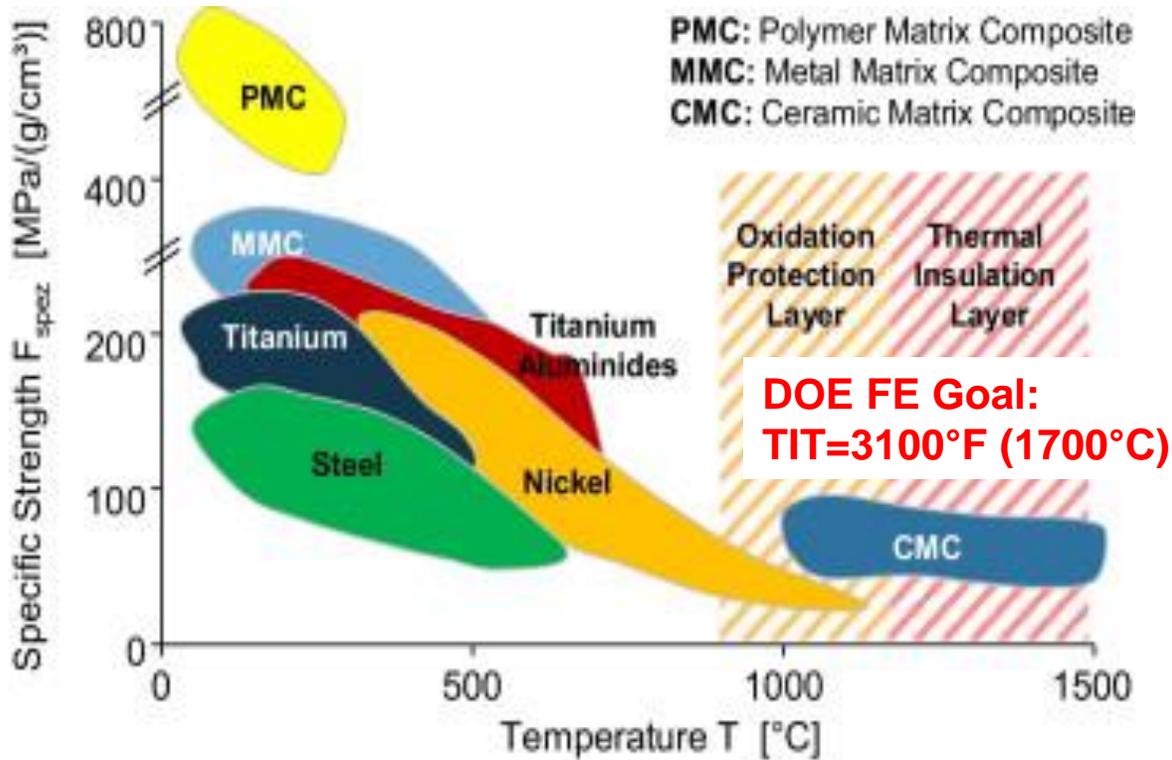
|   | Year 1 | Year 2 | Year 3 |
|---|--------|--------|--------|
| <b>Task 1 - Project Management and Planning</b>                                   |        |        |        |
| 1.1 - Update project management plan  |        |        |        |
| 1.2 - Update project schedule   |        |        |        |
| 1.3 - Update resource management  |        |        |        |
| <b>Task 2 - Fabrication and Testing of CMC for Turbine Vanes</b>                  |        |        |        |
| 2.1 - Design of a turbine vane  |        |        |        |
| 2.2 - Process development and optimization  |        |        |        |
| 2.3 - Additive manufacturing of turbine vanes                                     |        |        |        |
| <b>Task 3 - Materials and Development of CMC Turbine Vanes Testing</b>            |        |        |        |
| 3.1 - Turbine vane testing: operation 1   |        |        |        |
| 3.2 - Turbine vane testing: operation 2   |        |        |        |
| 3.3 - Turbine vane testing: operation 3   |        |        |        |
| <b>Task 4 - Performance Development and Evaluation of New Vanes for Hot Spots</b> |        |        |        |
| 4.1 - Turbine vane testing: operation 4   |        |        |        |
| 4.2 - Characterization of vanes in comparison to other materials                  |        |        |        |

**Project plan: execute simultaneous materials/design/process engineering to regularly produce vane test articles**



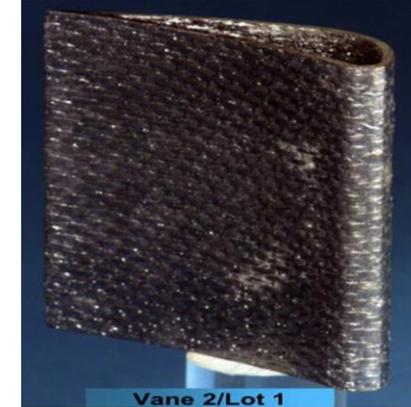
**Current progress**

To achieve DOE firing temperature targets and push higher thermodynamic efficiency, even advanced CMC materials will require some cooling



Klocke, et al., CIRP Annals, 2014

Melt-infiltrated woven CMC



<https://technology.nasa.gov/patent/LEW-TOPS-25>

Binder-jet CMC with pass-through cooling

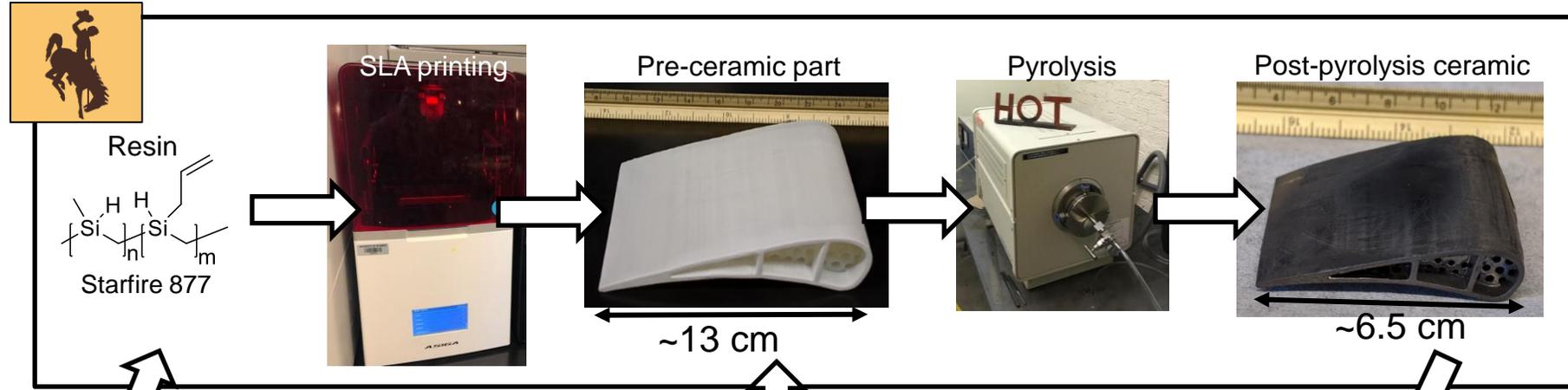


<https://ntrs.nasa.gov/search.jsp?R=20160010285> 2019-02-28T01:25:33+00:00Z

**Project goal: enable complex cooling features in CMC-like materials for realistic shapes and confirm cooling benefit**

# Our approach is to develop novel chemistries and manufacturing processes, coupled with design optimization, to create internally cooled ceramic vanes that can be tested

Dr. Carl Frick



Dr. Mike Hickner

$$\begin{array}{c} \text{CH}_2-\text{O}-\text{C}(=\text{O})-\text{CH}_2 \\ | \quad | \\ \text{CH}_2 \quad \text{CH}_3 \\ | \quad | \\ \text{CH}_3-\text{Si}-\text{O}-\left( \text{Si}-\text{O} \right)_m-\left( \text{Si}-\text{O} \right)_n-\text{Si}-\text{CH}_3 \\ | \quad | \quad | \quad | \\ \text{CH}_3 \quad \text{CH}_3 \quad \text{CH}_3 \quad \text{CH}_3 \end{array}$$

Dr. Ray Fertig

Dr. Steve Lynch

The project has five technical tasks that address **process engineering**, **material development**, **modeling**, and **validation of cooling technologies**

### **Task 1: Project management**

Success criteria: Project Coordinator delivers all required reports to NETL

### **Task 2: Design, fabrication, and testing of SiOC/SiC baseline vane**

Success criteria: project team is able to obtain surface temperatures on baseline SiOC/SiC internally cooled vane test articles tested in the transonic cascade at Penn State

### **Task 3: Modeling and optimization of ceramic turbine vane designs**

Success criteria: mechanical and thermal stresses reduced by 10% from baseline to optimized; increase overall cooling effectiveness by 0.05 relative to baseline

### **Task 4: Development of new precursor chemistries**

Success criteria: demonstrate that new precursors have equivalent or better material properties than baseline formulation

### **Task 5: Integrate new precursor chemistries into AM process**

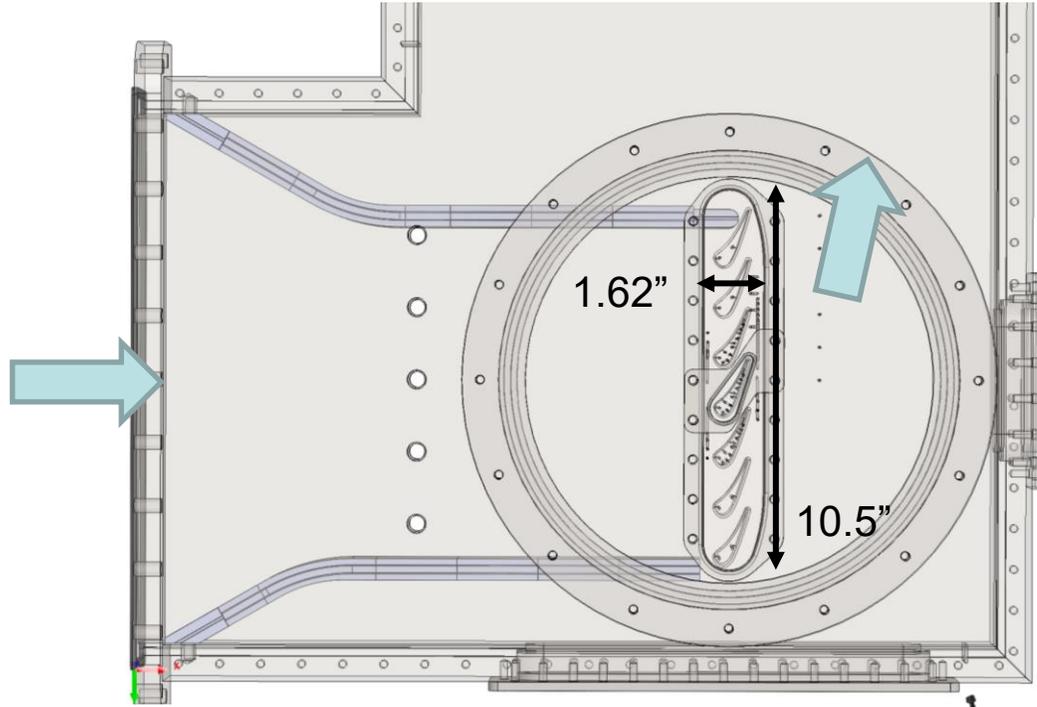
Success criteria: project team is able to deliver a baseline vane part for transonic cascade testing using new chemistry

### **Task 6: Fabricate and test optimized vanes with new chemistries**

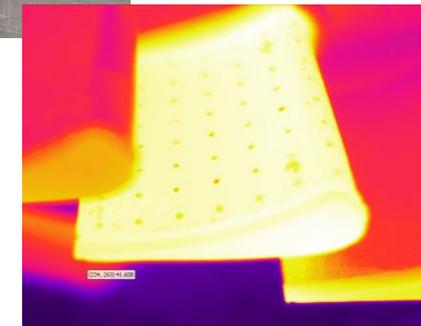
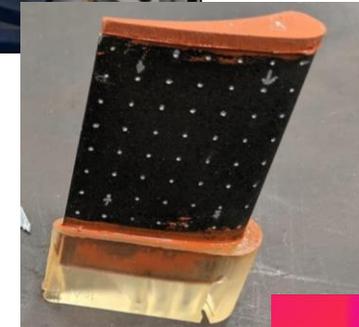
Success criteria: project team obtains surface temperatures on optimized AM vane and shows overall cooling effectiveness increase relative to baseline

# Task 2.1: Fabrication, assembly, and testing procedures for AM fabricated ceramic vanes have been developed for Penn State's high speed linear cascade

Linear cascade and test hardware (1<sup>st</sup> stage vane, NASA C3X)



Calibration vane with spatial markers visible in IR

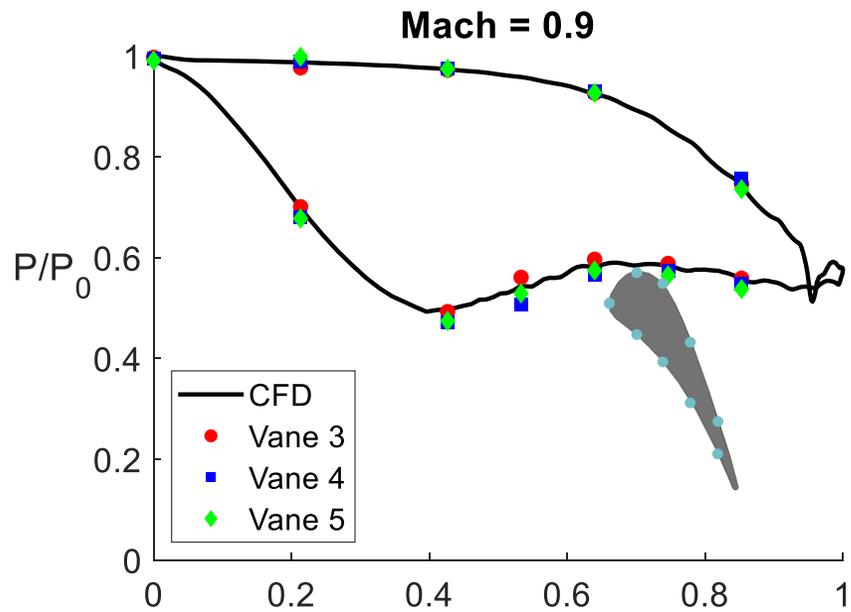


AM ceramic test insert

AM-fabricated coolant routing

Coolant

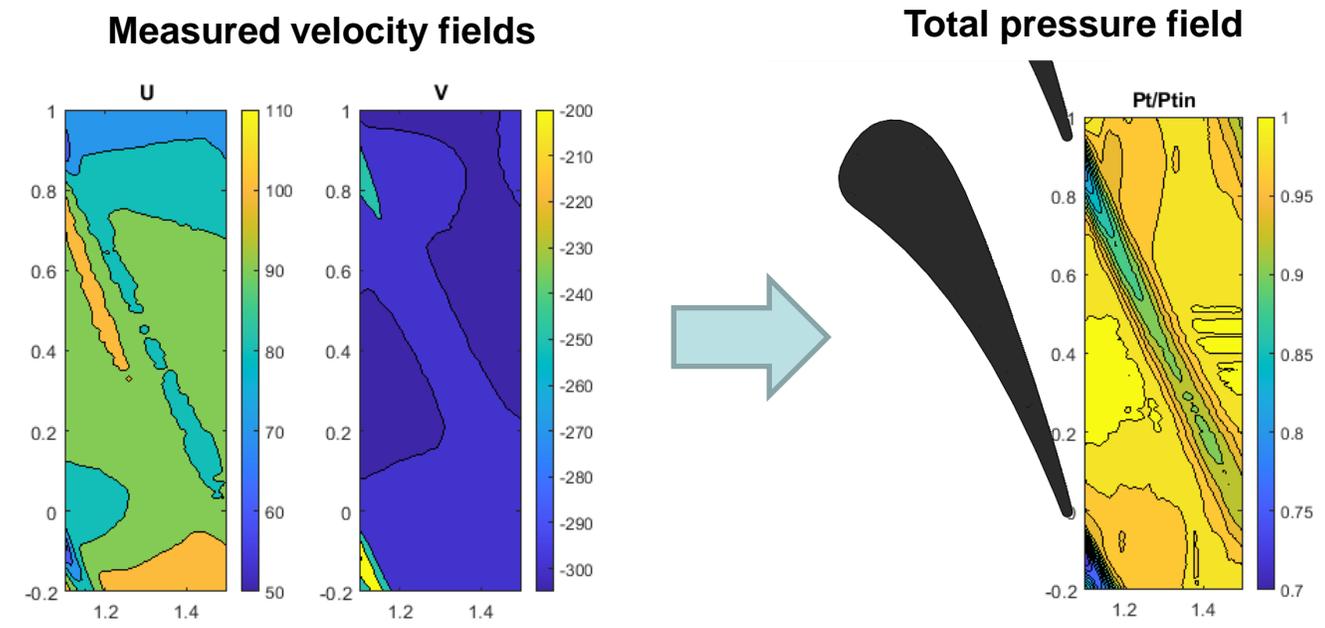
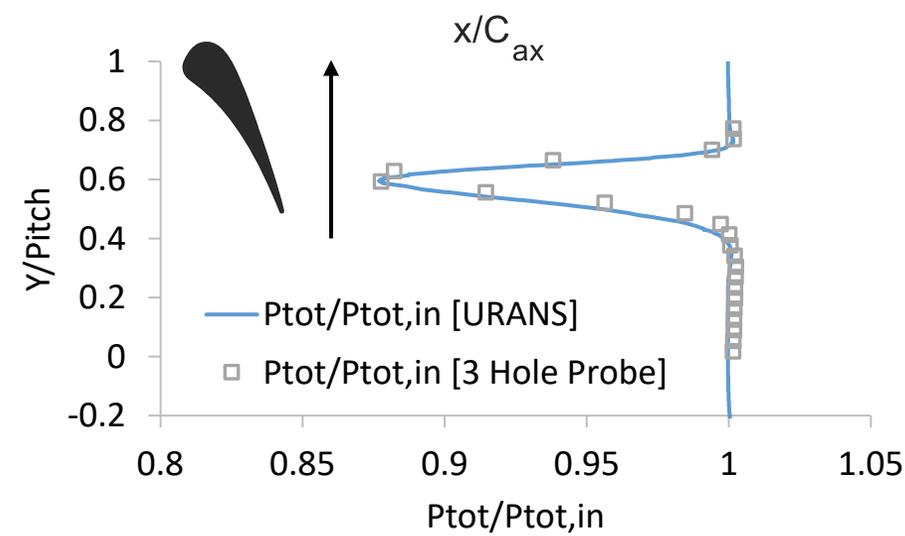
# Task 2.1: The aerodynamics of the high speed cascade were benchmarked, and a novel total pressure measurement technique using PIV was developed for aerodynamic characterization



Momentum equation

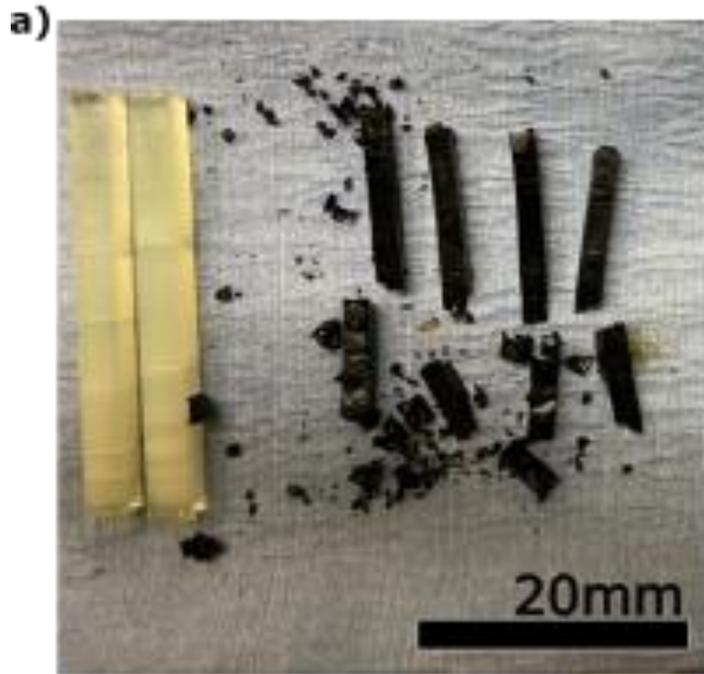
From measured velocity fields (PIV)

$$-\frac{\partial \bar{p}}{\partial x_i} = \rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \rho \frac{\partial \bar{u}'_i \bar{u}'_j}{\partial x_j} - \mu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}$$

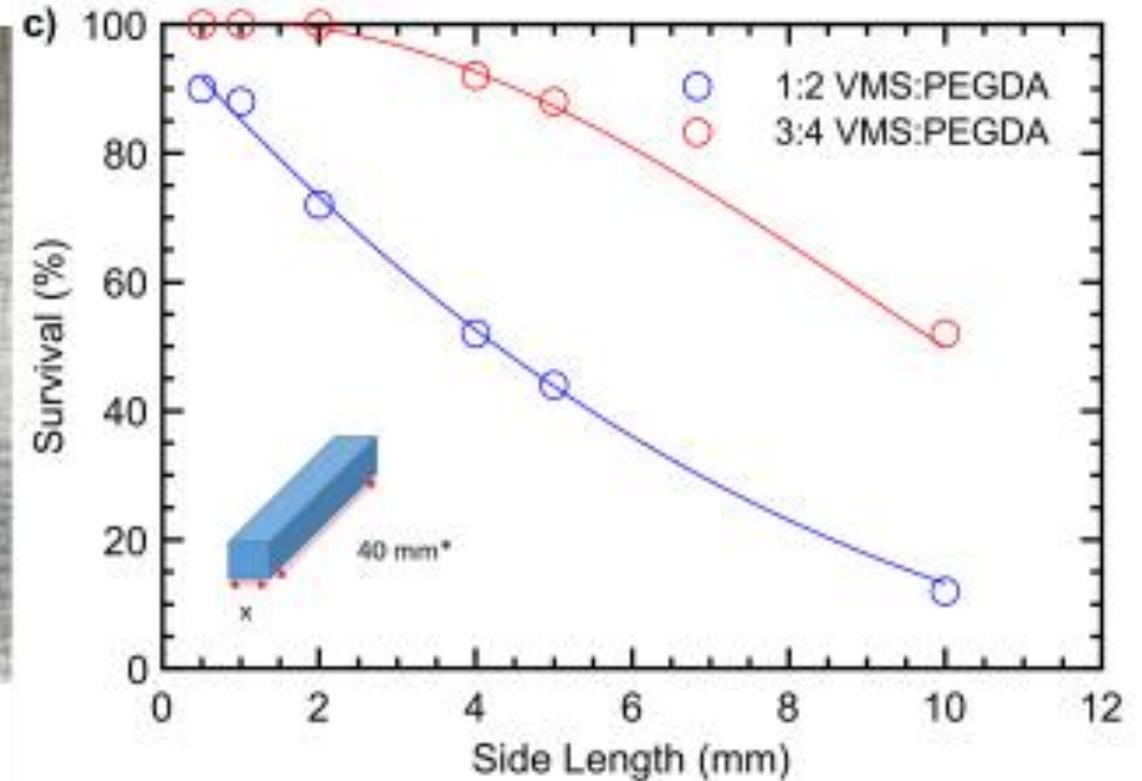
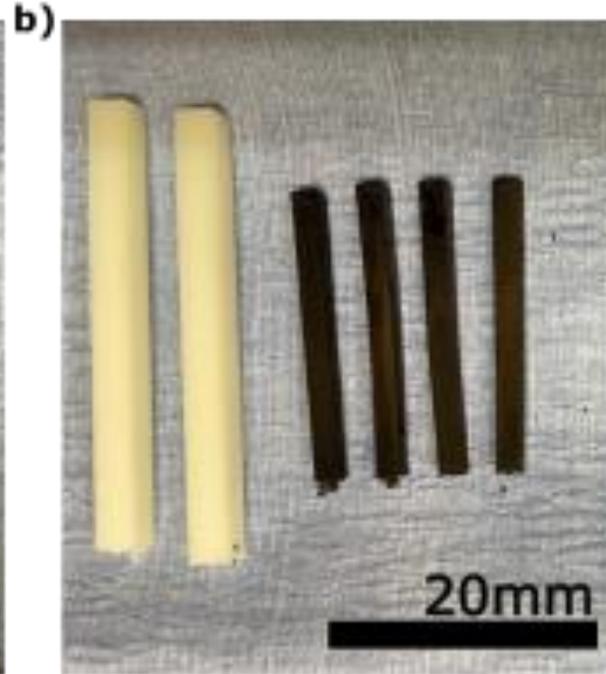


# Task 2.2: New baseline resin formulations were developed with more pyrolysis success in the feature size range of interest

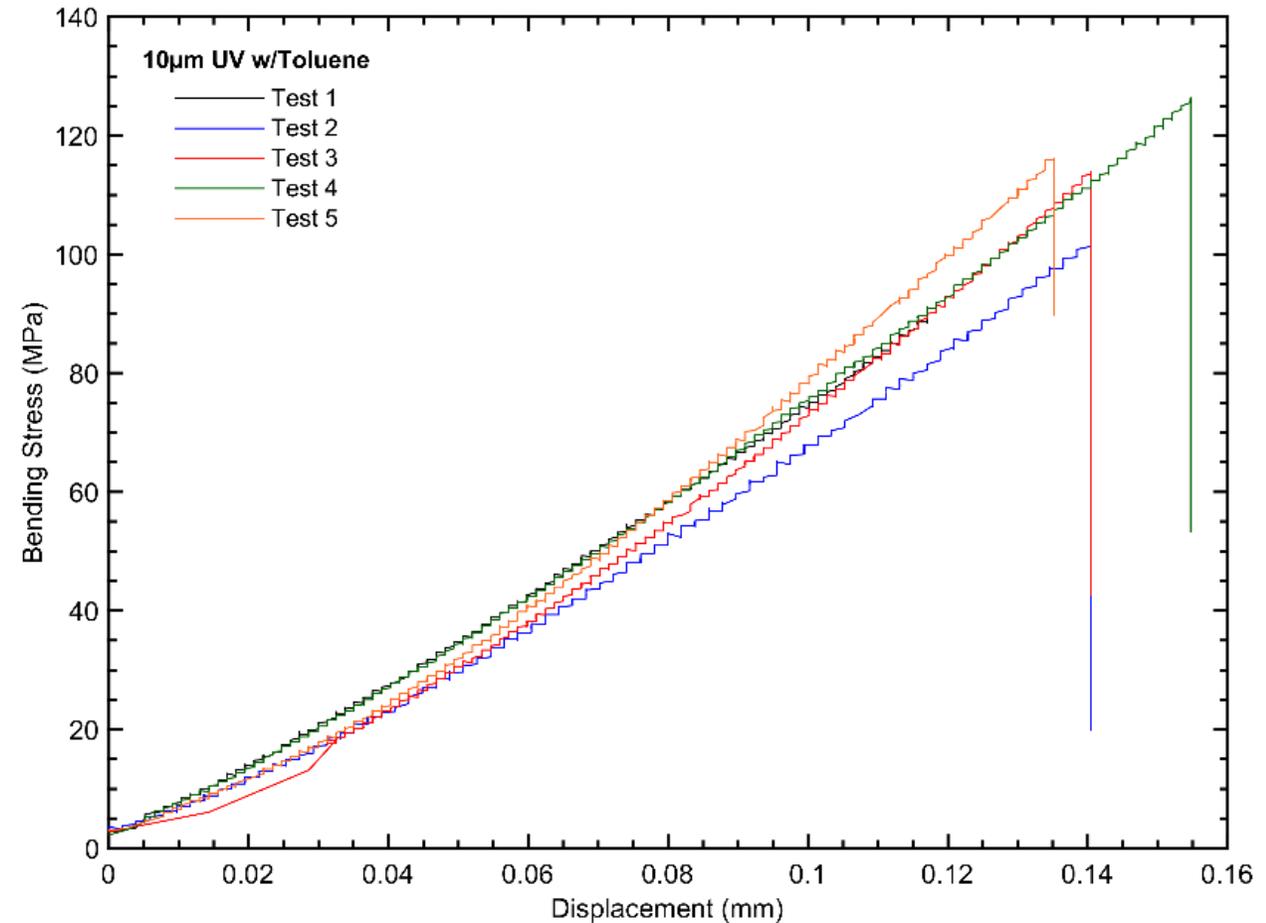
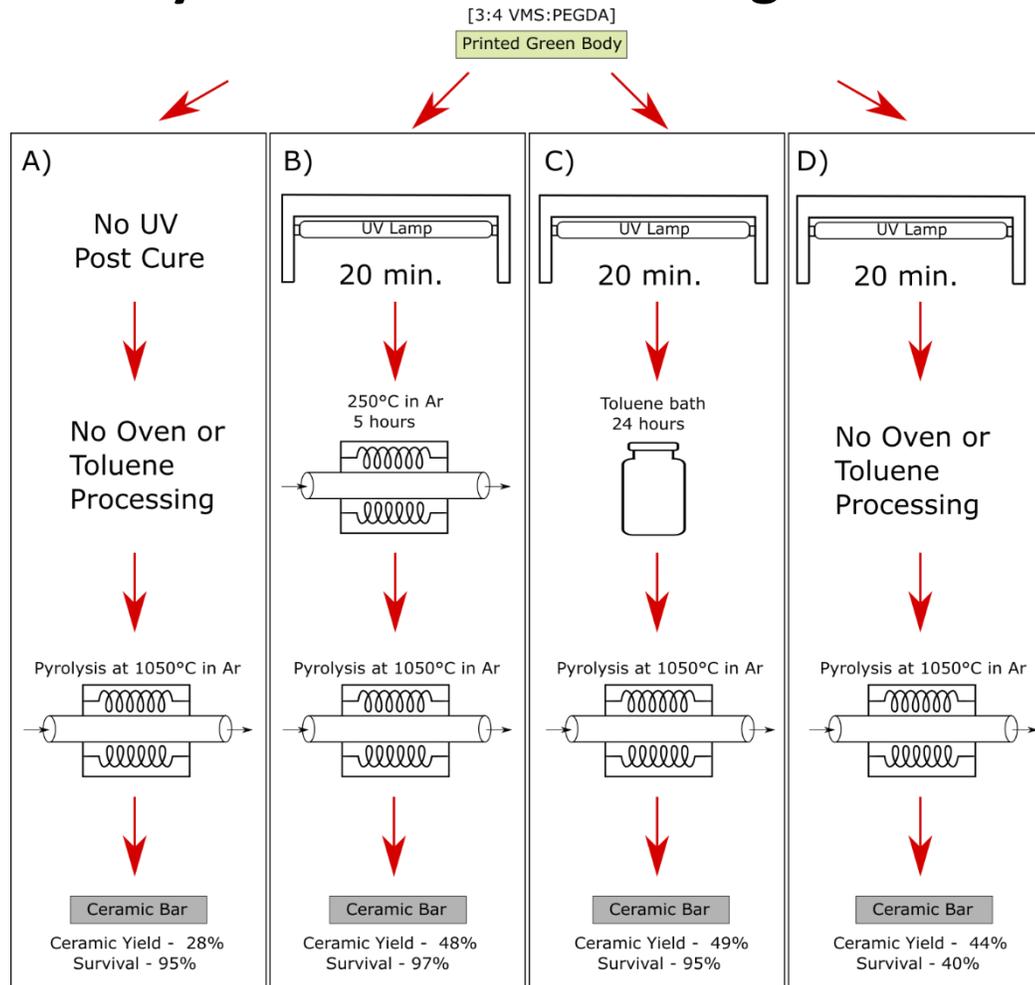
Prior formulation:  
1:2 wt% VMS:PEGDA



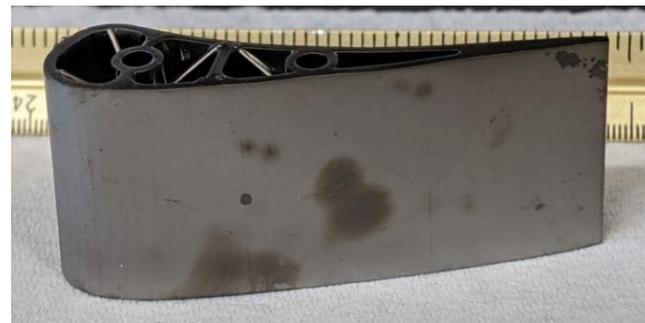
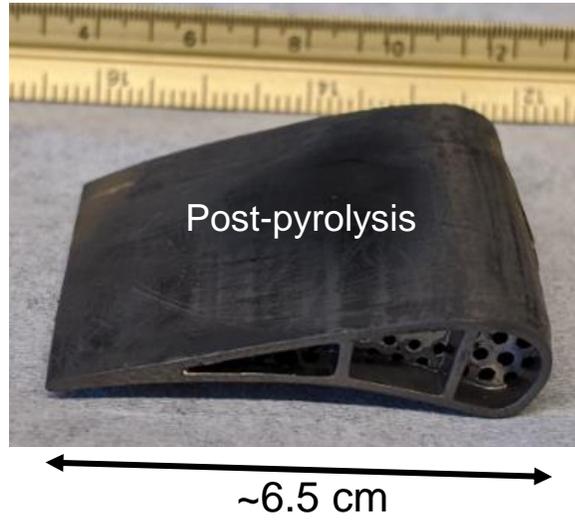
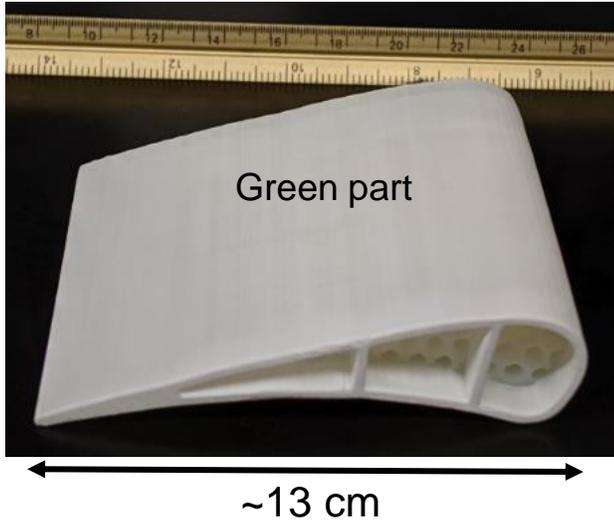
New formulation:  
3:4 wt% VMS:PEGDA



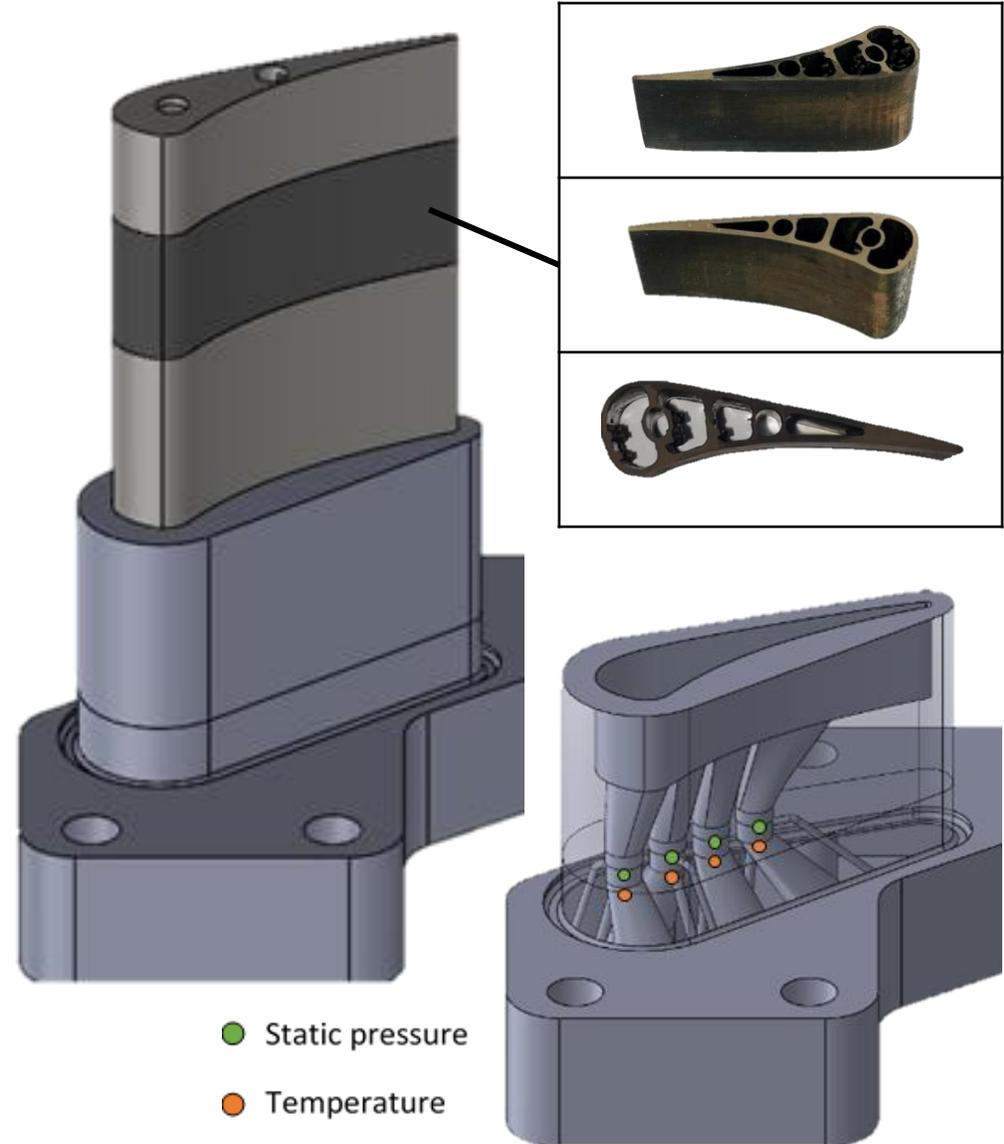
# Task 2.2: Creation of a new post-processing procedure for green bodies resulted in even higher survivability and ultimate bending stress of over 100 MPa for test bars



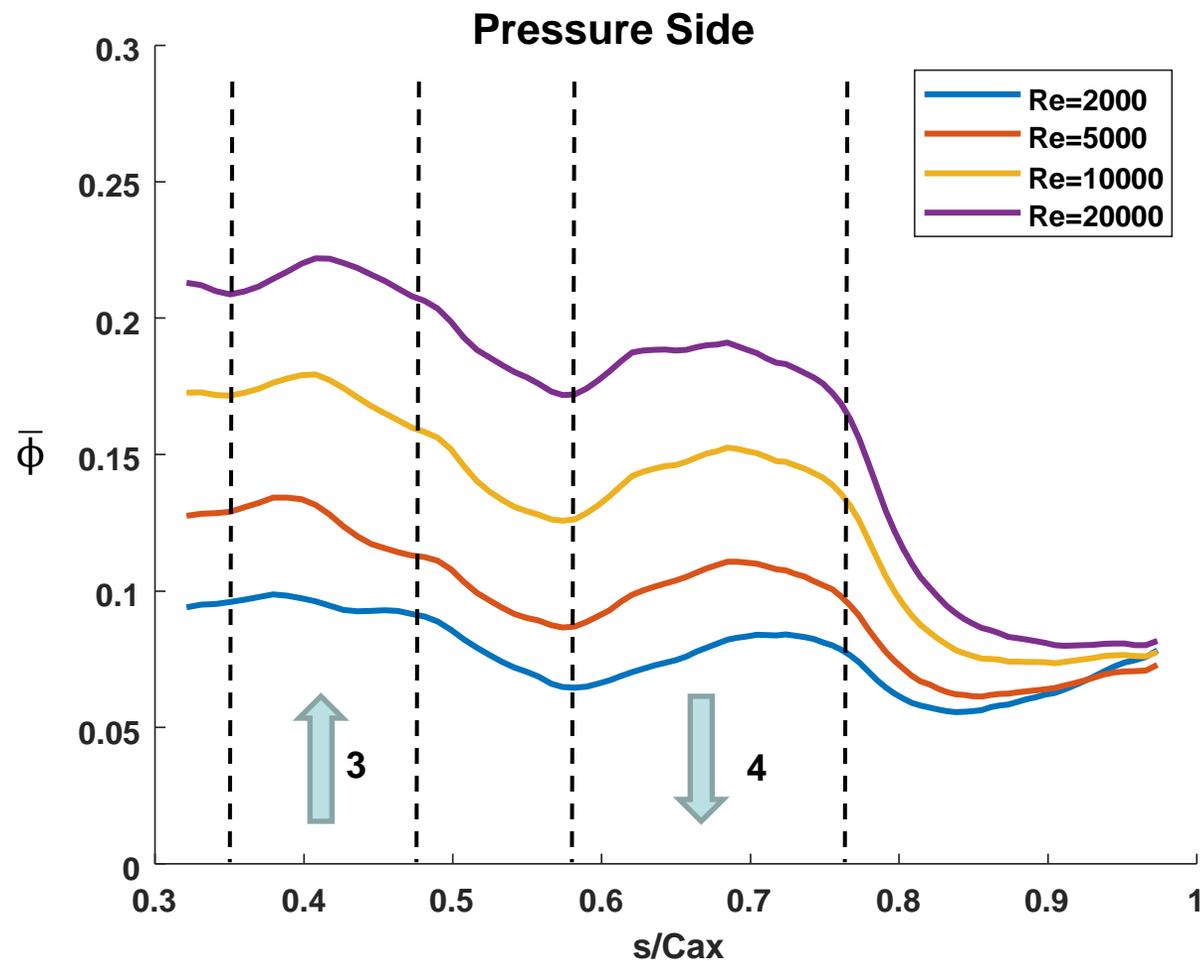
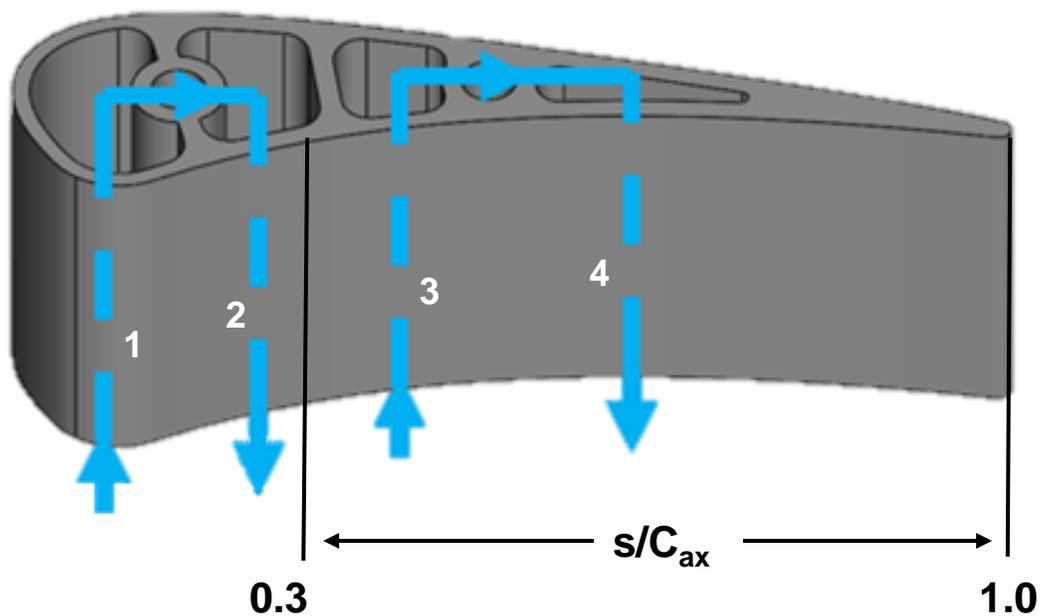
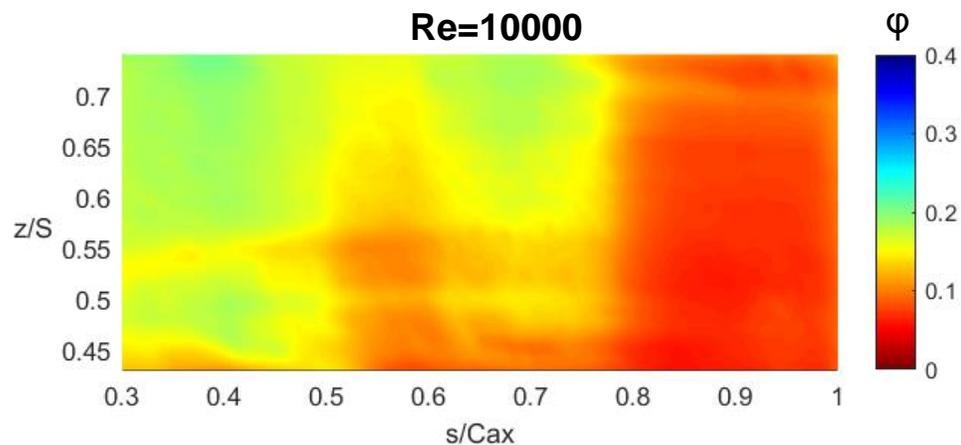
## Task 2.2: The resulting process improvements were able to generate airfoils with good dimensional tolerance and high survivability in pyrolysis



## Task 2.3: IR measurements coupled with embedded temperature & pressure instrumentation in the blade's coolant channels allow for cooling performance evaluation of the ceramic

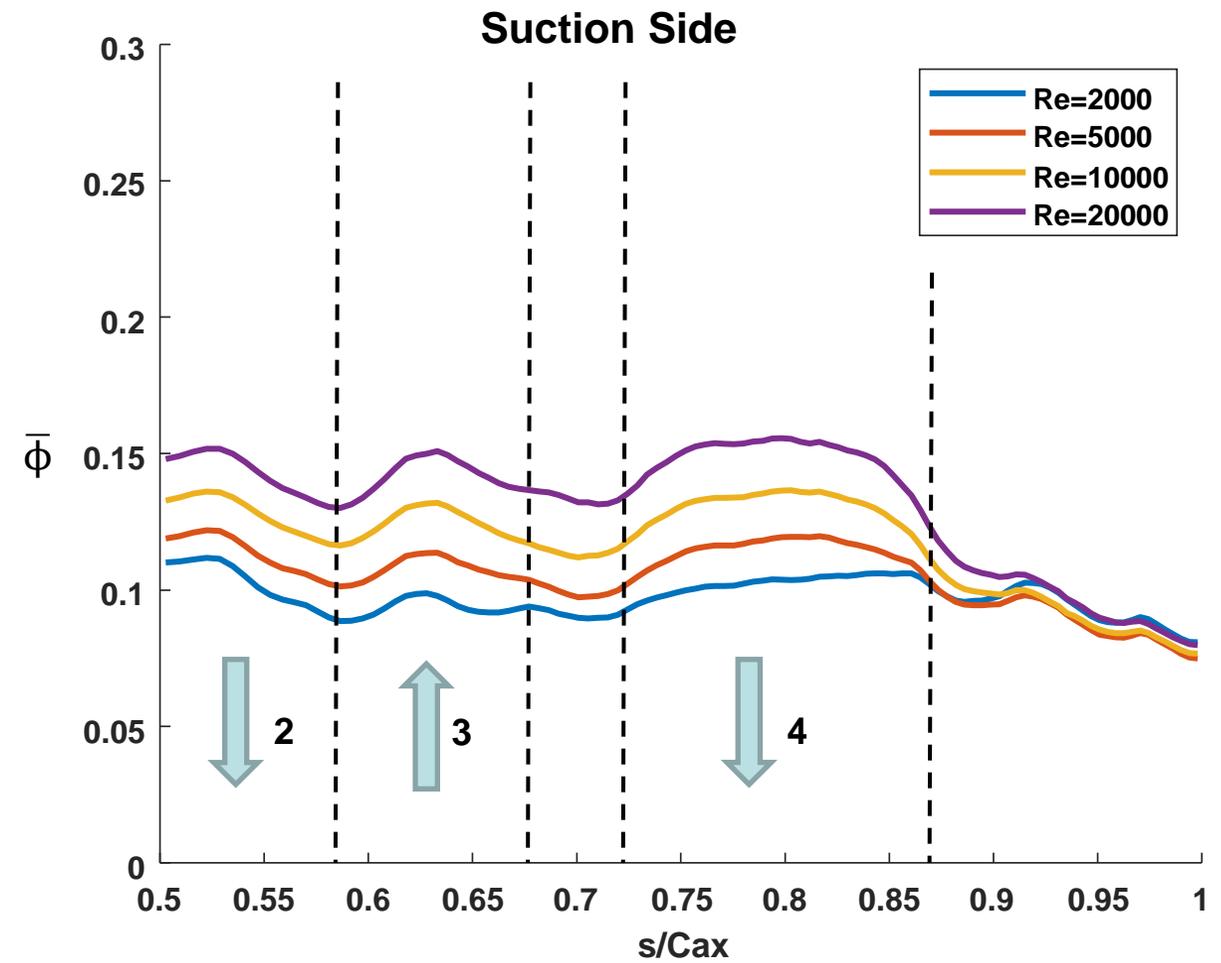
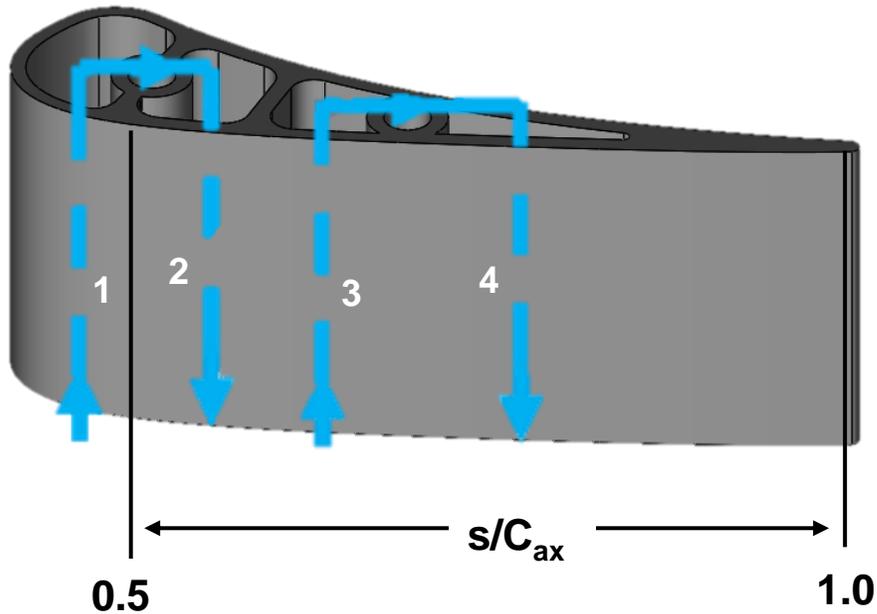
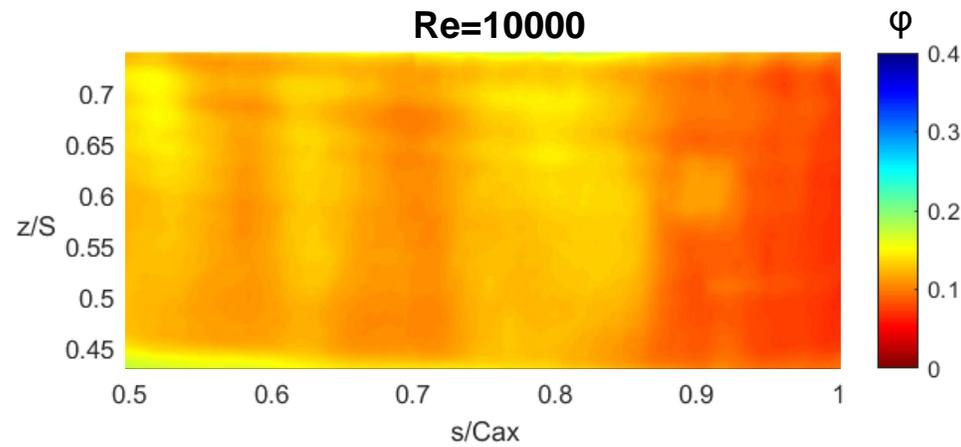


# Task 2.3: Laterally averaged cooling effectiveness of the baseline geometry increases with coolant flow rate & decreases across the coolant bypass loop as seen from the pressure side



$$\phi = \frac{T_{\infty} - T_w}{T_{\infty} - T_{c,avg 1,3}}$$

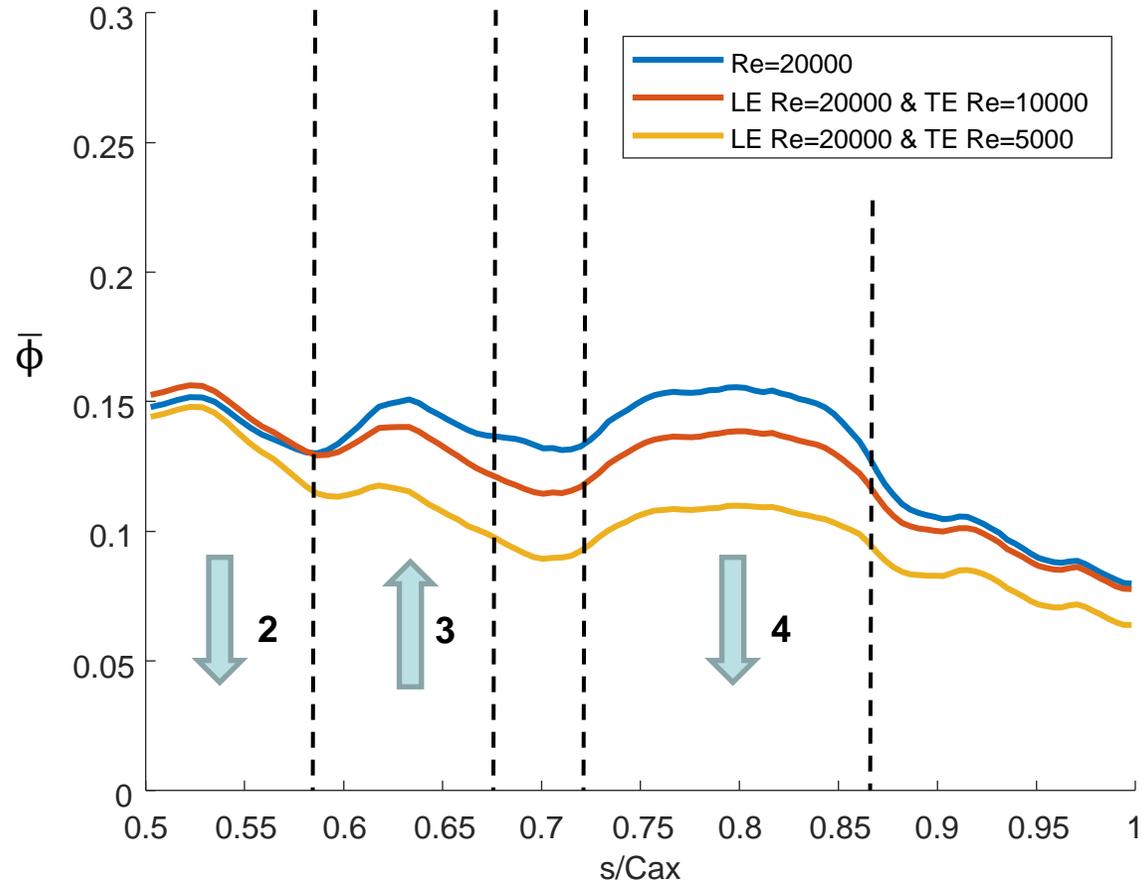
# Task 2.3: The cooling effectiveness is lower on the suction side and does not change as much with internal cooling flow rate due to higher external convection coefficients



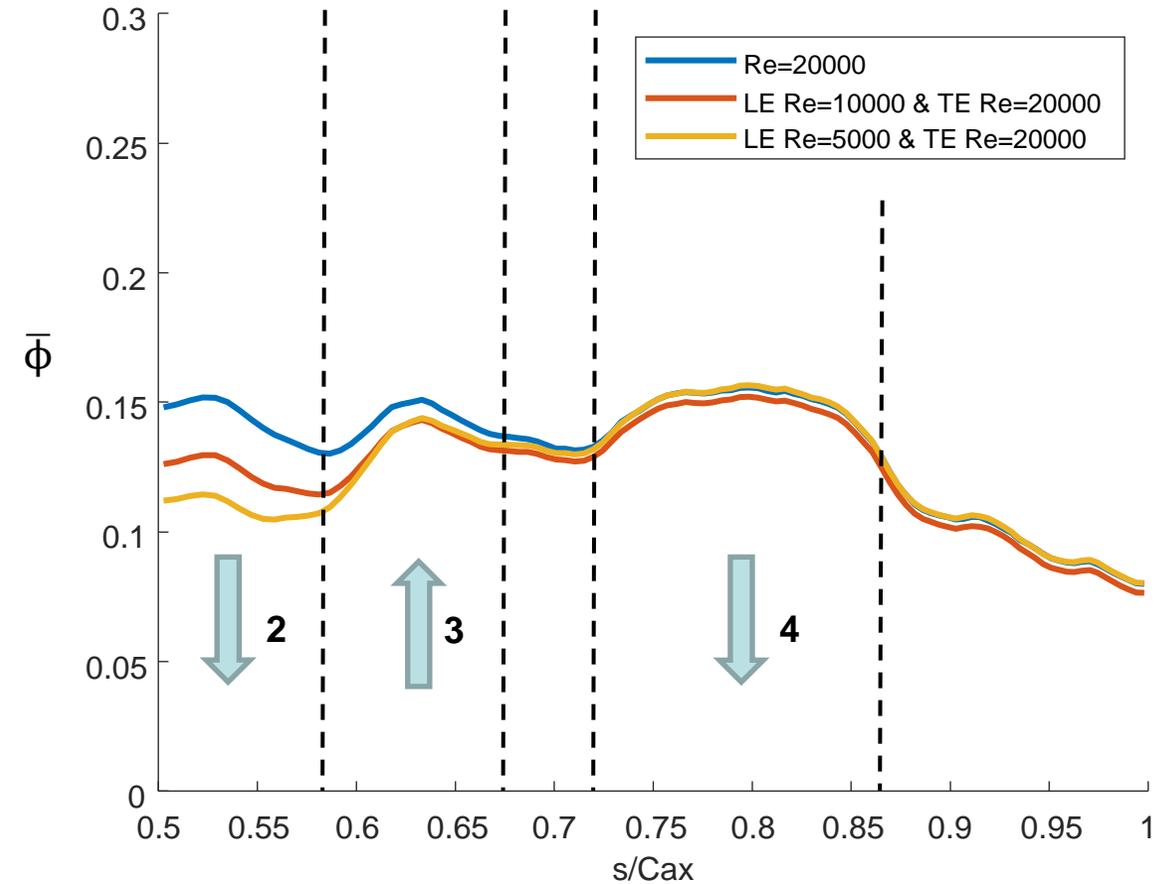
$$\phi = \frac{T_{\infty} - T_w}{T_{\infty} - T_{c,avg\ 1,3}}$$

# Task 2.3: Independently varying each coolant channel's Reynolds number shows the localized effect of internal cooling on the vane suction side temperatures

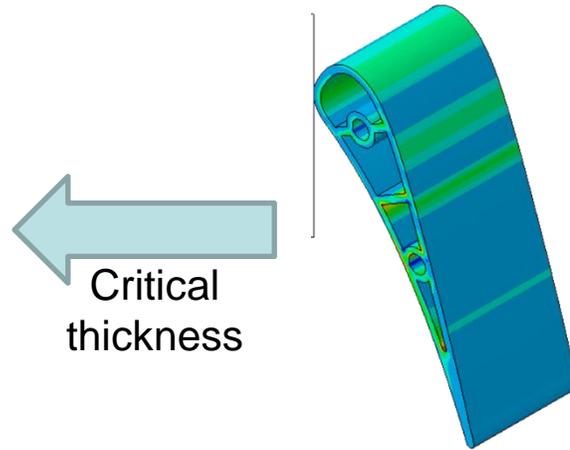
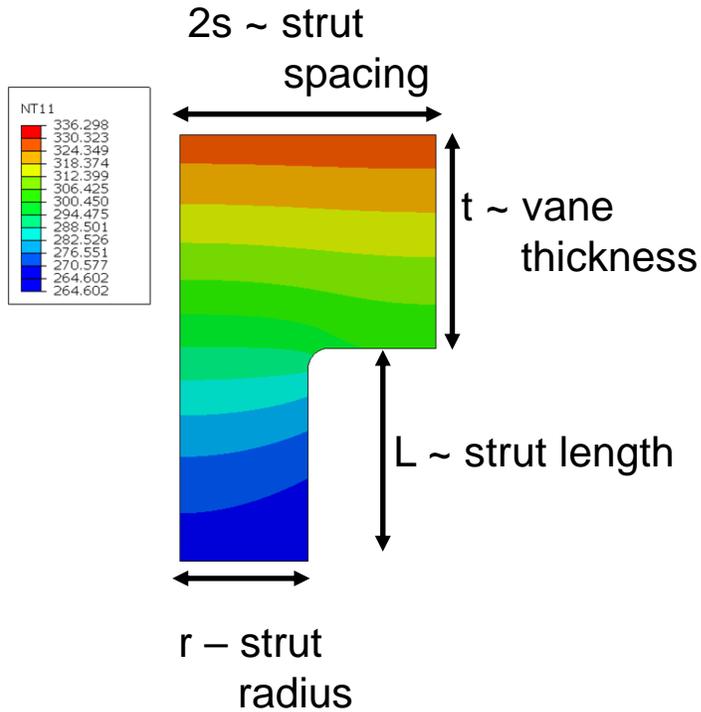
### Varying trailing edge cavity (TE) flow rate



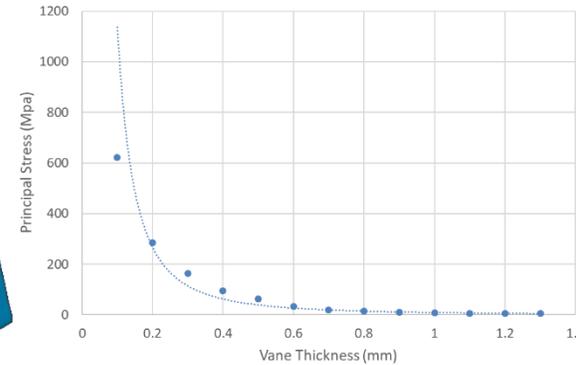
### Varying leading edge cavity (LE) flow rate



# Task 3.1-3.2: We have developed a multi-fidelity process for thermo-mechanical design of optimal vane internal cooling architecture to maximize cooling efficiency



Max. Stress vs. Wall Thickness



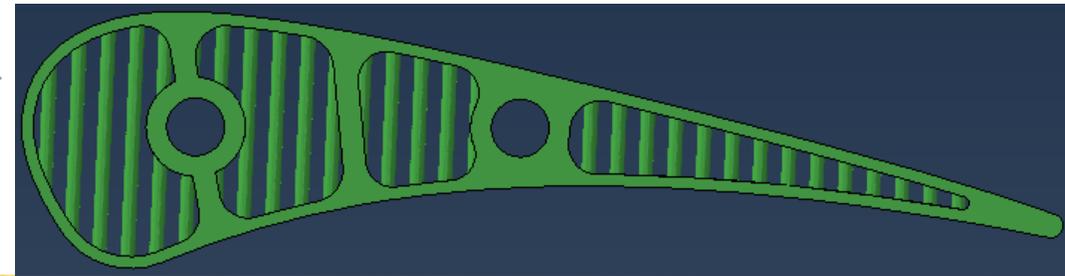
## Baseline Mechanical Model

- Used to predict wall thickness to peak stress relationship for limiting thickness in axisymmetric model
- Demonstrated that thermal stresses were negligible compared with mechanical stresses → optimize mechanical and thermal sequentially

Convection coefficient + spacing/radius + limitations on printing



## Optimized Vane Model

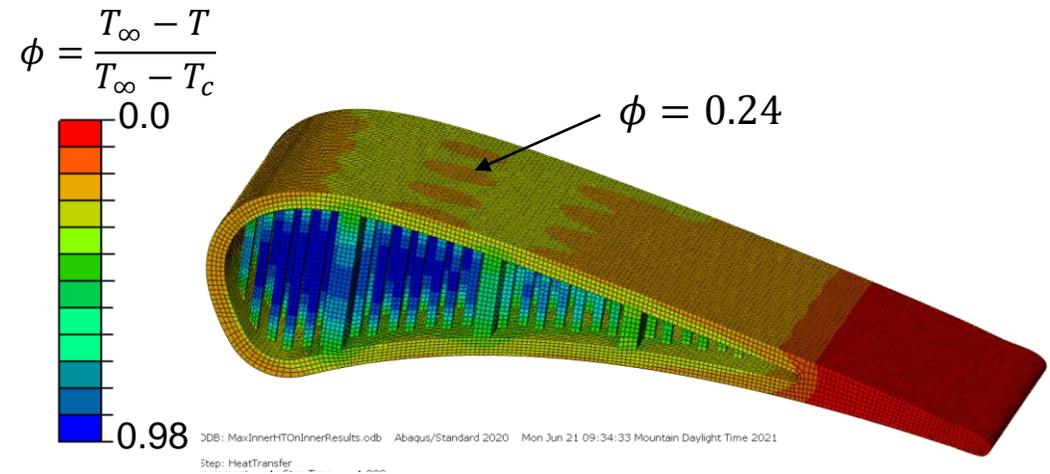
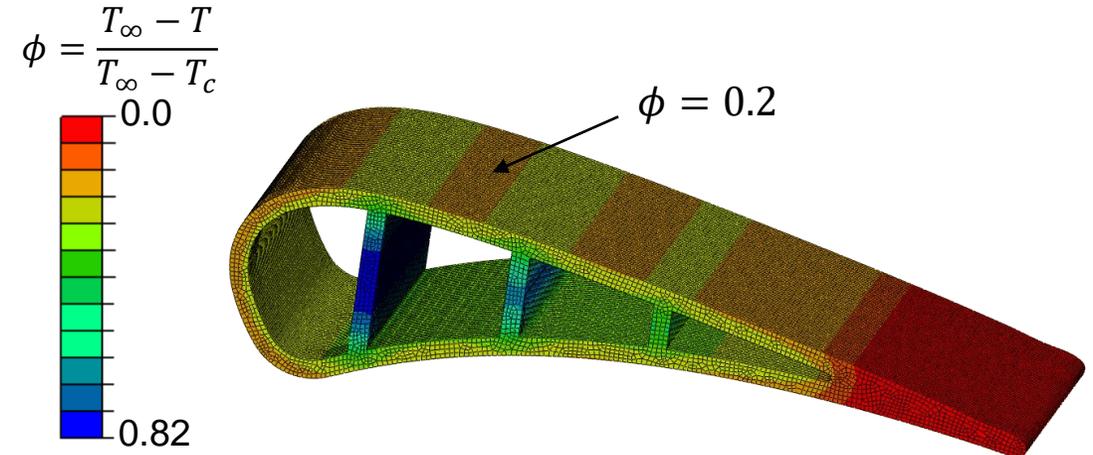
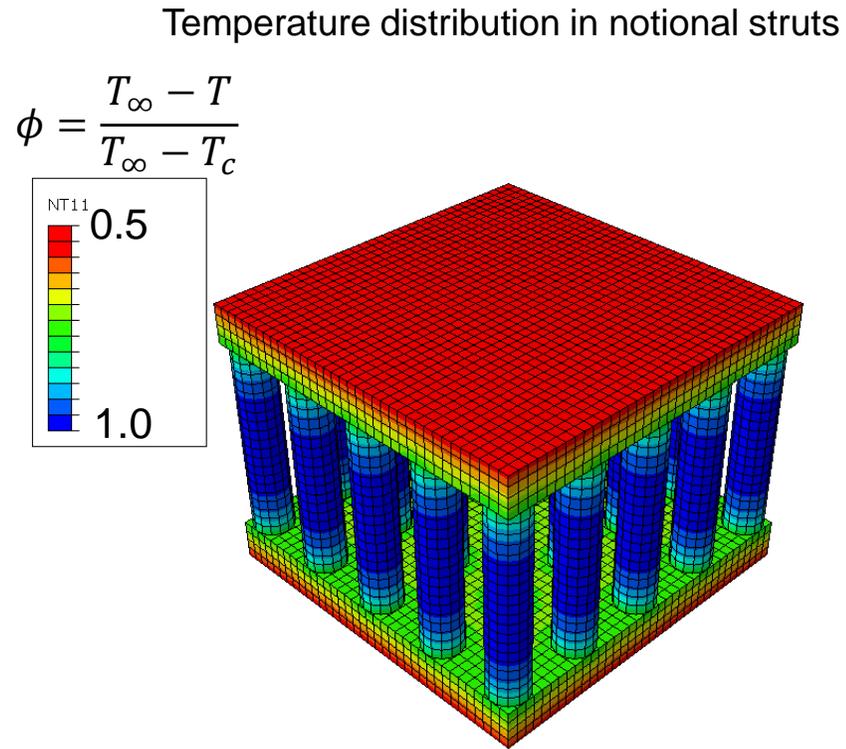


## Axisymmetric Thermal Model

- Wall thickness limited by mechanical stress
- Optimized pin fin parameters (spacing, radius, length) using convex methods
- Utilized correlations for convection coefficients for pin arrays

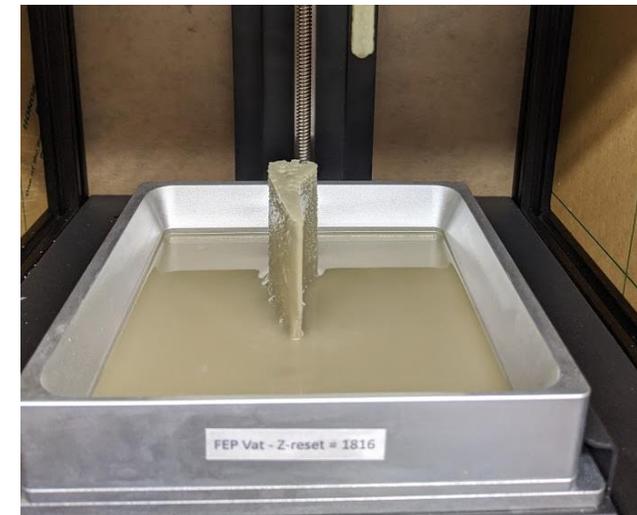
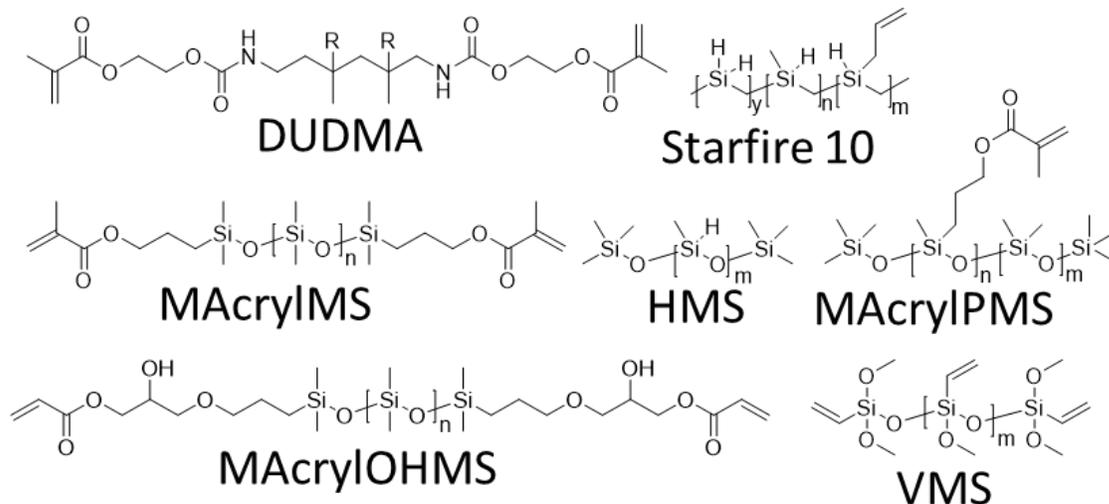


# Task 3.2: A design with multiple internal struts increases the vane outer surface overall cooling effectiveness nearly to the success criteria target of $\Delta\phi=+0.05$



# Task 4.1: Several new resin formulations were developed using acrylate siloxanes (SiOC precursor) to attempt to increase ceramic yield and decrease shrinkage

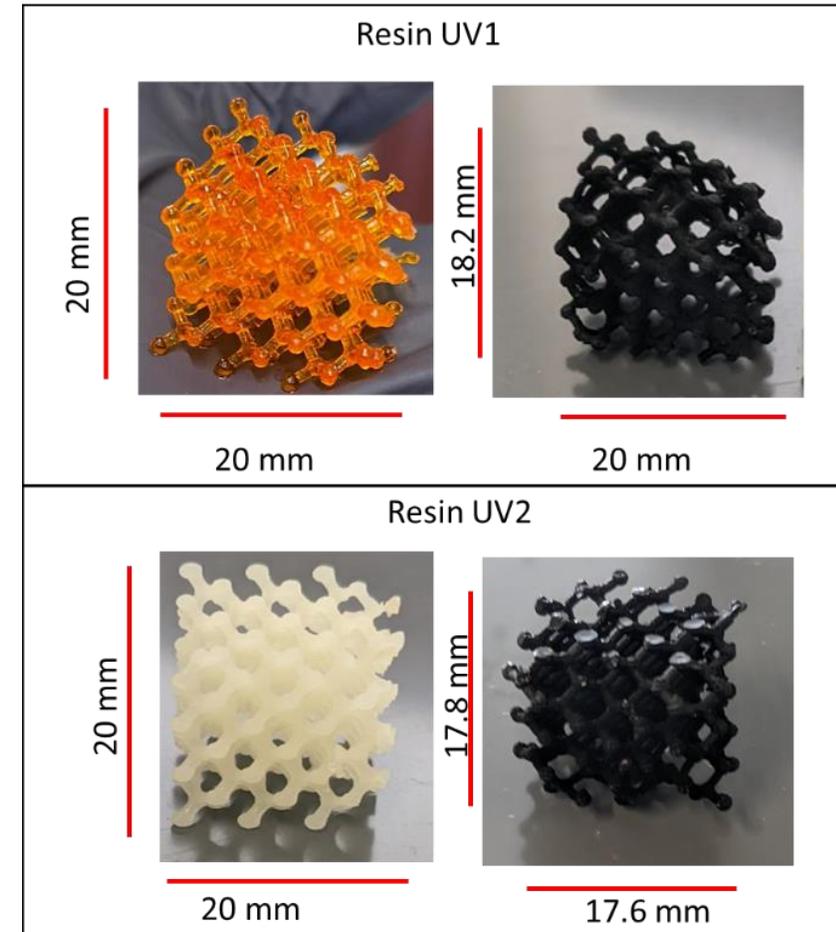
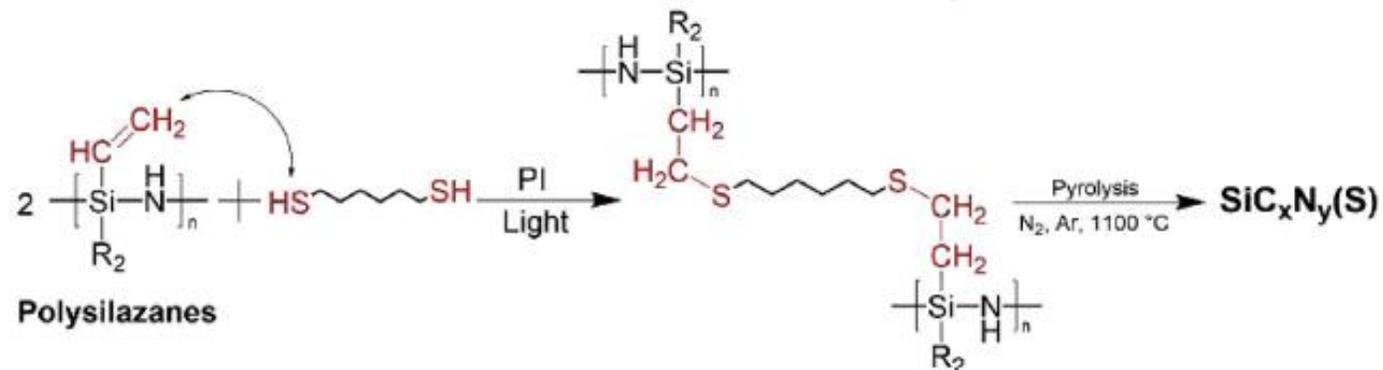
| Resin 10                 | wt%         | Resin 16                 | wt%         | Resin 28                 | wt%         |
|--------------------------|-------------|--------------------------|-------------|--------------------------|-------------|
| VMS                      | 38.6        | VMS                      | 35.71       | Starfire 10              | 50          |
| MAcrylPMS                | 40          | HMS                      | 35.71       | DUDMA                    | 25          |
| AcrylOHMS                | 20          | AcrylOHMS                | 27.45       | PPGGDMA                  | 25          |
| PhTrimPO                 | 1           | PhTrimPO                 | 0.26        | PhTrimPO                 | 0.3         |
| TMTBOHB                  | 0.3         | TiCp2Cl2                 | 0.04        | TMTBOHB                  | 0.7         |
| Sic whiskers             | 0.1         | SiC Whisker              | 0.82        | TiCp2Cl2                 | 0.195       |
| <b>Average shrinkage</b> | <b>26.1</b> | <b>Average shrinkage</b> | <b>25.2</b> | <b>Average shrinkage</b> | <b>23.2</b> |
| <b>Ceramic yield</b>     | <b>39.7</b> | <b>Ceramic yield</b>     | <b>48.4</b> | <b>Ceramic yield</b>     | <b>48.3</b> |



# Task 4.1: We also investigated Durazane (polysilazane, SiN precursor) with potential for higher ceramic yield and reduced shrinkage

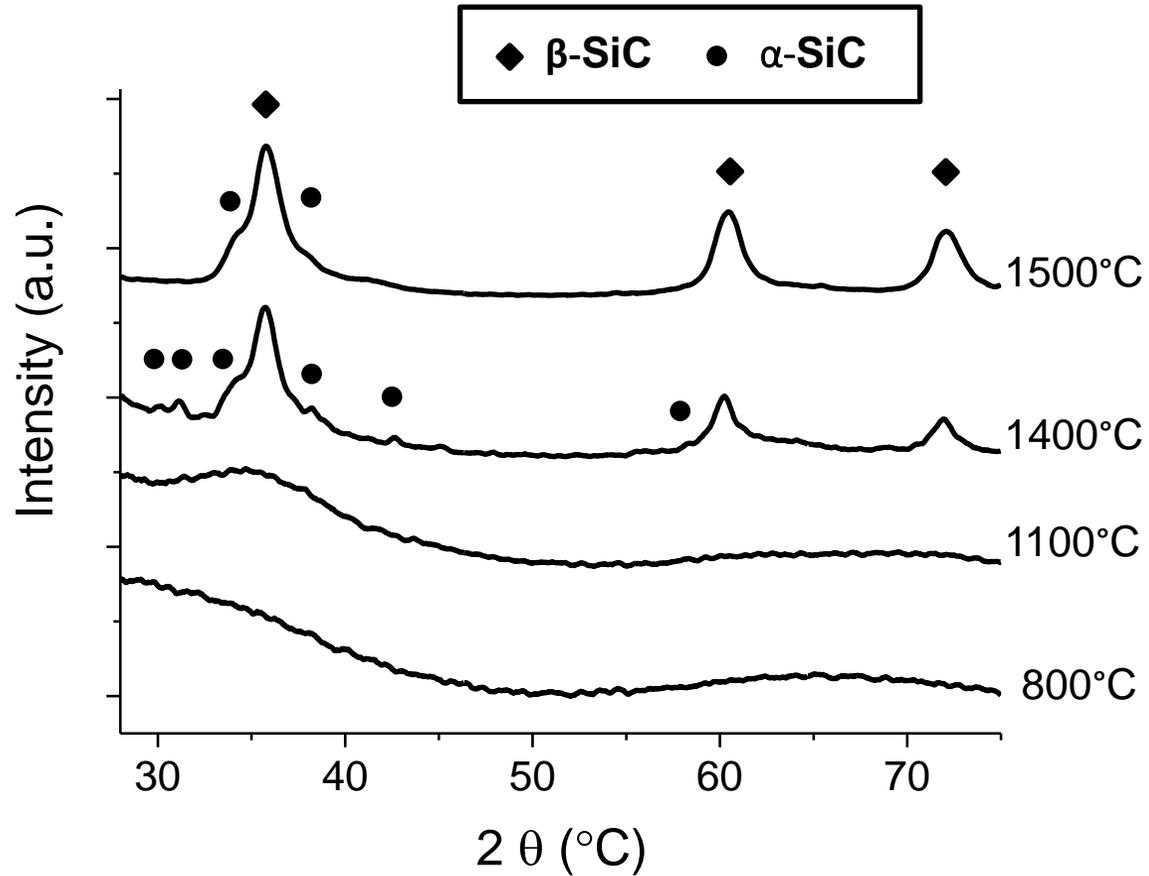
| Base resin | Crosslinkers | Amount of crosslinker | Expected composition | Shrinkage % (volumetric) | Ceramic yield % |
|------------|--------------|-----------------------|----------------------|--------------------------|-----------------|
| Durazane   | Hexandithiol | 10%                   | SiCN                 | 15-17                    | ~ 50-55         |
|            | Tri thiol    | 12%                   | SiCN                 | 22-24                    | ~ 60            |
|            | Tetrathiol   | 10%                   | SiCN                 | 20-25                    | ~ 60            |
|            | DUDMA        | 20%                   | SiOC                 | 30-35                    | ~ 32-35         |

Click reaction for crosslinking Durazane with hexandithiol provides best materials to date (~15% shrinkage, vs >35%)

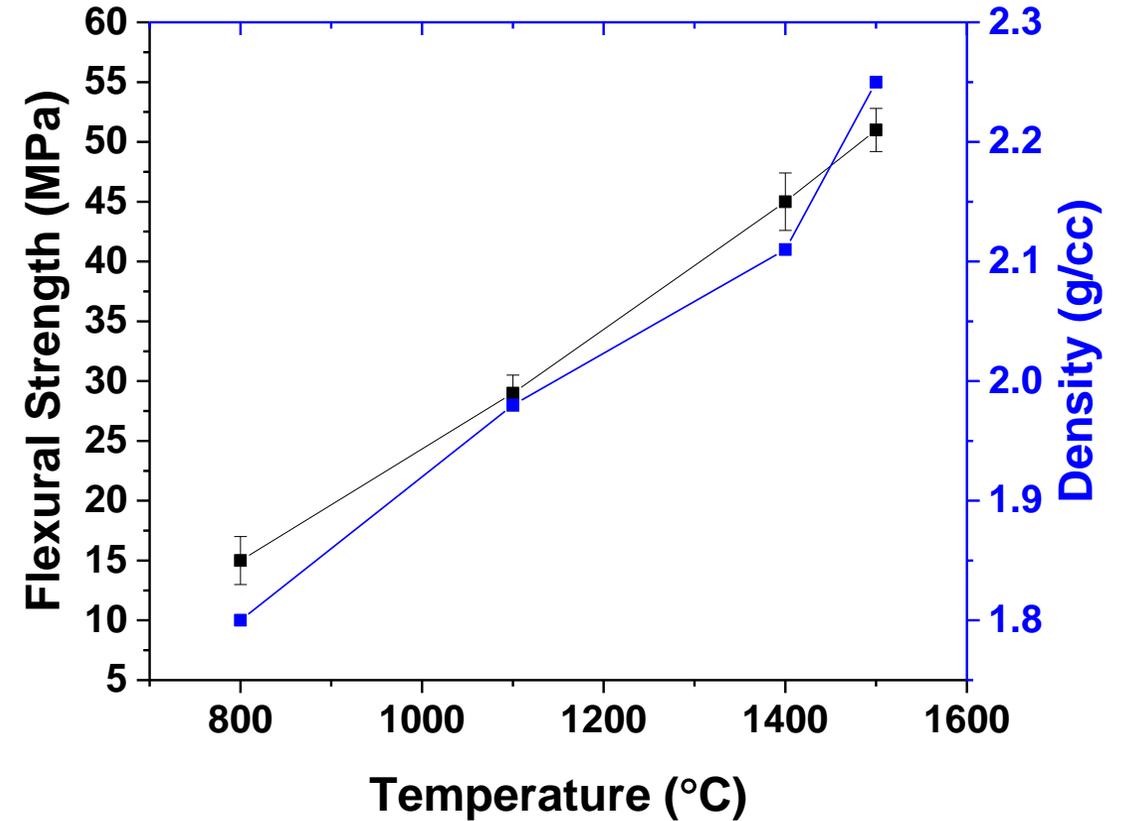


# Task 4.2: We have started to investigate the effects of higher temperature pyrolysis on chemistry and mechanical strength

X-Ray Diffraction (peaks indicate crystalline phases of SiC)

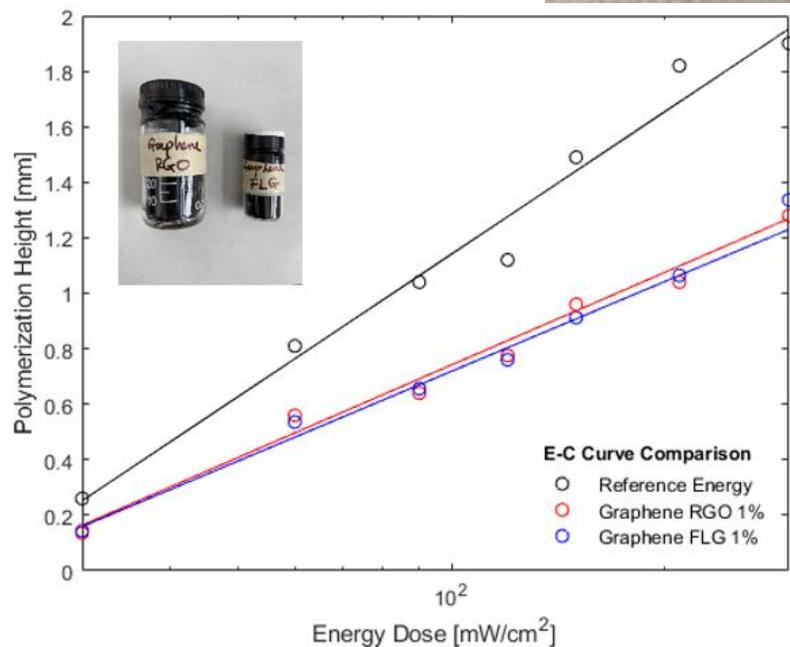
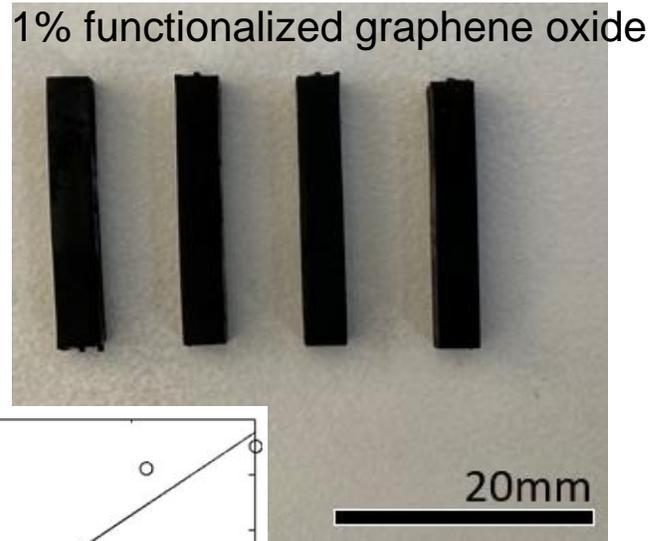
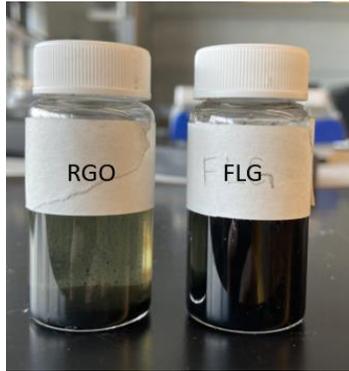


Flexural strength of bars under 3 point bend



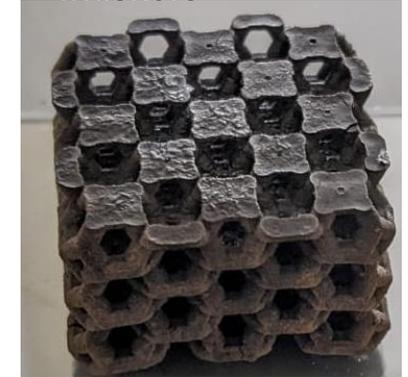
# Task 4.2: High weight percent of additives have been investigated for PDC formulations but they do decrease feature resolution and influence the UV curing process

## Polysiloxane formulation (SiOC precursor)



## Durazane formulation (SiN precursor)

5% silicon nitride whiskers



0% silicon nitride whiskers



10% silicon nitride whiskers



# Task 5: We have fabricated vanes using new PDC chemistries but have some limitations on printer/furnace size available, preventing us from making full size parts in all formulations

Durazane (SiN precursor)

Green body

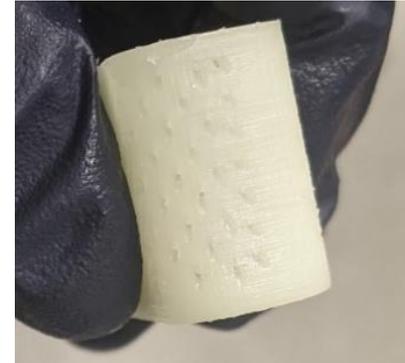


After pyrolysis

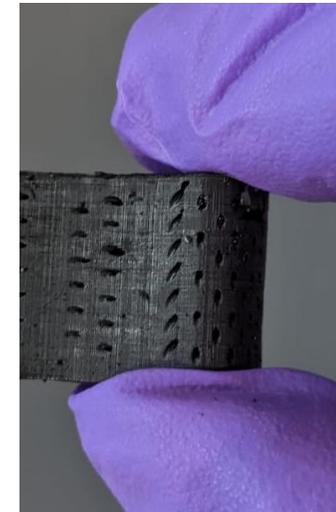


Starfire (SiC precursor)

Green body

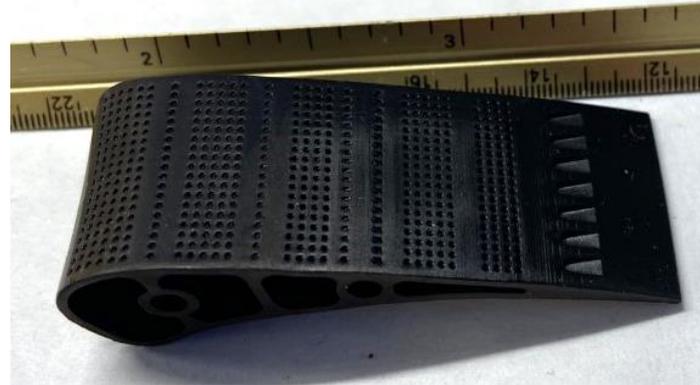
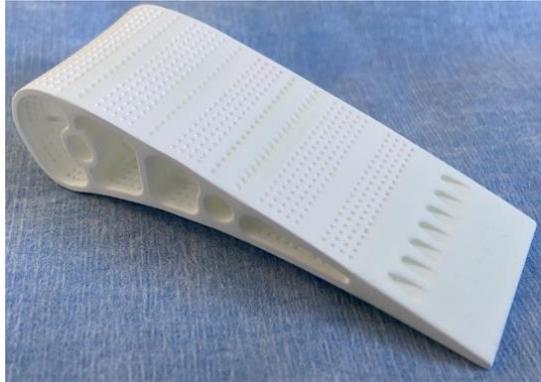


After pyrolysis

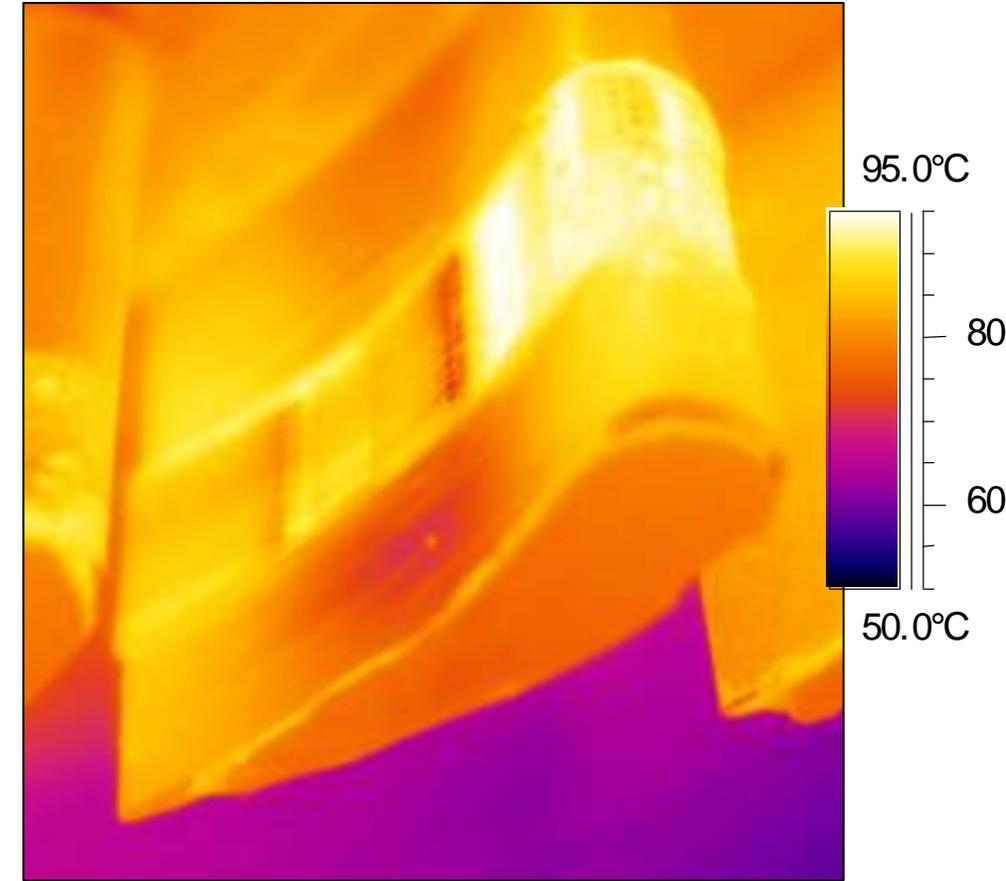


# Task 6.1: Advanced cooling designs (full coverage film cooling, transpiration cooling) have been printed and have survived pyrolysis; are being tested now in the high speed cascade

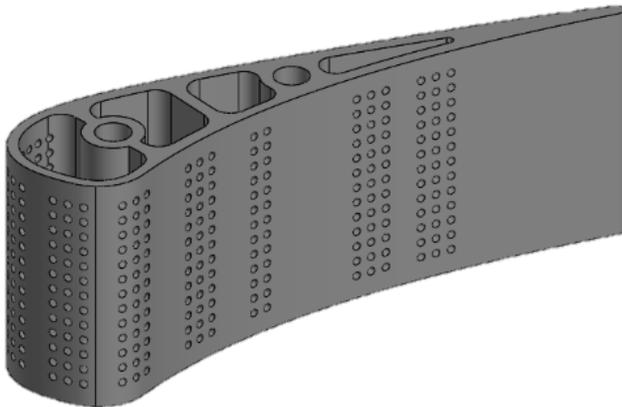
Advanced transpiration cooling design



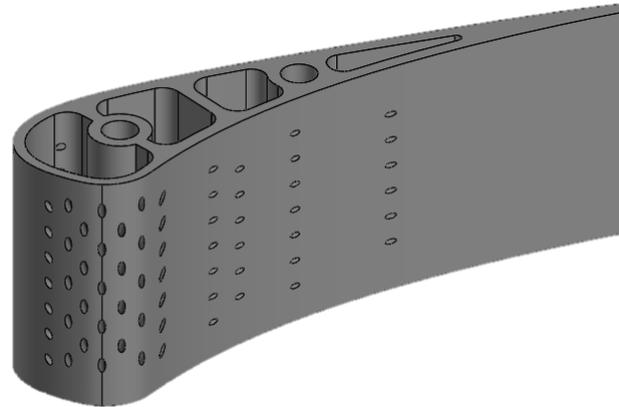
Advanced transpiration cooling design in high speed cascade



Transpiration and film cooled designs



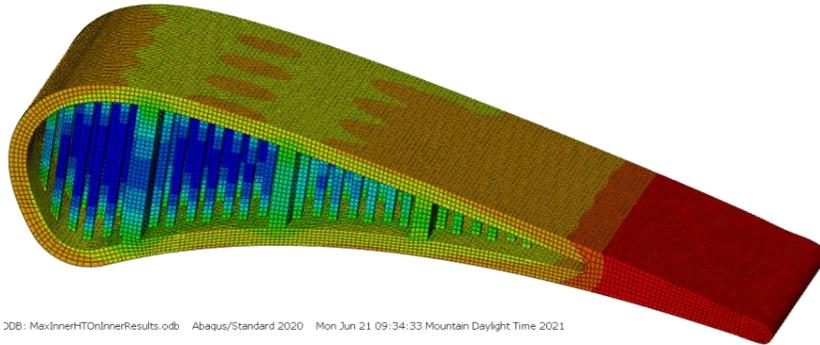
Based on design presented by Min, Huang, Parbat, Yang, & Chyu, J. Turbomach. 2019



Adapted from design presented by Dyson McClintic, Bogard, & Bradshaw, ASME Paper GT2013-94928

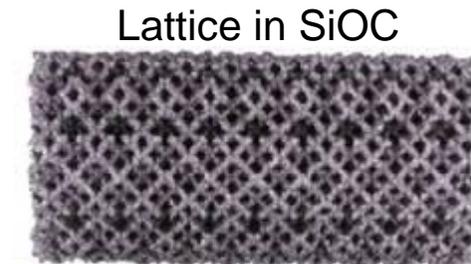
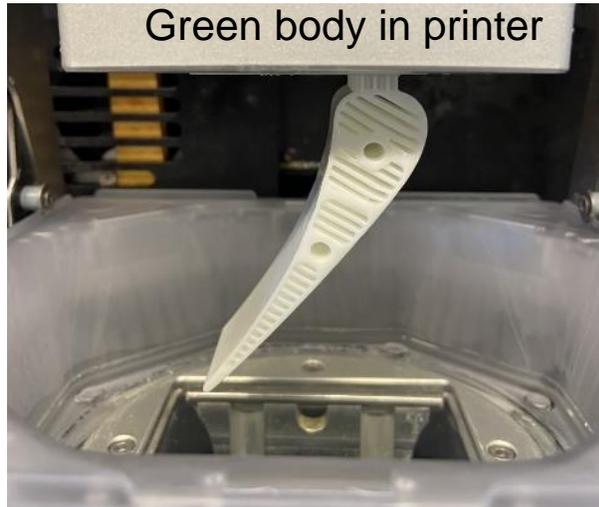
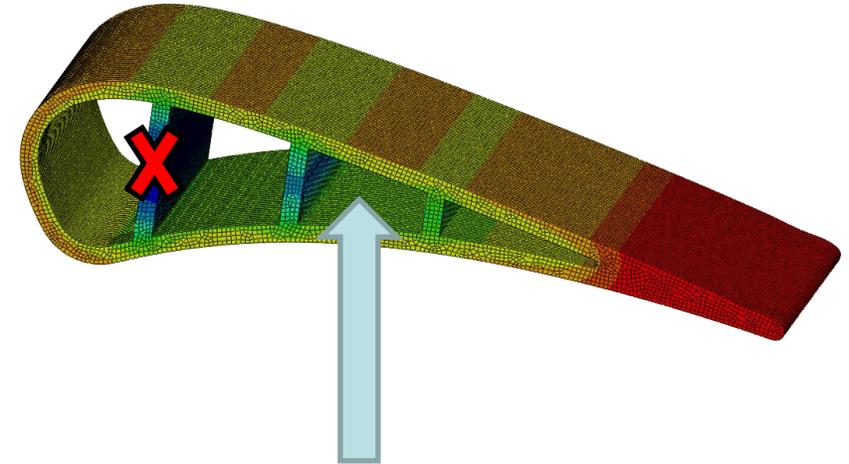
# Task 6.1: We have started to fabricate the first thermo-mechanically optimized geometry but want to try more designs

First optimized design: thin outer wall, high pin density



JDB: MaxInnerHTOnInnerResults.odb Abaqus/Standard 2020 Mon Jun 21 09:34:33 Mountain Daylight Time 2021  
Step: HeatTransfer

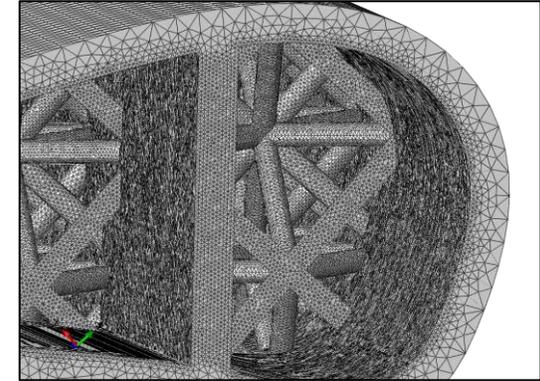
Future design: add internal lattice, remove or simplify internal struts



# We have a one-year extension with two tasks to incorporate more advanced designs in the processes developed here

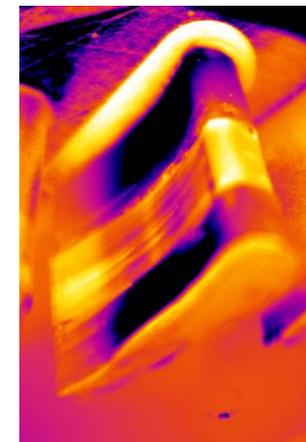
## Task 7 (UWYO):

- Design lattice-based internal cooling scheme that further improves cooling effectiveness and structural rigidity
- Evaluate using FEA code at PSU test conditions AND at representative gas turbine conditions



## Task 8 (UWYO/PSU):

- Fabricate and test lattice based design, compare to prior designs



Color key: process engineering, material development, modeling, validation of cooling technologies

# To date, several students on the project have received graduate degrees and we have disseminated work through conferences, company reviews, and invited seminars

- Publications to date:
  - Rusted & Lynch, “Determining Total Pressure Fields from Velocimetry Measurements in a Transonic Turbine Flowfield”, ASME Paper GT2021-59388
- Publications under review:
  - Brinckmann, Young, Fertig, Frick, “Effect of Print Direction on Mechanical Properties of 3D Printed Polymer-Derived Ceramics and their Precursors”, Additive Manufacturing Letters
- Students graduated to date:
  - PSU: Yifan Deng (PhD), Alex Rusted (MS)
  - UWYO: Jackson Rambough (MS), Stefan Brinckmann (PhD), Jennifer Hankins (MS)
- Other outreach:
  - PSU
    - Invited presentation (virtual) for New Mexico State University Graduate Seminar
    - American Institute for Aeronautics & Astronautics: “Frontiers in Gas Turbine Technology” seminar
  - UWYO
    - Interaction with NETL (Dr. Christopher Matranga) on Graphene oxide additives for PDCs

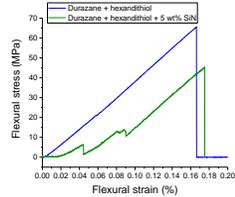
# The project has several milestones that are being tracked for progress

| Task/<br>Subtask | Milestone Title & Description   | Planned<br>Completion | Actual<br>Completion |
|------------------|---|-----------------------|----------------------|
| 2.1              | M1: Baseline cooled vane design complete                                | Feb 2020              | Feb 2020             |
| 2.2              | M2: Fabrication of baseline cooled SiOC/SiC vane complete               | Dec 2020              | Dec 2020             |
| 3.1              | M3: Topology optimization code for mechanical loading complete          | Dec 2020              | Dec 2020             |
| 4.1              | M4: First generation SiC precursor chemistry delivered from PSU to UWYO | Dec 2020              | Dec 2020             |
| 3.2              | M5: Topology optimization code for thermal stress/cooling complete      | Aug 2021              | Oct 2021             |
| 4.2              | M6: Detailed property characterization for SiC precursor complete       | Aug 2021              | Oct 2021             |
| 5.2              | M7: Fabrication of baseline cooled vane with new chemistry complete     | Aug 2021              | Oct 2021             |
| 5.3              | M8: Comparison of SiOC/SiC to SiC/SiC vane temperatures                 | Nov 2021              |                      |
| 6.2              | M9: Comparison of baseline SiC/SiC vane temperature to optimized design | Aug 2022              |                      |

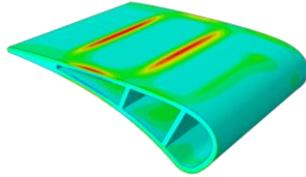
Color key: process engineering, material development, modeling, validation of cooling technologies



# In conclusion, we have demonstrated resin and manufacturing procedures to fabricate ceramic airfoils with complex internal cooling features



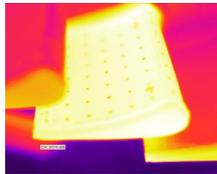
Development of resin improvements and post-processing steps has improved mechanical properties, ceramic yield, and pyrolysis survivability



Thermomechanical optimization has indicated we can achieve target improvements in cooling over baseline levels



Complex geometries including transpiration and film cooled configurations are possible with the AM PDC processes developed



Evaluation of ceramic airfoils in a high speed environment will help understand the cooling potential of novel AM ceramic designs

## Questions?

# Appendix



# The project schedule is designed to produce improved airfoil cooling geometries and chemistries at regular intervals throughout

|   | 2019   |    | 2020 |    |        | 2021 |    |    |        | 2022 |     |     |    |
|---|--------|----|------|----|--------|------|----|----|--------|------|-----|-----|----|
|   | Q3     | Q4 | Q1   | Q2 | Q3     | Q4   | Q1 | Q2 | Q3     | Q4   | Q1  | Q2  | Q3 |
|   | Year 1 |    |      |    | Year 2 |      |    |    | Year 3 |      |     |     |    |
|   | Q1     | Q2 | Q3   | Q4 | Q5     | Q6   | Q7 | Q8 | Q9     | Q10  | Q11 | Q12 |    |
| <b>Task 1 - Project Management and Planning</b>                                     |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 1.1 -- Update project management plan, assess risk, assess project resources        |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 1.2 -- Planning meetings/monthly teleconferences                                    |        |    |      |    |        |      |    |    |        |      |     |     |    |
| <b>Task 2 – Fabrication and Testing of SiOC/SiC Prototype Vane</b>                  |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 2.1 -- Design of a baseline vane geometry   |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 2.2 – Process characterization and fabrication of baseline vane geometry            |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 2.3 -- Aerothermal testing of baseline vane   |        |    |      |    |        |      |    |    |        |      |     |     |    |
| <b>Task 3 – Modeling and Optimization of CMC Turbine Vane Designs</b>               |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 3.1 -- Topology and morphology optimization for enhanced mechanical resilience      |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 3.2 -- Topology and morphology optimization for cooling design                      |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 3.3 -- Coupled cooling and resilience optimization for vane design                  |        |    |      |    |        |      |    |    |        |      |     |     |    |
| <b>Task 4 – Precursor Development and Optimization of New Resins for SiC Matrix</b> |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 4.1 – Develop new precursor resins for advanced matrices                            |        |    |      |    |        |      |    |    |        |      |     |     |    |
| 4.2 – Characterization of materials in composites and during pyrolysis              |        |    |      |    |        |      |    |    |        |      |     |     |    |



# The project schedule is designed to produce improved airfoil cooling geometries and chemistries at regular intervals throughout

|  | 2019   |    | 2020 |    | 2021   |    |    |    | 2022   |     |     |     |    |
|--|--------|----|------|----|--------|----|----|----|--------|-----|-----|-----|----|
|  | Q3     | Q4 | Q1   | Q2 | Q3     | Q4 | Q1 | Q2 | Q3     | Q4  | Q1  | Q2  | Q3 |
|  | Year 1 |    |      |    | Year 2 |    |    |    | Year 3 |     |     |     |    |
|  | Q1     | Q2 | Q3   | Q4 | Q5     | Q6 | Q7 | Q8 | Q9     | Q10 | Q11 | Q12 |    |
| <b>Task 5 – Integrate SiC Precursor Chemistry into AM Process</b>        |        |    |      |    |        |    |    |    |        |     |     |     |    |
| 5.1 -- Engineering of AM process for SiC precursor                       |        |    |      |    |        |    |    |    |        |     |     |     |    |
| 5.2 – Fabrication of baseline test vane using SiC matrix                 |        |    |      |    |        |    |    |    |        |     |     |     |    |
| 5.3 -- Aerothermal testing of baseline vane with SiC/SiC                 |        |    |      |    |        |    |    |    |        |     |     |     |    |
| <b>Task 6 – Fabricate and Test Optimized Vane with SiC/SiC Chemistry</b> |        |    |      |    |        |    |    |    |        |     |     |     |    |
| 6.1 -- Fabricate thermomechanically optimized vane                       |        |    |      |    |        |    |    |    |        |     |     |     |    |
| 6.2 -- Perform aerothermal testing of optimized vane                     |        |    |      |    |        |    |    |    |        |     |     |     |    |

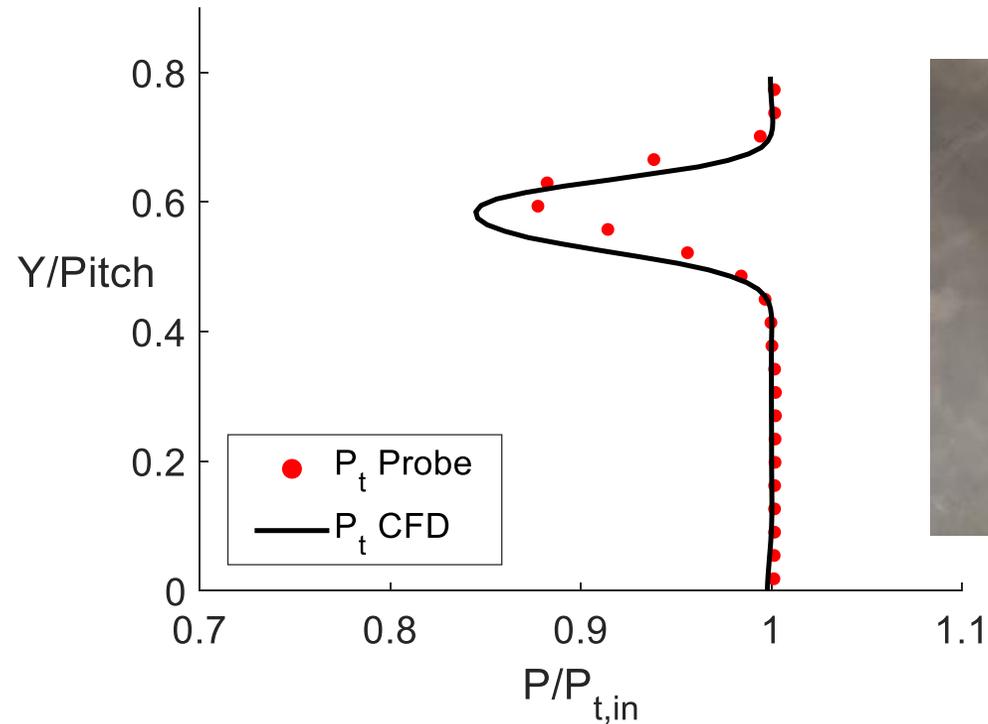
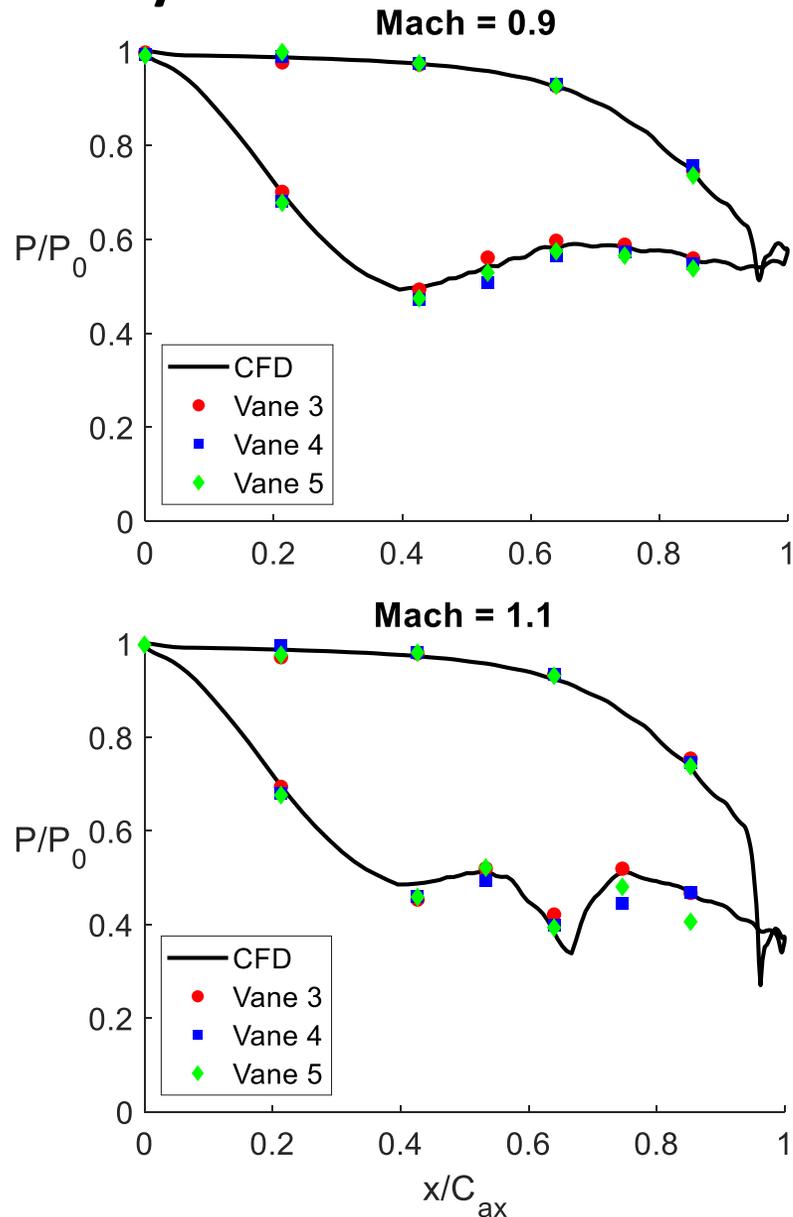
# We expect financial and schedule risks will be easily manageable with outlined research plan

| Perceived Risk  | Risk Rating      |        |         | Mitigation/Response Strategy   |
|---|------------------|--------|---------|--|
|   | Probability      | Impact | Overall |  |
|   | (Low, Med, High) |        |         |  |
| <b>Financial Risks:</b>   |                  |        |         |  |
| High cost for polymer precursor materials (Task 2, 4, 5, 6)                   | Low              | Med    | Low     | Fewer extra test parts will be produced and materials will be conserved.   |
| Cost of vane test hardware is high (Tasks 2.3, 5.3, 6.2)                      | Low              | Low    | Low     | Fewer extra test parts will be produced, or will be sourced through lower-cost vendors.  |
| <b>Schedule Risks:</b>  |                  |        |         |  |
| Unable to schedule time in transonic testing facility (Tasks 2.3, 5.3, 6.2)   | Low              | Low    | Low     | There is plenty of buffer in schedule to account for conflicts with other facility users   |
| Shape optimization tool development takes longer than planned (Task 3.1-3.3)  | Low              | Low    | Low     | Existing structural optimizers can be used as a backup, or a design can be developed manually and model can be validated with that design. |
| Novel SiC precursor material development takes longer than planned (Task 4.1) | Med              | Low    | Low     | Intermediate precursor recipes can be provided early to build expertise.   |

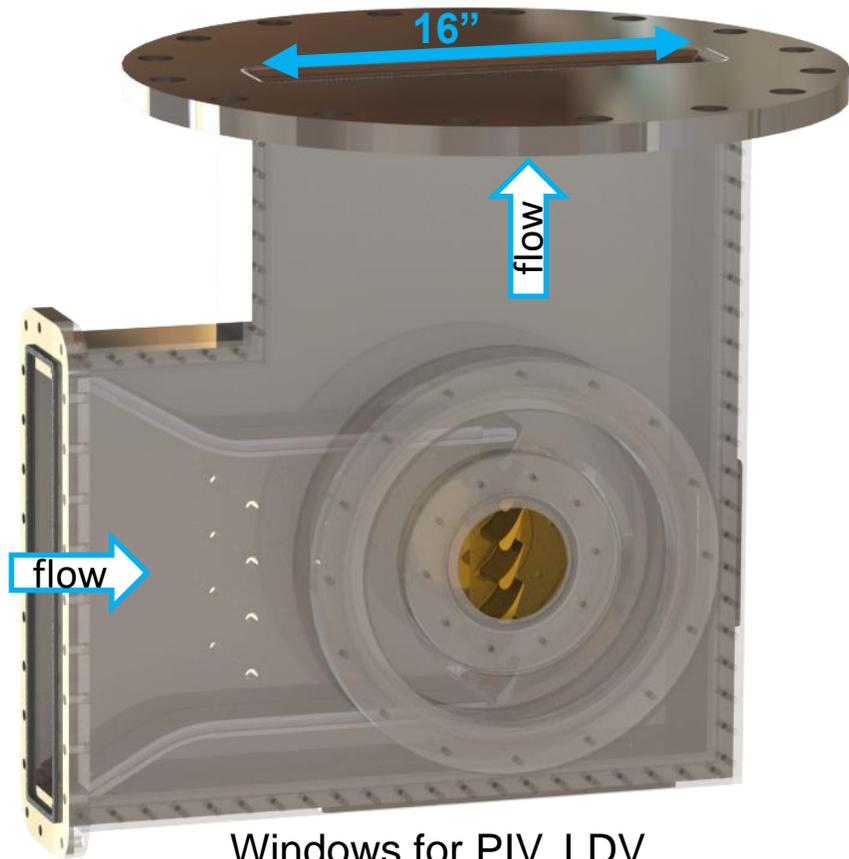
# There are some technical risks for this low-TRL project but high payoff if the technology is successful

| Perceived Risk   | Risk Rating      |        |         | Mitigation/Response Strategy   |
|--|------------------|--------|---------|--|
|  | Probability      | Impact | Overall |  |
|  | (Low, Med, High) |        |         |  |
| <b>Technical Risks:</b>  |                  |        |         |  |
| Unable to fabricate cooled SiOC CMC vane with large internal cavity (Task 2.2) | Low              | Med    | Med     | UWYO has fabricated turbine-like test parts previously, but if thin walls are an issue, can resort to thicker walls. If film cooling is a problem, can be removed from design without impact to technical goals. |
| Unable to integrate new SiC material into SLA process (Task 5.1)               | Low              | Med    | Med     | 3D printable variants of the SiC precursor formula would be investigated although may not have the same desired material capabilities of SiC.  |
| Unable to fabricate optimized vane shape using SiC/SiC (Task 6.1)              | Med              | Low    | Low     | This risk is avoided early by creating intermediate test prints as novel designs are developed, so that final design fits within manufacturing constraints.  |
| <b>Management, Planning, and Oversight Risks:</b>                              |                  |        |         |  |
| None   |                  |        |         | Organizational team is collaborative and all PI's have management experience.  |
| <b>ES&amp;H Risks:</b>   |                  |        |         |  |
| None   |                  |        |         | Co-PI's have expertise and facilities to handle volatile chemicals and hot furnace objects.  |
| <b>External Factor Risks:</b>  |                  |        |         |  |
| None identified  |                  |        |         |  |

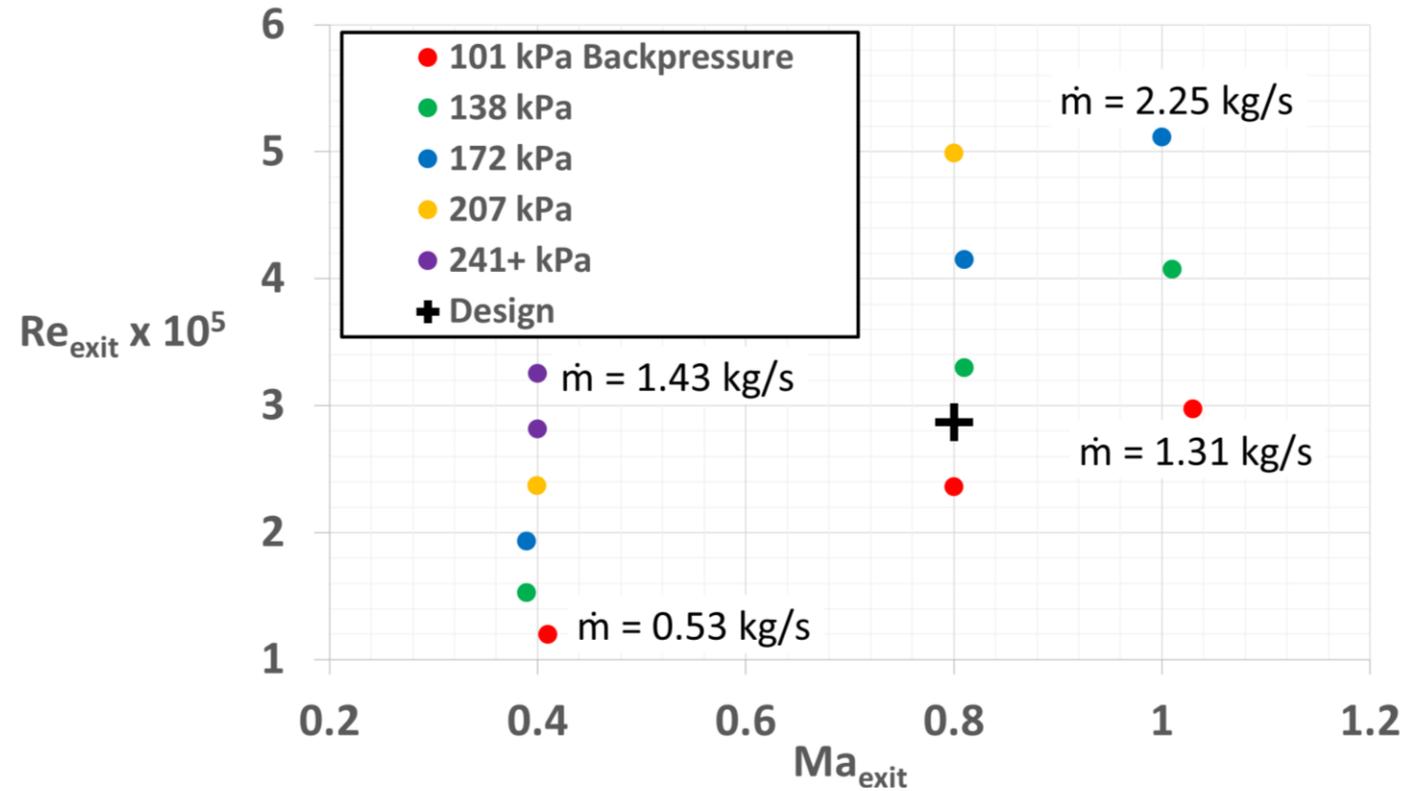
# Task 2.1: Aerodynamic measurements have been taken to benchmark the cascade and vane assembly without CMCs



# Task 2.1: The vane assembly will be tested in a new high speed cascade facility at Penn State capable of relevant Mach and Reynolds numbers, as well as cooled air capability



Windows for PIV, LDV, and IR measurements

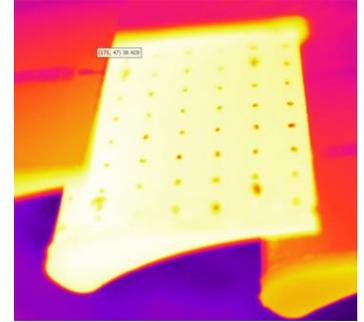
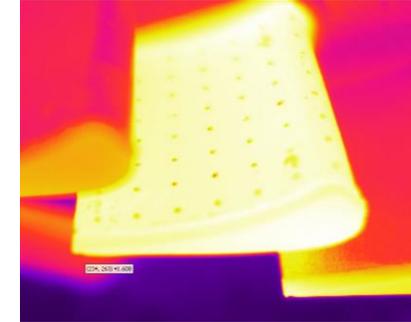
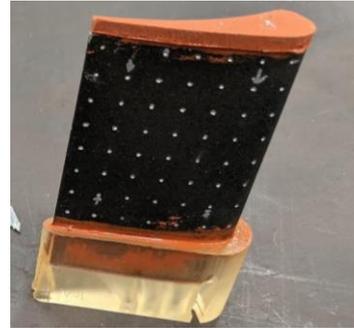


# Task 2.1: We have developed in-situ infrared image spatial/temperature calibration for cooling performance, as well as novel total pressure measurement for aerodynamic performance

Infrared camera in facility



Calibration vane with spatial markers visible in IR

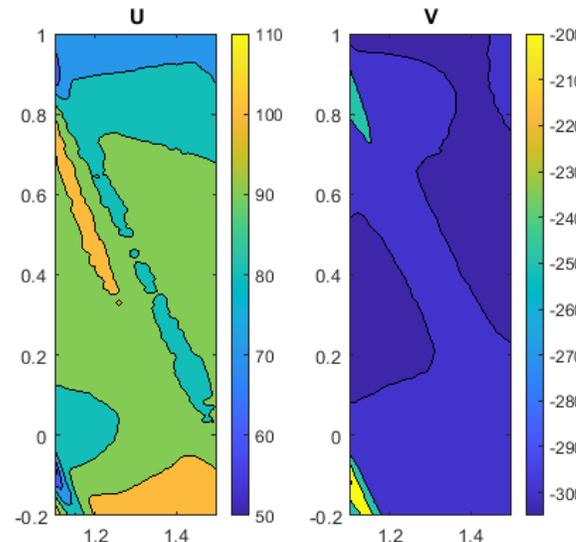


Momentum Equation

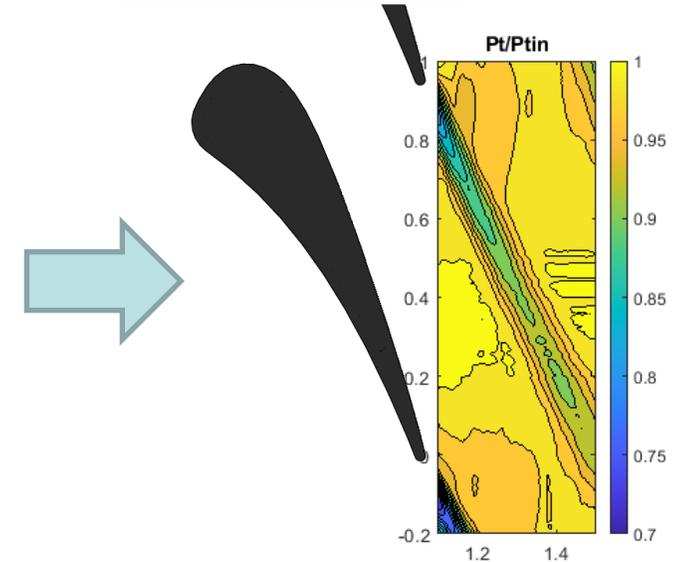
$$-\frac{\partial \bar{p}}{\partial x_i} = \rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \rho \frac{\partial \overline{u'_i u'_j}}{\partial x_j} - \mu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}$$

From measured velocity fields (PIV)

Measured velocity fields



Total pressure field



**Independently varying each coolant channel's Reynold's number shows a divergence in cooling effectiveness values as the difference between them increases.**

