Outline

Motivation

Research Objectives

Systematic Approach to Assessment

Results and Accomplishments

Summary
Motivation

• Recent drives to increase the efficiency of existing fossil energy (FE) power plants and the development of Advanced Ultrasupercritical (A-USC) power plants, have led to designs with steam pressures above 4000 psi and temperatures exceeding 1400°F.
Motivation

• The existing FE fleet has an average age of 40 years.
• The Department of Energy has outlined a strategy of life extension for US coal-fired power plants where many plants will operate for up to 30 additional years of service.

In Service Hours….
30 Years = 262,974 hours
40 Years = 350,634 hours
70 Years = 613,607 hours

Uncertainty ↑
Temperature ↑
Stress ↓

Creep-Rupture of 9Cr-1Mo Tube
300,000 hours
During Life Assessment, the integrity of components is assessed and the remaining service life estimated.

Deterioration of Component
- Creep
- Fatigue
- Creep-Fatigue
- Embrittlement & SCC
- Corrosion
- Erosion
- Wear
- Performance (HR, Output)

Change of Operating Circumstances
- Expected operation in future
- Decision support system
- Prolongation of overhaul interval
- Reduction of operating cost

Daily Operation Results
- Overhaul Inspection Results
- Life Assessment
- Performance Assessment
- Economical Assessment

Planning Modernization & Upgrading Program
- Repair
- Replacement
- Refurbishment
- Re-Powering
- Etc.

Based on Mitsubishi’s Life Extension Program
Motivation

• An immense number of models have been developed to predict the deformation, damage evolution, and rupture of structural alloys subjected to Creep and Creep-Fatigue.

There are Many More!…
Research Objectives

• Of primary concern to FE practitioners is a determination of which constitutive models are the “best”, capable of reproducing the mechanisms expected in an intended design accurately; as well as what experimental datasets are proper or “best” to use for fitting the constitutive parameters needed for the model(s) of interest.

RO1

Development of Aggregated Experimental Databases of Creep and Creep-Fatigue Data

RO2

Computational Validation and Assessment of Creep and Creep-Fatigue Constitutive Models for Standard and Non-Standard Loading Conditions
The Team

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Jack F Chessa, Project Co-PI
Systematic Approach to Assessment

Example for Creep Deformation

Aggregate Datasets with Uncertainty

Interpolation & Extrapolation

Model Fit to Datasets

Analytical Fit

Global Optimization

MACHO

Material Constant Heuristic Optimization

Performance

Model Uncertainty

$NSME, Z_{CRMS}$
Systematic Approach to Assessment

Task 1: Locate, Digitize, Sort, Store Experimental Data

Task 2: Uncertainty and Integrity of Experimental Database

Task 3: Mathematical Analysis and FEA of the Models

Task 4: Calibration & Validation – Fit, Interpolation, Extrapolation of the Models

Task 5: Post-Audit Validation of the Models

Task 6: Disparate Data problem and Design Maps

Task 7: Metamodeling: Finding the “best” model

Task 8: Multiaxial Representative Function
Task 1: Locate, Digitize, Sort, and Store Data

Creep Data
Creep-rupture
Minimum creep strain rate
Time to creep strain
Creep deformation
Stress relaxation

Creep-Fatigue Data
Tensile Hold Tests

Ideal creep rupture curve

Ideal minimum creep strain rate curve

Ideal creep deformation and damage evolution
Task 1: Locate, Digitize, Sort, and Store Data
### Task 1: Locate, Digitize, Sort, and Store Data

#### Table 3 – Data collected by data source

<table>
<thead>
<tr>
<th>Source</th>
<th>Creep Deformation</th>
<th>Stress Relaxation</th>
<th>Min. Strain Rate</th>
<th>Time to Cr. Strain</th>
<th>Creep Rupture</th>
<th>Mono. Tensile</th>
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Task 2: Uncertainty Analysis (Data Pre-processing)

NMSE reduced from 164.25 to 16.65.
Task 2: Uncertainty Analysis (Metadata)

- **Metadata** include: form, thermomechanical processing, source, chemistry, geometry, laboratory code, etc.

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<th>Data points</th>
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<tr>
<td>Bar and Plate</td>
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<tr>
<td>Pipe</td>
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<td>Plate</td>
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<td>Tube</td>
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<td>Hot extruded</td>
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<td>Hot extruded and cold drawn</td>
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<td>Hot rolled</td>
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<td>Quenched</td>
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<tr>
<td>Rotary pierced and cold drawn</td>
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<table>
<thead>
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<th>Source</th>
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<td>NIMS online database</td>
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Data parsing

$$Z = t_{r,exp} - t_{r,LMP}$$

$$\log(Z) = \log(P_{exp}) - \log(P_{LMP})$$

<table>
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<tr>
<th>Form</th>
<th>Rupture time (hrs)</th>
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<td>Bar</td>
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<tr>
<td>Plate</td>
<td>2.57E7</td>
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<tr>
<td>Tube</td>
<td>28.2E7</td>
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</table>
Data parsing

<table>
<thead>
<tr>
<th>Rupture Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(100MPa, 500°C)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TMP</th>
<th>Rupture time (hrs)</th>
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</thead>
<tbody>
<tr>
<td>HE &amp; CD</td>
<td>14.5E7</td>
</tr>
<tr>
<td>HR</td>
<td>5.05E7</td>
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<tr>
<td>Quenched</td>
<td>105E9</td>
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</table>
Figure 1 – Rupture prediction of LM, MS, and CD against (a) 50% data cull between $t_r, \text{max}/10$ and the longest experimental time and (b) 10% data cull from the lowest stress data.
Task 3: Mathematical Analysis

Stress-Rupture
- Eight commonly used TTP models
- Four newly developed model

Creep-deformation
- Omega model
- Theta Projection

Minimum Strain rate
- Norton Power Law
- McVetty Law
- Four additional model

Continuum Damage Mechanics
- Kachanov-Rabotnov
- Sin-hyperbolic
- Liu-Murakami

Multiaxial Representative Stress Functions
- Hayhurst
- Huddleston
- Additional five model
Rupture, Deformation, and Steady-state models

Table: Creep-Rupture Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Year</th>
<th>Parametric equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larson-Miller</td>
<td>1952</td>
<td>$P_{LMD} = T(\log(t_r) + t_s)$</td>
</tr>
<tr>
<td>Manson-Haferd</td>
<td>1953</td>
<td>$P_{MH} = \frac{\log(t_r) - \log(t_a)}{T - T_a}$</td>
</tr>
<tr>
<td>Manson-Brown</td>
<td>1953</td>
<td>$P_{MB} = \frac{\log(t_r) - \log(t_a)}{(T - T_a)^n}$</td>
</tr>
<tr>
<td>Orr-Sherby-Dorn</td>
<td>1954</td>
<td>$P_{OD} = \log(t_r) - Q/RT$</td>
</tr>
<tr>
<td>Manson-Snecup</td>
<td>1959</td>
<td>$P_{MS} = \log(t_r) - BT$</td>
</tr>
<tr>
<td>Graham-Wallog</td>
<td>1955</td>
<td>$P_{GW} = \log(t_r) (T - T_a)^n$</td>
</tr>
<tr>
<td>Chity-Duval</td>
<td>1963</td>
<td>$P_{CD} = mT - \log(t_r)$</td>
</tr>
<tr>
<td>Goldhoff-Sherby</td>
<td>1968</td>
<td>$P_{GS} = \frac{\log(t_r) - \log(t_a)}{1/T - 1/T_a}$</td>
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<tr>
<td>Modified Manson-Haferd</td>
<td>--</td>
<td>$P_{MHD} = \frac{\log(t_r) - \log(t_a)}{T}$</td>
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<tr>
<td>Modified Graham-Wallog</td>
<td>--</td>
<td>$P_{GWM} = \frac{\log(t_r)}{(1/T - 1/T_a)^n}$</td>
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<tr>
<td>Modified Chity-Duval</td>
<td>--</td>
<td>$P_{CDM} = mT - \log(t_r)$</td>
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<tr>
<td>Modified Goldhoff-Sherby</td>
<td>--</td>
<td>$P_{GSF} = \frac{\log(t_r) - \log(t_a)}{(1/T - 1/T_a)^n}$</td>
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Table: Creep deformation

<table>
<thead>
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<th>Omega model</th>
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<tbody>
<tr>
<td>$\varepsilon = \theta_1 [1 - \exp(-\theta_2 t)]$</td>
</tr>
<tr>
<td>$+ \theta_3 [\exp(\theta_4 t) - 1]$</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_t = \theta_3 \theta_4 \exp(\theta_4 t)$</td>
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<table>
<thead>
<tr>
<th>Theta model</th>
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<tr>
<td>$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp(\varepsilon \Omega)$</td>
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<tr>
<td>$\omega = \frac{t}{t_r} = \frac{\dot{\varepsilon} \Omega t}{1 + \dot{\varepsilon} \Omega t}$</td>
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Table: Minimum-creep-strain-rate model

<table>
<thead>
<tr>
<th>Source</th>
<th>Creep law</th>
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<tbody>
<tr>
<td>Norton, 1929</td>
<td>$\dot{\varepsilon}_{cr} = A(\sigma / \sigma_0)^n$</td>
</tr>
<tr>
<td>Soderberg, 1936</td>
<td>$\dot{\varepsilon}_{cr} = A{\exp(\sigma / \sigma_0) - 1}$</td>
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<tr>
<td>McVetty, 1943</td>
<td>$\dot{\varepsilon}_{cr} = A\sinh(\sigma / \sigma_0)$</td>
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<td>Dorn, 1955</td>
<td>$\dot{\varepsilon}_{cr} = A\exp(\sigma / \sigma_0)$</td>
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<td>JHK, 1963</td>
<td>$\dot{\varepsilon}_{cr} = A_1(\sigma / \sigma_0)^n + A_2(\sigma / \sigma_0)^n$</td>
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<tr>
<td>Garofalo, 1965</td>
<td>$\dot{\varepsilon}_{cr} = A{\sinh(\sigma / \sigma_0)}_t^n$</td>
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Figure: Turbine blade inspection
CDM models

<table>
<thead>
<tr>
<th>Strain rate and Min. Strain rate</th>
<th>Damage Rate and Rupture Life</th>
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<tbody>
<tr>
<td><strong>Kachanov-Rabotnov (KR) model</strong></td>
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<tr>
<td>$\dot{\varepsilon}_{cr} = A \left( \frac{\sigma}{1-\omega} \right)^n$</td>
<td>$\dot{\omega} = \frac{M_K \sigma^x}{(1-\omega)^k}$</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{\min} = A\sigma^n$</td>
<td>$t_r = \left[ (\phi_K + 1)M_K \sigma^x \right]^{-1}$</td>
</tr>
<tr>
<td>$A, n = $ Norton power law constants</td>
<td>$M_K, \chi, \phi_K = $ tertiary creep damage constants</td>
</tr>
</tbody>
</table>

| **Liu-Murakami (LM) model** |                              |
| $\dot{\varepsilon}_{cr} = A\sigma^n \exp(\rho \omega^{3/2})$ | $\dot{\omega} = \frac{M_L \left[ 1 - \exp(\phi_L) \right]}{\phi_L} \sigma^q \exp(\phi_L \omega)$ |
| $\dot{\varepsilon}_{\min} = A\sigma^n$ | $t_r = \left[ M_L \sigma^q \right]^{-1}$ |
| $\rho = (2n + 2) / (\pi \sqrt{1 + 3/n})$ | $M_L, q, \phi_L = $ tertiary creep damage constants |

| **Sin-hyperbolic (Sinh) model** |                              |
| $\dot{\varepsilon} = B \sinh \left( \frac{\sigma}{\sigma_s} \right) \exp(\lambda \omega^{3/2})$ | $\dot{\omega} = \frac{M_S \left[ 1 - \exp(\phi_S) \right]}{\phi_S} \sinh \left( \frac{\sigma}{\sigma_i} \right) \exp(\phi_S \omega)$ |
| $\dot{\varepsilon}_{\min} = B \sinh \left( \frac{\sigma}{\sigma_s} \right)$ | $t_r = \left[ M_S \sinh \left( \frac{\sigma}{\sigma_i} \right) \right]^{-1}$ |
| $B, \sigma_s = $ secondary creep constants | $M_S, \sigma_i, \phi_S = $ tertiary creep damage constants |
| $\lambda = \ln \left( \dot{\varepsilon}_{final} / \dot{\varepsilon}_{\min} \right)$ |                              |
## Representative Stress Functions

<table>
<thead>
<tr>
<th>Author</th>
<th>Representative Stress Function</th>
<th>Dyson, Webster and Cane</th>
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<tbody>
<tr>
<td>Sodbyrev</td>
<td>( \sigma_{rep} = \alpha \sigma_1 + (1 - \alpha) \sigma_{vm}, \ 0 \leq \alpha \leq 1 )</td>
<td>( \sigma_{rep} = \left( \frac{\sigma_1}{\sigma_{VM}} \right)^{\gamma/v} \sigma_{VM}, \ 0 \leq \gamma \leq v )</td>
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<tr>
<td>Hydrostatic</td>
<td>( \sigma_{rep} = 3 \beta \sigma_m + (1 - \beta) \sigma_{vm}, \ 0 \leq \beta &lt; 1 )</td>
<td>Hydrostatic</td>
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<tr>
<td>Hayhurst</td>
<td>( \sigma_{rep} = \alpha \sigma_1 + 3 \beta \sigma_m + (1 - \alpha - \beta) \sigma_{vm} ) ( , \ 0 \leq \alpha + \beta \leq 1 )</td>
<td>Combined</td>
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<tr>
<td>Hydrostatic</td>
<td>( \sigma_{rep} = \left( \frac{3 \sigma_m}{\sigma_{VM}} \right)^{\gamma/v} \sigma_{VM}, \ 0 \leq \gamma &lt; v )</td>
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<tr>
<td>Combined</td>
<td>( \sigma_{rep} = \left( \frac{\sigma_1}{\sigma_{VM}} \right)^{\gamma/v} \left( \frac{3 \sigma_m}{\sigma_{VM}} \right)^{\mu/v} \sigma_{VM}, \ 0 \leq \gamma + \mu &lt; v )</td>
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### Huddleston

\[
\sigma_{rep} = \frac{3}{2} S_1 \left( \frac{2 \sigma_{VM}}{3 S_1} \right)^{\alpha} \exp \left[ b \cdot \left( \frac{J_1}{S_s} - 1 \right) \right]
\]

\[
S_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}, \ (S_1 = \sigma_1 - J_1 / 3), \ (J_1 = \sigma_1 + \sigma_2 + \sigma_3)
\]
Task 4: Global optimization software

**Input:**
- Model name
- Stress function-type
- Initial Guess constants

**Assessment:**
- NMSE (Normalized Mean Squared Error)
- Inflection point
- Physical realism

**Output:**
- Calibrated material constant.
- Stress parameter function.
- Most suitable model.

Other group are in Excel function form. Requires higher human involvement.

**MATLAB process flow chart**

- **Start**
  - Input: $\Delta T, \Delta \sigma, t_f, T_s, \sigma_s, t_s$
  - $T = T_{\text{min}}, \sigma = \sigma_{\text{min}}, t = 0$
  - $T = T_{\text{max}} \geq T$
  - Get material constants at $T$ and store at designated array
  - $\sigma_{\text{max}} \geq \sigma$
    - YES
  - Get $\dot{\varepsilon}_{\text{min}}$ and $t_f$ at $(T, \sigma)$. Store at designated array
  - $t_f \geq t$
    - YES
  - Get $\varepsilon$ and $\omega$ at $(T, \sigma)$. Store at designated array
  - $\sigma = \sigma + \sigma_s$
    - NO
  - $\omega = \omega_s$ and get $\varepsilon$ at $(T, \sigma)$. Store at designated array
  - $T = T + T_f$
  - $\sigma = \sigma + \sigma_s$
    - NO
  - $\sigma = \sigma + \sigma_s$
    - YES
  - $\omega = \omega_s$ and get $\varepsilon$ at $(T, \sigma)$. Store at designated array
  - $T = T + T_f$
  - $\sigma = \sigma + \sigma_s$
  - $\omega = \omega_s$
  - $\varepsilon = \varepsilon + \varepsilon$
  - $\omega = \omega + \omega$

- **Stop**
  - Output: $\dot{\varepsilon}_{\text{min}}, t_f, \varepsilon, \omega$

**Other equations and calculations**:
- Temperature-range: $\Delta T = T_{\text{max}} - T_{\text{min}}$
- Stress-range: $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$
- Target time: $t_f$
- Rupture time: $t_r = f(T, \sigma)$
- Temperature, Stress, and time step: $T_s, \sigma_s, t_s$
- Current Temperature, Stress and time: $T, \sigma, t$
- Minimum strain rate: $\dot{\varepsilon}_{\text{min}} = f(T, \sigma)$
- Material constants: $f(T)$
- Damage: $\omega$, Critical damage: $\omega_c$
- Strain: $\varepsilon$
- $X, Y = X$ and $Y$-coordinate array
- $\varepsilon$, $\sigma$, and $\omega$ (for each data point)
Analytic fit (Stress-rupture)

304 SS, MS model

Hastelloy X, Chitty-Duval

316 SS, MCD model
**Minimum Creep Strain Rate**

Norton Power Law (KR) \( \dot{\varepsilon}_{\text{min}} = A\sigma^n \)

McVetty (Sinh) \( \dot{\varepsilon}_c = B \sinh(\sigma/\sigma_s) \)

**KR limitations:** The KR minimum creep strain rate predictions are linear on a log-log scale and thus are not able to accurately model the sigmoidal behavior observed in the experimental data.

**Sinh advantage:** The Sinh minimum creep strain rate predictions bend on a log-log scale and are able to accurately model the sigmoidal behavior.

Comparison of Norton and McVetty model against 304 SS data
Comparison of various data model against P91 data
Creep deformation

Theta Projection (tertiary strain rate)
\[ \varepsilon = \theta_1[1 - \exp(-\theta_2 t)] + \theta_3[\exp(\theta_4 t) - 1] \]
\[ \dot{\varepsilon}_t = \theta_3 \theta_4 \exp(\theta_4 t) \]

Omega Model
\[ \dot{\varepsilon} = \dot{\varepsilon}_0 \exp(\varepsilon \Omega) \]
\[ \omega = \frac{t}{t_r} = \frac{\varepsilon \Omega t}{1 + \varepsilon \Omega t} \]

Sin-Hyperbolic Model
\[ \dot{\varepsilon} = A \sinh\left(\frac{\sigma}{\sigma_s}\right) \exp\left(\lambda \omega^{3/2}\right) \]
\[ \dot{\omega} = M \left[1 - \exp(-\phi)\right] \sinh\left(\frac{\sigma}{\sigma_t}\right) \exp(\phi \omega) \]

Ref: [4]
Comparison of Sinh and KR model against 304 SS creep strain data

Comparison of Sinh and KR model damage evolution against 304 SS analytic damage
Comparison of Sinh, LM and KR model against 316 SS creep strain data


**Task 5: Post-Audit Verification**

Post-audit validation with additional data that is not used in calibration

### Stress-Rupture

<table>
<thead>
<tr>
<th>Stress, $\sigma$ (MPa)</th>
<th>Rupture life, $t_r$ (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>80</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>10</td>
</tr>
<tr>
<td>500</td>
<td>10</td>
</tr>
<tr>
<td>600</td>
<td>10</td>
</tr>
</tbody>
</table>

### Minimum-creep-strain-rate

<table>
<thead>
<tr>
<th>Stress, $\sigma$ (MPa)</th>
<th>Minimum Creep Strain Rate, $\epsilon_{\text{min}}$ (%/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>10</td>
<td>$10^{0}$</td>
</tr>
<tr>
<td>25</td>
<td>$10^{1}$</td>
</tr>
<tr>
<td>50</td>
<td>$10^{2}$</td>
</tr>
<tr>
<td>100</td>
<td>$10^{3}$</td>
</tr>
<tr>
<td>200</td>
<td>$10^{4}$</td>
</tr>
<tr>
<td>300</td>
<td>$10^{5}$</td>
</tr>
<tr>
<td>400</td>
<td>$10^{6}$</td>
</tr>
<tr>
<td>500</td>
<td>$10^{7}$</td>
</tr>
<tr>
<td>600</td>
<td>$10^{8}$</td>
</tr>
</tbody>
</table>

![Stress-Rupture Graph](image1)

![Minimum-creep-strain-rate Graph](image2)
Task 6: Disparate Data problem

Gap exist in creep data.
Data is not available in the regime or form of interest.
Creep deformation extrapolation using short-term data may lead to unreliable prediction.
Cross-calibration to any and every form of data can improve extrapolation.

Data exists in diaspora
Unreliable extrapolation
Limited up to certain multiple of the longest rupture life,

\[
\begin{align*}
\tau &= 10^n \\
\sigma &= 3, 4, 5, 6, 8, 20, 30, 40, 50, 60, 80, 200, 300, 400, 500, 600, 1000 \\
\end{align*}
\]
Disparate Data problem: Improved Extrapolation

Calibration procedure for CDM models to disparate creep data

<table>
<thead>
<tr>
<th>Model Selection</th>
<th>Select the creep model of interest. (Kachanov-Rabotnov, Sinh, etc.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segregation</td>
<td>Segregate the model into equations related to each type of creep data. (Minimum-Creep-Strain-Rate, Stress-Rupture, Creep Deformation, etc.)</td>
</tr>
<tr>
<td>Calibration</td>
<td>Calibrate the material constants of the equations using the creep data.</td>
</tr>
<tr>
<td>Regression</td>
<td>Use regression analysis to convert material constants into temperature-dependent functions, $f_i(T)$</td>
</tr>
<tr>
<td>Validation</td>
<td>Take the pre-calibrated model and compare it to additional data not used in the calibration process.</td>
</tr>
<tr>
<td>Design</td>
<td>Make interpolative and extrapolative prediction of creep behavior. Plot these predictions as Creep Design Maps.</td>
</tr>
</tbody>
</table>
Development of a Design Map

- Tensile properties
- The variable of interest
- Design envelope
Extrapolation and Interpolation: Design Maps

Minimum-creep-strain-rate design maps

Mcvety law

Norton law

Temperature, $T$ (°C)

Stress, $\sigma$ (MPa)

0.2% Yield strength

UTS

$\dot{\varepsilon}_{\text{min}}$

$<10^{-6}$

$10^{-5}$

$10^{-4}$

$10^{-3}$

$10^{-2}$

$10^{-1}$

$10^{0}$

$10^{1}$

$10^{2}$

$>10^{2}$
Extrapolation and Interpolation: Design Maps

The area above the inflection lines represents unrealistic prediction zone.

Minimum-creep-strain-rate Inflection
Design Maps

Stress-Rupture design maps

(a) Kachanov-Rabotnov

(b) Sin-Hyperbolic

Temperature, $T$ (°C)

0.2% Yield strength

UTS
Design Maps

Deformation design maps

Damage design maps
Task 7: Metamodelling: Finding the “best” model

A “metamodel” can be described as a combinational model, derived from rearranging, modifying, and/or expanding the functional relationships between different models.

- Stress-Rupture
  - Eight commonly used TTP models
  - Four newly developed model

- Creep-deformation
  - Omega model
  - Theta Projection
  - Sin-hyperbolic model

- Continuum Damage Mechanics
  - Kachanov-Rabotnov
  - Sin-hyperbolic
  - Liu-Murakami

- A detailed explanation of stress-rupture metamodelling will be presented.
- Summary and progress of the other metamodelling group is reported.
### Metamodeling: Stress-Rupture

**Metamodel:**

\[ P_{RS} = \frac{\log(t_r) - \alpha_0 - \alpha_1 T'}{(T' - \alpha_2')^n} \]

**Parent model:**

- **Larson-Miller**

\[ P_{LMP} = T(\log(t_r) + t_a) \]

### New models

<table>
<thead>
<tr>
<th>Model</th>
<th>Year</th>
<th>Parametric equation</th>
<th>log-stress equation</th>
<th>Material constants</th>
<th>Characteristics</th>
<th>Metamodel condition</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larson-Miller</td>
<td>1952</td>
<td>( P_{LMP} = T(\log(t_r) + t_a) )</td>
<td>( \log(t_r) = \frac{P_{LMP}}{T} - t_a )</td>
<td>( P_{LMP} = (\sigma) ), ( t_a )</td>
<td>Cy, L, NP, Sp</td>
<td>( \alpha_2 = \alpha_1 = 0 ), ( r = -1, q = 1 )</td>
<td>11</td>
</tr>
<tr>
<td>Manson-Haferd</td>
<td>1953</td>
<td>( P_{MH} = \frac{\log(t_r) - \log(t_a)}{T - t_a} )</td>
<td>( \log(t_r) = P_{MH}(T - T_a) + \log(t_a) )</td>
<td>( P_{MH} = (\sigma), (\sigma') ), ( T_a, t_a )</td>
<td>Cxy, L, NP, Sp</td>
<td>( \alpha_0 = 0 ), ( r = q = 1 )</td>
<td>3</td>
</tr>
<tr>
<td>Manson-Brown</td>
<td>1953</td>
<td>( P_{MB} = \frac{\log(t_r) - \log(t_a)}{(T - T_a)^n} )</td>
<td>( \log(t_r) = P_{MB}(T - T_a)^n + \log(t_a) )</td>
<td>( P_{MB} = (\sigma), (\sigma'), T_a, t_a, n )</td>
<td>Cxy, NL, NP, Swv</td>
<td>( \alpha_2 = 0, r = 1, q = n )</td>
<td>12</td>
</tr>
<tr>
<td>Orr-Sherby-Dorn</td>
<td>1954</td>
<td>( P_{OSD} = \log(t_r) - Q / RT )</td>
<td>( \log(t_r) = Q / RT + P_{OSD} )</td>
<td>( P_{OSD} = (\sigma), Q, R )</td>
<td>NC, L, P, Sp</td>
<td>( \alpha_0 = 0, r = -1, q = 0 )</td>
<td>13</td>
</tr>
<tr>
<td>Manson-Succop</td>
<td>1959</td>
<td>( P_{MS} = \log(t_r) - BT )</td>
<td>( \log(t_r) = BT + P_{MS} )</td>
<td>( P_{MS} = (\sigma) ), B</td>
<td>NC, L, P, Sp</td>
<td>( \alpha_2 - \alpha_0 = 0 ), ( r = 1, q = 0 )</td>
<td>14</td>
</tr>
<tr>
<td>Graham-Walles</td>
<td>1955</td>
<td>( P_{GW} = \frac{\log(t_r)}{(T - T_a)} )</td>
<td>( \log(t_r) = P_{GW}(T - T_a)^n )</td>
<td>( P_{GW} = (\sigma), T_a, n )</td>
<td>Cxy, NL, NP, Swv</td>
<td>( \alpha_2 = \alpha_1 = 0 ), ( r = 1, q = 0 )</td>
<td>15</td>
</tr>
<tr>
<td>Chitty-Duval</td>
<td>1963</td>
<td>( P_{CD} = mT - \log(t_r) )</td>
<td>( \log(t_r) = mT - P_{CD} )</td>
<td>( P_{CD} = (\sigma), m = q \sigma^b )</td>
<td>NC, NP, L, Sn</td>
<td>( \alpha_2 = \alpha_0 = 0 ), ( r = 1, q = 0 )</td>
<td>16</td>
</tr>
<tr>
<td>Goldhoffs-Sherby</td>
<td>1968</td>
<td>( P_{GS} = \frac{\log(t_r) - \log(t_a)}{1/T - 1/T_a} )</td>
<td>( \log(t_r) = P_{GS}(1/T - 1/T_a) + \log(t_a) )</td>
<td>( P_{GS} = (\sigma), T_a, t_a )</td>
<td>Cxy, L, NP, Sp</td>
<td>( \alpha_2 = 0, r = -1, q = 1 )</td>
<td>17</td>
</tr>
</tbody>
</table>

**Modified**

- **Manson-Haferd**

\[ P_{MH'} = \frac{\log(t_r) - \log(t_a)}{T} \]

\[ \log(t_r) = P_{MH'} T + \log(t_a) \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Year</th>
<th>Parametric equation</th>
<th>log-stress equation</th>
<th>Material constants</th>
<th>Characteristics</th>
<th>Metamodel condition</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Manson-Haferd</td>
<td></td>
<td>( P_{MH'} = \frac{\log(t_r) - \log(t_a)}{T} )</td>
<td>( \log(t_r) = P_{MH'} T + \log(t_a) )</td>
<td>( P_{MH'} = (\sigma) ), ( t_a )</td>
<td>Cy, L, NP, Sn</td>
<td>( \alpha_2 = \alpha_1 = 0 ), ( r = 1, q = 1 )</td>
<td>--</td>
</tr>
<tr>
<td>Modified</td>
<td></td>
<td>( P_{MGW'} = \frac{\log(t_r)}{(1/T - 1/T_a)^n} )</td>
<td>( \log(t_r) = P_{MGW'}(1/T - 1/T_a)^n )</td>
<td>( P_{MGW'} = (\sigma), T_a, n )</td>
<td>Cy, NL, NP, Swv</td>
<td>( \alpha_2 = \alpha_0 = 0 ), ( r = -1, q = 0 )</td>
<td>--</td>
</tr>
<tr>
<td>Modified</td>
<td></td>
<td>( P_{CD'} = \frac{m}{T} - \log(t_r) )</td>
<td>( \log(t_r) = mT - P_{CD} )</td>
<td>( P_{CD'} = (\sigma), m = q \sigma^b )</td>
<td>NC, NP, L, Sp</td>
<td>( r = -1, q = 0 ), ( \alpha_0 = \alpha_2 = 0 )</td>
<td>--</td>
</tr>
<tr>
<td>Modified</td>
<td></td>
<td>( P_{GS'} = \frac{\log(t_r) - \log(t_a)}{(1/T - 1/T_a)^n} )</td>
<td>( \log(t_r) = P_{GS'}(1/T - 1/T_a)^n + \log(t_a) )</td>
<td>( P_{GS'} = (\sigma), T_a, t_a )</td>
<td>Cxy, NL, NP, Swv</td>
<td>( r = -1, q = n ), ( \alpha_0 = 0 )</td>
<td>--</td>
</tr>
</tbody>
</table>
Mirror Models

Larson-Miller
\[ \log(t_r) = P_{LMP} / T - t_a \]
\[ P_{LMP} = f(\sigma_i) \]

Modified-Manson-Haferd
\[ \log(t_r) = P_{MMH} T + \log(t_a) \]
\[ P_{MMH} = f(\sigma_i) \]

Orr-Sherby-Dorn
\[ \log(t_r) = Q / RT + P_{OSD} \]
\[ P_{OSD} = f(\sigma_i) \]

Manson-Succop
\[ \log(t_r) = BT + P_{MS} \]
\[ B = \text{constant} \]

Goldhoff-Sherby
\[ \log(t_r) = P_{GS} (1/T - 1/T_a) + \log(t_a) \]
\[ P_{GS} = f(\sigma_i) \]

Manson-Haferd
\[ \log(t_r) = P_{MH} (T - T_a) + \log(t_a) \]
\[ P_{MH} = f(\sigma_i) \]

Modified Graham-Walles
\[ \log(t_r) = P_{GW} (1/T - 1/T_a)^n \]
\[ n = \text{even integer} \]

Graham-Walles
\[ \log(t_r) = P_{GW} (T - T_a)^n \]
\[ n = \text{even integer} \]
Mirror Models (Cont.)

<table>
<thead>
<tr>
<th>Mirror Pairs</th>
<th>Larson-Miller</th>
<th>Mod-Manson-Haferd</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>Goldhoff-Sherby</td>
<td>Manson-Haferd</td>
</tr>
<tr>
<td>(3)</td>
<td>Mo-Goldhoff-Sherby</td>
<td>Manson-Brown</td>
</tr>
<tr>
<td>(4)</td>
<td>Orr-Sherby-Dorn</td>
<td>Manson-Succop</td>
</tr>
<tr>
<td>(5)</td>
<td>Mod-Graham-Walles</td>
<td>Graham-Walles</td>
</tr>
<tr>
<td>(6)</td>
<td>Mod-Chitty-Duval</td>
<td>Chitty-Duval</td>
</tr>
</tbody>
</table>

Table – List of the “mirror pairs” that have similar mathematical form whose isostress lines are inverse/mirror image due to the temperature variable $T$ being inverted.
The **stress-parameter function** is a mathematical expression of the master curve such that a temperature invariant parameter can be obtained for a given stress.

\[
P_{LM} = 11492 - 23.24\sigma \\
LM_P = 18760 - 6.8\sigma - 3288.4\log(\sigma) \\
P_{LM} = 15231 - 53.4\sigma + 0.16\sigma^2 \\
P_{LM} = 81863\sigma^{0.027} - 60076 \\
P_{LM} = 9928.2 + 5825.6e^{-0.012\sigma} \\
P_{LM} = 0.027 \\
\]

Alloy P91

Larson-Miller model fit using five Stress-parameter function
### Stress-parameter function

#### Table – Effect of typical stress-parameter functions during extrapolation on Larson-Miller

<table>
<thead>
<tr>
<th>Function type</th>
<th>Equation</th>
<th>Prediction type</th>
<th>Inflection?</th>
<th>$\frac{dP_i}{d\sigma} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$a + b\sigma$</td>
<td>High Weakening</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>Logarithm</td>
<td>$a + b\sigma + c\log(\sigma)$</td>
<td>Stable</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>Ploy-nominal</td>
<td>$a + b\sigma + c\sigma^2$</td>
<td>Weakening</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>$a + b\sigma^c$</td>
<td>Stable</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>Exponential</td>
<td>$a + be^{c\sigma}$</td>
<td>Weakening</td>
<td>YES</td>
<td></td>
</tr>
</tbody>
</table>

- Force fit to a particular function may lead to poor prediction.
- A flexible option to choose function may improve prediction accuracy.
Model fit to alloy P91 data

- Stress, \( \sigma \) (MPa)
- Rupture life, \( t_r \) (hr)
- Temperature: 550°C, 600°C, 650°C, 700°C
Model fit to alloy P91 data

- **Rupture life, \( t_r \) (hr)**
  - 10
  - 1
  - 10
  - 2
  - 10
  - 3
  - 10
  - 4
  - 10
  - 5
  - 10
  - 6

- **Stress, \( \sigma \) (MPa)**
  - 20
  - 30
  - 40
  - 50
  - 60
  - 80
  - 200
  - 300
  - 10
  - 100

- **Avg. Experimental Rupture Time (hr)**
  - 10
  - 1
  - 10
  - 2
  - 10
  - 3
  - 10
  - 4
  - 10
  - 5
  - 10
  - 6

- **Predicted Rupture Time (hr)**
  - 10
  - 1
  - 10
  - 2
  - 10
  - 3
  - 10
  - 4
  - 10
  - 5
  - 10
  - 6

- **Manson-Succop**
- **Orr-Sherby-Dorn**
- **Graham-Walles**
- **M-Graham-Walles**
- **Chitti-Duval**
- **M-Chitti-Duval**
Calibrated Material constants and Stress-Parameter functions

<table>
<thead>
<tr>
<th>Model</th>
<th>Material constants</th>
<th>Master curve/Stress-parameter function $P_i = f(\sigma)$ with maximum $R^2$</th>
<th>Max. $R^2$</th>
<th>Inflection status</th>
<th>Overall NMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMH</td>
<td>$\alpha_0 = 25.0$</td>
<td>$P_{MMH} = 2.9E - 7\sigma^2 - 1.2E - 4\sigma - 0.03$</td>
<td>0.97</td>
<td>YES</td>
<td>2.46</td>
</tr>
<tr>
<td>LM</td>
<td>$\alpha_0 = -19.0$</td>
<td>$P_{LM} = 22000 - 4680 \log(\sigma)$</td>
<td>0.98</td>
<td>NO</td>
<td>3.16</td>
</tr>
<tr>
<td>MH</td>
<td>$\alpha_0 = 16.6, \alpha_2 = 210$</td>
<td>$P_{MH} = -1E - 4\sigma - 0.025$</td>
<td>0.98</td>
<td>NO</td>
<td>6.05</td>
</tr>
<tr>
<td>GS</td>
<td>$\alpha_0 = 32.0, \alpha_2 = 248$</td>
<td>$P_{GS} = -0.05\sigma^2 + 27.1\sigma + 10435$</td>
<td>0.98</td>
<td>YES</td>
<td>1.60</td>
</tr>
<tr>
<td>MB</td>
<td>$\alpha_0 = 14.0, \alpha_2 = 25.0$</td>
<td>$P_{MB} = 3E - 10\sigma^2 - 2E - 7\sigma - 2E - 5$</td>
<td>0.99</td>
<td>NO</td>
<td>1.49</td>
</tr>
<tr>
<td>MGS</td>
<td>$\alpha_0 = 520, \alpha_2 = 12.4$</td>
<td>$P_{MGS} = 0.31\sigma^2 - 12.8\sigma - 82076$</td>
<td>0.99</td>
<td>YES</td>
<td>1.70</td>
</tr>
<tr>
<td>MS</td>
<td>$\alpha_1 = -0.036$</td>
<td>$P_{MS} = -3.29 \ln(\sigma) + 39.22$</td>
<td>0.98</td>
<td>NO</td>
<td>5.49</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2 = 5.00$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OSD</td>
<td>$\alpha_1 = 12100$</td>
<td>$P_{MB} = 2E - 4\sigma^2 - 7E - 2\sigma - 2E - 12.48$</td>
<td>0.99</td>
<td>YES</td>
<td>8.69</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2 = 5.00$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW</td>
<td>$\alpha_2 = 1005$</td>
<td>$P_{GW} = 5E - 8\ln(\sigma) - 3E - 7$</td>
<td>0.94</td>
<td>NO</td>
<td>66.8</td>
</tr>
<tr>
<td>MGW</td>
<td>$\alpha_2 = 900$</td>
<td>$P_{MGW} = 7E8\sigma^{-0.975}$</td>
<td>0.98</td>
<td>NO</td>
<td>64.6</td>
</tr>
<tr>
<td>CD</td>
<td>$\alpha_1 = f(\sigma)$</td>
<td>$P_{CD} = -3E - 4\sigma^2 + 0.2\sigma + 5.1$</td>
<td>0.99</td>
<td>YES</td>
<td>42.8</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2 = 1E4\log(\sigma) + 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCD</td>
<td>$\alpha_1 = f(\sigma)$</td>
<td>$P_{MCD} = -11.1\ln(\sigma) + 19.2$</td>
<td>0.99</td>
<td>YES</td>
<td>81.6</td>
</tr>
</tbody>
</table>

- For most model, polynomial function produce the best goodness-of-fit.
- Polynomial function may exhibit inflection during extrapolation.
- Logarithmic and power function does not exhibit inflection during extrapolation.
- Inflection may be avoided by selecting next good-fit function at the cost of accuracy.
### Normalized Mean Square Error

- Normalized Mean Square Error: \[ NMSE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{X_{\text{sim},i} - X_{\text{exp},i}}{X_{\text{exp},i}} \right)^2 \]

**Figure - Comparison of the cumulative creep rupture NMSE of the models**

- The Manson-Brown and Goldhoff Sherby produces the lowest NMSE.
Guideline to model selection

- ASME (American Society of Mechanical Engineers)
- ECCC (European Creep Collaborative Committee)
- ASTM (American Society for Testing and Materials)

Data pre-processing:
- Inclusion of ongoing data
- Exclusion of outlier data
- Removal of low stress data

Procedure:
- Follow Metamodelling procedure

The purpose of modeling:
- Interpolation
- Extrapolation

Temperature feasibility:
- Point of convergence

Stress feasibility:
- Inflection point stress

Material behavior:
- Strengthening
- Weakening
- Stable behavior

Post-processing:
- Check physical realism
- Goodness-of-fit
Creep Deformation Metamodel

Table 6 – Combinational model summary

<table>
<thead>
<tr>
<th>No</th>
<th>Base model</th>
<th>Creep Strain Rate</th>
<th>Damage Theta-Omega Identity, $I_{\theta\Omega} = (\dot{\varepsilon}_0 / \theta_3 \theta_4)$</th>
<th>Life Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Theta</td>
<td>$\dot{\varepsilon} = (\dot{\varepsilon}<em>0 / I</em>{\theta\Omega}) \exp(\theta_4 t)$</td>
<td>$\omega(t) = \frac{t}{t_r}$</td>
<td>$t_r = \frac{1}{\theta_4} \ln \left( \frac{\varepsilon_r \theta_4 I_{\theta\Omega}}{\varepsilon_0} + 1 \right)$</td>
</tr>
<tr>
<td>M2</td>
<td>Omega</td>
<td>$\dot{\varepsilon} = I_{\theta\Omega} \theta_3 \theta_4 \exp(\Omega \varepsilon)$</td>
<td>$\omega(t) = \frac{t}{t_r}$</td>
<td>$t_r = \left( \frac{1}{I_{\theta\Omega} \theta_3 \theta_4 \Omega} \right)$</td>
</tr>
</tbody>
</table>

Omega-Sinh Identity, $I_{\Omega S} = \dot{\varepsilon}_0 / A \sinh(\sigma / \sigma_s)$

| M3 | Omega      | $\dot{\varepsilon} = I_{\Omega S} A \sinh(\sigma / \sigma_s) \exp(\Omega \varepsilon)$ | $\omega(t) = \frac{t}{t_r}$ | $t_r = \frac{1}{I_{\Omega S} A \sinh(\sigma / \sigma_s) \Omega}$ |
| M4 | Sinh       | $\dot{\varepsilon} = (\dot{\varepsilon}_0 / I_{\Omega S}) \exp(\lambda \omega^{3/2})$ | $\omega(t) = -\frac{1}{\phi} \ln \left[ 1 - \left[ 1 - \exp(-\phi) \right] \frac{t}{t_r} \right]$ | $t_r = \left( \frac{1}{\dot{\varepsilon}_0 \cdot \Omega} \right)$ |

Sinh-Theta Identity, $I_{S\theta} = \theta_3 \theta_4 / A \sinh(\sigma / \sigma_s)$

| M5 | Sinh       | $\dot{\varepsilon} = (\theta_3 \theta_4 / I_{S\theta}) \exp(\lambda \omega^{3/2})$ | $\omega(t) = -\frac{1}{\phi} \ln \left[ 1 - \left[ 1 - \exp(-\phi) \right] \frac{t}{t_r} \right]$ | $t_r = \frac{1}{\theta_4} \ln \left( \frac{\varepsilon_r}{\theta_3} + 1 \right)$ |
| M6 | Theta      | $\dot{\varepsilon} = I_{S\theta} A \sinh(\sigma / \sigma_s) \exp(\theta_4 t)$ | $\omega(t) = \frac{t}{t_r}$ | $t_r = \frac{1}{\theta_4} \ln \left( \frac{\varepsilon_r \theta_4}{I_{S\theta} A \sinh(\sigma / \sigma_s) + 1} \right)$ |
# CDM Metamodel

## KR-LM metamodell

\[
I_{t_{r},KR-LM} = \ln \left( \frac{M_K \sigma^z (\phi_K + 1)}{M_L \sigma^y} \right) - \ln \left( \frac{1 - \exp(-\phi_L)}{\phi_L} \right) - \ln(\phi_K + 1)
\]

\[
\omega_L = (-\phi_K / \phi_L) \ln(1 - \omega_K) + I_{t_{r},KR-LM} / \phi_L
\]

## LM-Sinh metamodell

\[
I_{t_{r},LM-Sinh} = \ln \left( \frac{M_L \sigma^p}{M_S \text{Sinh}(\sigma / \sigma_i)} \right) - \ln \left( \frac{1 - \exp(-\phi_S)}{\phi_S} \right) + \ln \left( \frac{1 - \exp(-\phi_L)}{\phi_L} \right)
\]

\[
\omega_S = (\phi_L / \phi_S) \omega_L + I_{t_{r},LM-Sinh} / \phi_S
\]

## Sinh-KR metamodell

\[
I_{t_{r},Sinh-KR} = \ln \left( \frac{M_K \sigma^z (\phi_K + 1)}{M_S \text{Sinh}(\sigma / \sigma_i)} \right) - \ln \left( \frac{1 - \exp(-\phi_S)}{\phi_S} \right) - \ln(\phi_K + 1)
\]

\[
\omega_S = (-\phi_K / \phi_S) \ln(1 - \omega_K) + I_{t_{r},Sinh-KR} / \phi_S
\]
Creep Deformation Metamodeling process

Figure: Graphical approach to model transformation (a) Theta-Omega, (b), Omega-Sinh and (c) Sinh-Theta

\[ \varepsilon = \left( \frac{\theta}{\Omega} \right) t - \ln(I_{\mathrm{ce}}) / \Omega \]  
(1)

\[ \varepsilon = \left( \frac{\lambda}{\Omega} \right) \omega^{3/2} - \ln(I_{\mathrm{ce}}) / \Omega \]  
(2)

\[ \omega^{3/2} = \left( \frac{\theta}{\lambda} \right) t + \ln(I_{\mathrm{ce}}) / \lambda \]  
(1)

\[ \omega^{3/2} = \left( \frac{\theta}{\lambda} \right) t + \ln(I_{\mathrm{ce}}) / \lambda \]  
(2)

\[ \varepsilon_{\min} = 3.2 \times 10^{-4} \]

\[ \varepsilon_{\min} = 2.1 \times 10^{-4} \]

\[ \varepsilon = 0.27 \omega^{3/2} + 0.00 \]  
Omega-Sinh

\[ \varepsilon = 0.3 \omega^{3/2} + 0.0108 \]  
Sinh-Theta

\[ \varepsilon = \theta_{\text{min}} / I_{\text{ce}} - \ln(I_{\text{ce}}) / \Omega \approx 0 \]

\[ 1600^\circ F \]

\[ \text{slope } \dot{\varepsilon}_{\min} = \dot{\theta} / \Omega = 3.2 \times 10^{-4} \]  
Theta-Omega

\[ \text{slope } \dot{\varepsilon}_{\min} = \dot{\theta} / \Omega = 2.1 \times 10^{-4} \]  
(b)
Assessment

- Normalized Mean Squared Error
  \[ NMSE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{X_{sim,i} - X_{exp,i}}{X_{max}} \right)^2 \]

- Physical realism and numerical stability
- Conservatism

Figure: Model performance in predicting (a-c) creep ductility and (d-f) rupture life where the dotted lines indicate 2.5x of the maximum standard deviation in repeated tests. In the ratio plots, the dotted line indicates the ratio of 2.5x of the maximum standard deviation divided by the average obtained in repeated tests. The highlight areas indicate conservative predictions.
Task 8: Multiaxial Representative Stress Function

**Representative stress** $(\sigma_{rep})$
The stress applied to a plain bar that results in the same effective strain accumulation or rupture life as that obtained in a notched bar tested at the same temperature.

**Classic approaches**
- Rankine criteria, $\sigma_{p,max} > \sigma_{uts}$
- Tresca criteria, $\tau_{max} > \sigma_{uts}$
- Von-Mises criteria,

\[
\sigma_{vm} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}
\]

**Limitations of Classic approaches**
- Under equi-biaxial tension, $\sigma_x / \sigma_y = 1$ the rupture stress is non-conservative and identical to the uniaxial case.
Representative Stress Functions

\[ \sigma_{rep} = a \sigma_x + 3 \beta \sigma_m + (1 - \alpha - \beta) \sigma_m \quad 0 \leq \alpha + \beta \leq 1 \]

\[ \sigma_{rep} = \frac{3}{2} S_i \left( \frac{2 \sigma_{YM}}{3 S_i} \right)^b \exp \left[ b \left( \frac{J_1}{S_i} - 1 \right) \right] \]

Guideline to representative function selection

- ASME B&PV III (ASME Boiler and Pressure Vessel Code)
- RCC-MR (Design and Construction Rules for Power Generating Stations)
- ESIS (European Structural Integrity Society)

**Stress function**
- Either the Hayhurst or Huddleston function be used.
- Hayhurst function is simpler.
- Huddleston function is more flexible.

**Numerical stability**
- Check for infinite or imaginary condition leading to numerical instability.
- Hayhurst function is completely stable.
- Huddleston function is unstable at unstressed condition.

**Material parameter calibration**
- Uniaxial and Equi-biaxial tension creep tests.
- Regression analysis.

**Damage mechanism**
- Von-Mises stress: deformation.
- Hydrostatic stress: void growth.
- Principal stress: intergranular damage.

**Incompressibility**
- Constraints should be applied to enforce incompressibility.

Ref: [6]
Results and recommendations

- Performance of the individual models
- Performance of the Metamodels

- Recommendations
  - Guideline to TTP model selection
  - Guideline to representative function selection.
  - Guideline to minimum creep strain rate model selection.
  - Guideline to CDM model selection.

- A master guideline to adaptive approach towards creep modeling.

Figure: Comparison of combinational models using Siemens data (a-b) deformation, (c-d) damage prediction for Hastelloy X at 1200°F and 1600°F
A metamodel derived by combining and exploiting creep models where the submodels (existing and new models that are exploited to develop the metamodel) are special case can facilitate instantaneous and efficient evaluation of the submodels leading to the selection of “best” model.

Inclusion of disparate data into calibration process can significantly improve the prediction accuracy.
## Publications


### In progress:


Future Work

Software: Secondary Creep model

Material Constant Custom Input

Select Model
- Norton
- Soderberg
- Dorn
- McVetty
- Garofalo
- JHK

Constant Input
- $A$
- $n$
- $\sigma_0$
- $\sigma_1$
- $\sigma_2$

Source file: File

Apply

Use Data base

Execute

Ricardo Vega
Future Work

Software: Creep Deformation model

Material Constant Custom Input

Source file:

Select Model
- Omega
- Theta projection
- Sine-Hyperbolic
- Kachanov-Rabotnov

Constant Input

Use Data base

Execute

Jimmy Perez
Future Work

Software: Extrapolation and Design Maps

<table>
<thead>
<tr>
<th>Temperature Function</th>
<th>Stress Function</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Linear</td>
<td>Extrapolation</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>Logarithmic</td>
<td>Design map</td>
</tr>
<tr>
<td>Exponential</td>
<td>Exponential</td>
<td>Detail Analysis</td>
</tr>
<tr>
<td>Power</td>
<td>Power</td>
<td></td>
</tr>
<tr>
<td>Polynomial</td>
<td>Polynomial</td>
<td></td>
</tr>
</tbody>
</table>

Material Constant Custom Input

- Source File
- Temperature Range
- Stress Range
- Parameters
- Select model
  - Stress-rupture
  - Secondary creep
  - Deformation

Help!!

Apply

Check Inflection

Execute
Acknowledgement:

Department of Energy (DOE)
National Energy Technology Laboratory (NETL)
Award Number(s): DE-FE0027581

- Omer R. Bakshi
  - Federal Project Manager, Crosscutting Research, NETL, U.S. DOE

Thank you
Time-Temperature Parameter (TTP) Model

1. **TTP model**

2. Find initial guess constant, \( t_a \)
   (graphical approach)

3. Calculate \( P_{LMP} \) for each data point

4. Simultaneous calibration of constant, \( t_a \) and stress-parameter function.

5. Plot model with experimental data

\[
\log(t_r) = P_{LMP} / T - t_a
\]

Initial guess

\[
P_{LMP} = T(\log(t_r) + t_a)
\]

\[
\sigma_1 < \sigma_2 < \sigma_3
\]

\[P_{LMP} = f(\sigma_i)\]

\[
\log(t_r) = P_{f(\sigma_i)} / T - t_a
\]
## Metamodel

### Existing approach
  \[
  \log(t_r) = \left\{ \sum_{k=0}^{n} \beta_k (\log[\sigma]^k) \right\}(T - T_a) + \beta_5; \quad n = 2, 3, 4
  \]
- **Eno, 2008** (LM, MH, OSD, MS, and MRM)
  \[
  \log(t_r) = \beta_0 + \beta_1 \left( \frac{1}{T} \right) + \beta_2 \log(\sigma) + \beta_3 \log(\sigma) \left( \frac{1}{T} \right)
  \]
- **Seruga, 2011** (LM, MH, OSD, and MB)
  \[
  \log(t_r) = (T - T_a \cdot \langle q \rangle)^q \left( a_0 + a_1 \log(\sigma) + a_2 \log^2(\sigma) \right)
  \]
  \[
  + \log(t_a)T^{q-1}
  \]

### New approach: (Includes 12 TTP models)
- **New approach**
  \[
  P_{HS}(\sigma) = \frac{\log(t_r) - \alpha_0 - \alpha_1 T^r}{(T^r - \alpha_2^r)^q}
  \]

**Suitable for verified material behavior.**

**Flexible** to fit any material behavior.
Task 8: Computational Analysis (2D)

Ref: [10]

Eight node single element with boundary constraint

2D center-hole plate (a) dimensions, (b) ANSYS mesh ( =0.05 mm)
Task 8: Computational Analysis (3D)

**ANSYS FEM mesh of Bridgeman notch specimen**

Sinh damage at 250 MPa and 700°C (a) 50 hr, (b) 200 hr, (c) 400 hr, (d) 600 hr, (e) 700 hr, and (f) 832 hr