Simulation of the Cranfield CO_2 Injection Site with a Drucker-Prager Plasticity Model

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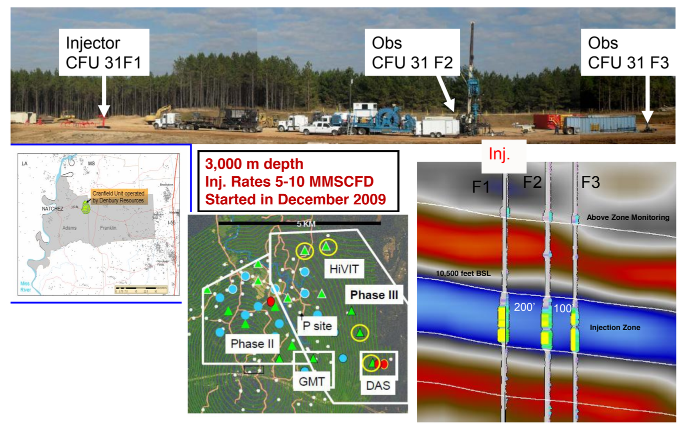
1. Introduction

- Coupled fluid flow and geomechanics simulations have strongly supported CO_2 injection planning and operations, for example those at the Cranfield site.
- Linear elasticity is the predominant solid material model used in simulations, but nonlinear constitutive models can take into account more complex rock formation behaviors.
- Plastic behavior can occur near wellbores, resulting in changes to rock porosity and permeability, which can impact flow behavior.
- The Druker-Prager plasticity model has been incorporated into IPARS (Integrated Parallel Accurate Reservoir Simulators developed at the Center for Subsurface Modeling, The University of Texas at Austin). It uses general hexahedral elements for flow and mechanics, and can solve large-scale problems in parallel.
- A Cranfield CO_2 injection model is set up according to the reservoir geological field data and rock plasticity parameters based on Sandia national lab experimental results.



Schematic of the Cranfield CO_2 sequestration project in western Mississippi, with wells monitored by the Bureau of Economic Geology.

2. Plasticity Model

Fluid Flow and Stress Equilibrium Equations

$$\frac{\partial(\rho(\phi_0+\alpha\varepsilon_v+\frac{1}{M}(p-p_0)))}{\partial t}+\nabla\cdot\left(\rho\frac{K}{\mu}(\nabla p-\rho g\nabla h)\right)-q=0$$

$$\nabla\cdot(\sigma''+\sigma_o-\alpha(p-p_0)I)+f=0$$

Hooke's Law and Strain-Displacement Relation

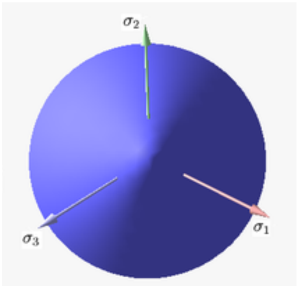
$$\sigma'' = D^e:(\varepsilon-\varepsilon^p)$$
$$\varepsilon = \frac{1}{2}(\nabla u+\nabla^T u)$$

Yield and Flow Functions (Druker-Prager)

$$Y = q+\theta\sigma_m-\tau_0$$
$$F = q+\gamma\sigma_m-\tau_0$$

Plastic Strain Evolution Equations

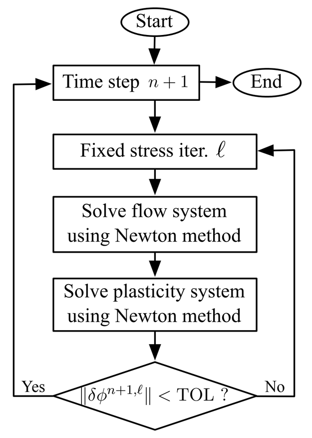
$$\dot{\varepsilon}^p = \lambda\frac{\partial F(\sigma'')}{\partial\sigma''}, \quad \text{at } Y(\sigma'')=0$$
$$\dot{\varepsilon}^p = 0, \quad \text{at } Y(\sigma'')<0$$



Druker-Prager Yield Surface.

Here ρ is fluid density, ϕ_0 is initial porosity, α is the Biot coefficient, ϵ_v is volumetric strain, M is the Biot modulus, p is fluid pressure, K is permeability, μ is fluid density, $g\nabla h$ is gravitational force, q are fluid sources/sinks, σ'' is effective stress, σ_0 is initial stress, f is solid body force, D^e is the Gassman tensor, ϵ is elastic strain, ϵ^p is plastic strain, u is displacement, λ is a consistency parameter, F is plastic flow function, Y is plastic yield function, q is the Von-Mises stress, θ and γ are the yield and flow function slopes, and τ_0 is the shear strength.

- Plastic model is non-linear. A Newton iteration is used to solve the mechanics residual equations on a global level, and a second Newton iteration is used to evaluate the material behavior on the element level. This leads to a consistent formulation, and our numerical results show quadratic Newton convergence.
- To solve an elastic model, we may set plastic strain $\epsilon^p = 0$, and the mechanics equation becomes linear.
- The coupled poro-plasticity system is solved using an iterative coupling scheme: the nonlinear flow and mechanics systems are solved sequentially using the fixed-stress splitting, and iterates until convergence is obtained in the fluid fraction. To the best of our knowledge, the application of this algorithm is new for plasticity.



- On a given Newton iteration, the mechanics linear system is solved using either the iterative multigrid solver library HYPRE, or the direct solver library SuperLU. The latter must be used when the systems are difficult to converge. However, both solvers are fully parallel.

3. Numerical Results

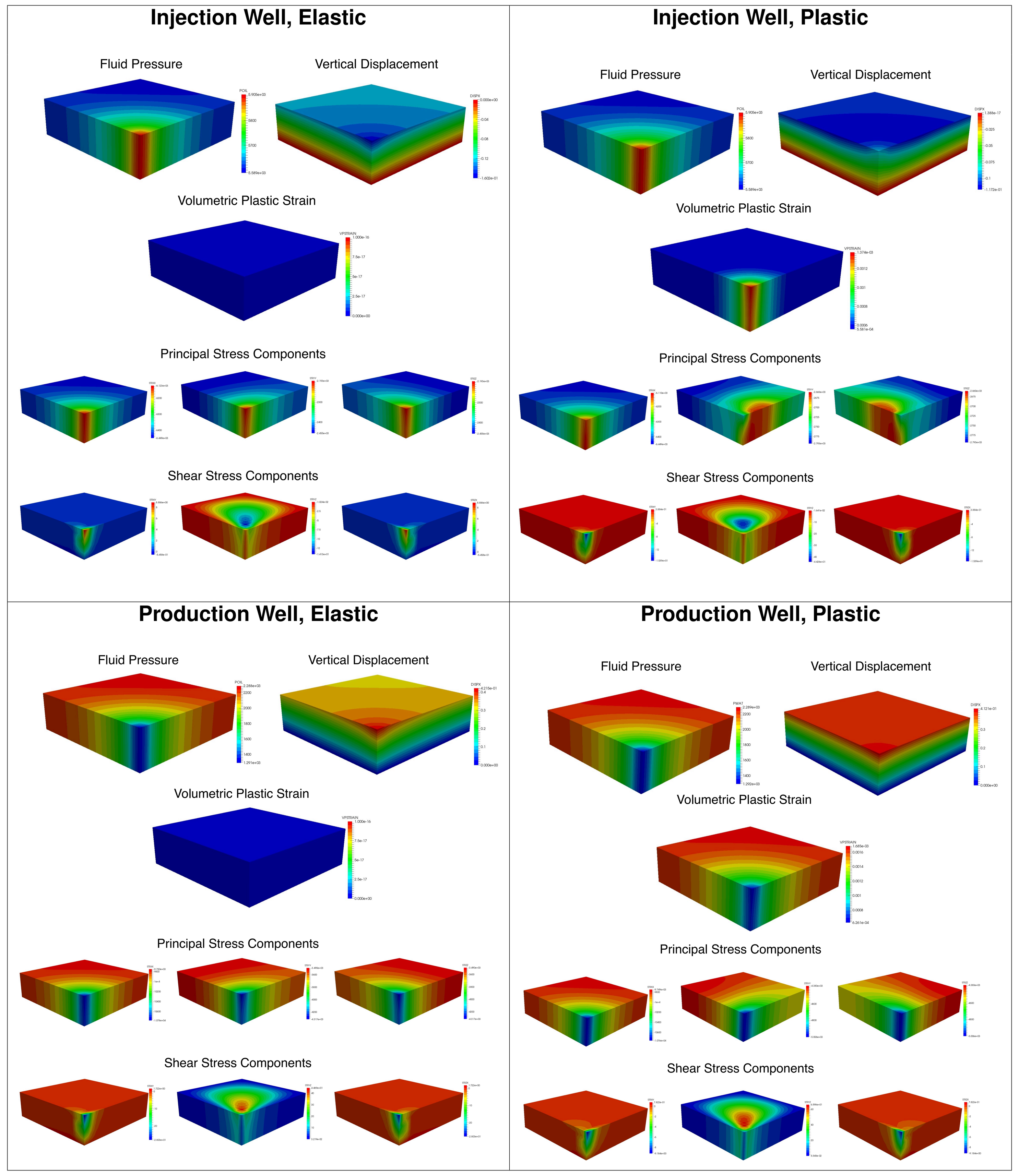
3.1 Geomechanical Data Obtained with Laboratory Experiments

- Boundary conditions: no flow; overburden = 12038 [psi] on top face, zero normal displacement on all other faces.
- Initial pressure = 4640 [psi], initial stress $\sigma_{xx} = -7395$, $\sigma_{yy} = \sigma_{zz} = 2755$ [psi].
- Calculate Young's Modulus with $E = 9KG/(3K+G)$ where K is the Bulk Modulus and G is the Shear Modulus as determined by unconfined compressible strength tests [3].

E	Young's modulus	375581 [psi]
ν	Poisson's ratio	0.25
α	Biot's coefficient	1.0
$1/M$	Biot's modulus	1e-6 [1/psi]
τ_0	Shear strength	4922 [psi]
θ	Yield function slope	0.95

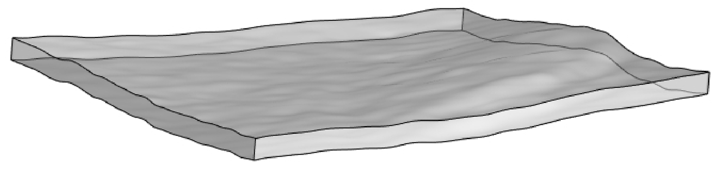
3.2 Comparison of Plasticity and Elasticity Models with Homogeneous Parameters and Rectangular Geometry

- Here we use a homogeneous porosity ($\phi = 0.2$) and permeability ($K = 64$ [md]) and rectangular geometry.
- Domain is $60 \times 1000 \times 1000$ feet discretized into $5 \times 20 \times 20$ elements.
- Simulation time is 40 days, and parallel computation is performed on 16 processors.

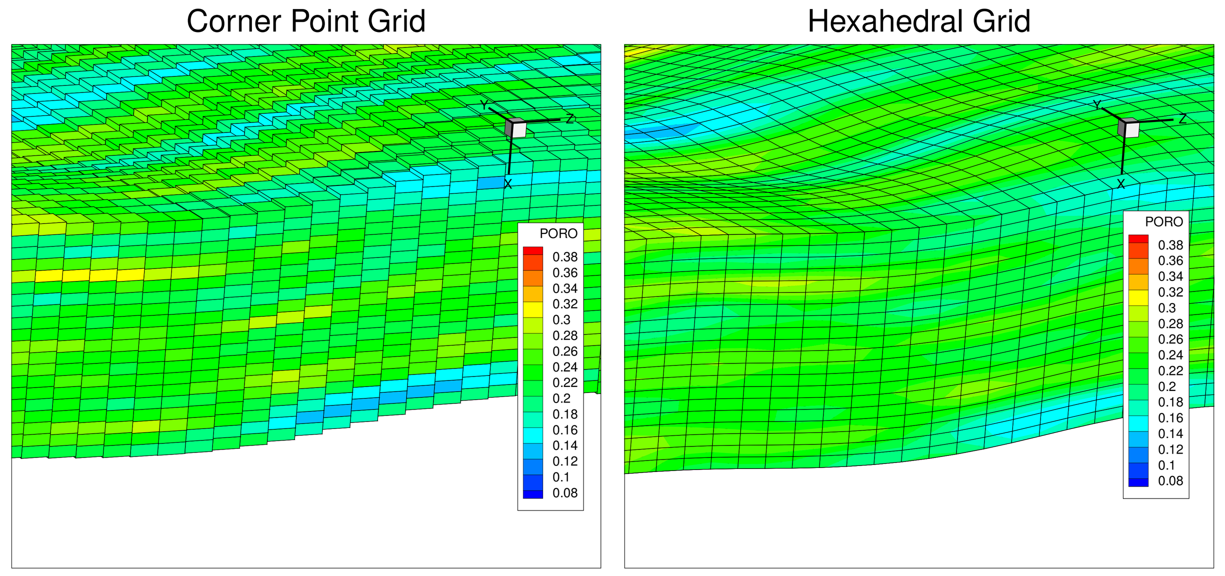


3.3 Elasticity Model with Heterogeneous Properties and Cranfield Geometry

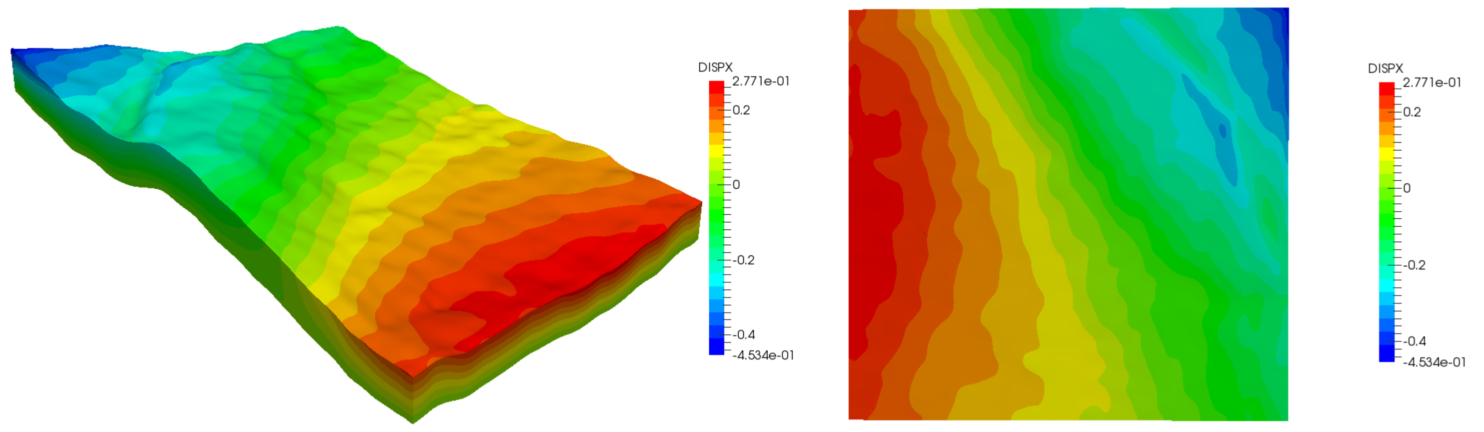
- Domain is $80 \times 9400 \times 8800$ feet discretized into $20 \times 188 \times 176$ elements.
- Cranfield depth data is available on each grid column (average depth 10,000 ft).
- Original Cranfield corner point grid was processed to form a smooth structured hexahedral grid, on which we can obtain a better quality solution.



Cranfield geometry data.



Closeup of heterogeneous Cranfield porosity data.



3D (left) and 2D (right) plots of the vertical displacement component at final time.

4. Conclusions and Future Work

Conclusions:

- Incorporating a plasticity model can more accurately predict the geomechanical response of CO_2 injection in the subsurface, and numerical results show significant differences versus an elastic model.
- Numerical tests with realistic parameters based on the Cranfield CO_2 injection site show plastic yielding may occur near the wellbore at a typical injection pressure.

Future Work:

- We are currently working towards running plasticity with heterogeneous parameters and actual geometry. To accomplish this, we must allow plastic yielding to occur at the initial time.
- Results reported here used a two-phase flow model running in a single-phase configuration. Next we will couple the plastic model with a fully compositional (multi-phase, multi-component) flow model.

References

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U.S. Department of Energy (DOE) Carbon Storage R&D Project Review Meeting, Pittsburgh, Pennsylvania, August 18–20, 2015