### Optimal Model Complexity in Geological Carbon Sequestration: A Design of Experiment (DoE) & Response Surface (RS) Uncertainty Analysis

Project Number: DE-FE-0009238

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### **Presentation Outline**

- Project goals and benefits;
- Detailed project objectives & success criteria;
- Accomplishments to date;
- Summary of results;
- Appendix (organization chart; Gantt chart; supplemental results).

## Benefit to the Program

#### Major goals:

Support industry's ability to predict  $CO_2$  storage capacity in geologic formations to within ±30% accuracy;

Develop and validate technologies to ensure 99% storage permanence.

#### **Project benefits:**

Facilitate the development and implementation of efficient workflows for modeling field-scale GCS in a variety of geochemically reactive environments, where formations exhibit multiple scales of permeability (*k*) heterogeneity.

### **Project Overview**: Goals and Objectives

- Develop, test, and verify the DoE and RS uncertainty analysis for a fully heterogeneous reference model (FHM) & increasingly lower resolution "geologic models" created from upscaling the FHM.
- Investigate the effect of increasing reservoir k variance and depth on the uncertainty outcomes including optimal heterogeneity resolution(s). At greater injection depths, investigate gravity-stable injection.
- Investigate the effect of mineral reactions on GCS, including mineral volume fractions, reactive rate constants, reactive surface areas, and the impact of different geochemical databases.

### Project Overview: Success Criteria

- At increasing depth, for both weakly and strongly heterogeneous systems, the geologic models can capture the FHM CO<sub>2</sub> behaviors within the full parameter space; → Reduced characterization cost;
- RS analytical models are successfully verified against full-physics reservoir simulations via HPC, thus prediction uncertainty of any outcome at any time can be assessed using the low-resolution model(s) running the efficient RS models. → Enhanced computation efficiency;
- Mineral storage analysis: seeking the most efficient composition for reactive storage 
   → Enhanced storage;
- Greater injection depth: within the uncertainty analysis framework, identify the combination(s) of favorable parameters & reservoir condition that give rise to gravity-stable flow. → Enhanced storage security.

## Accomplishments to Date

- High-resolution reservoir k heterogeneity (3.2 M grid cells)
   & geologic models of decreasing k resolutions;
- Permeability upscaling & single-phase flow verification;
- CO<sub>2</sub> modeling with PFLOTRAN & performance scaling on the petascale Yellowstone supercomputer at NWSC;
- Model comparison & DoE/RS analysis;
- CO<sub>2</sub> modeling considering mineral reactions.

### Sediment Experiment at SAFL





http://www.safl.umn.edu/

Project Leader: Prof. Chris Paola Founding: NSF & oil industry consortium

## Reservoir Heterogeneity Vs Geologic Models



A 1-unit homogeneous "formation" model is also created (not shown);

## **Upscaling Verification**



MRE

# Carbon Sequestration Modeling with Reactions

- Multicomponent-multiphase non-isothermal reactive flow and transport model;
- Massively parallel---based on the PETSc parallel framework; Peta-scale performance Highly scalable (run on over 265k cores)
- Supercritical CO<sub>2</sub>-H<sub>2</sub>O; Span-Wagner EOS for CO<sub>2</sub> density & fugacity coefficient Mixture density for dissolved CO<sub>2</sub> in brine (Duan et al., 2008) Viscosity of CO<sub>2</sub> (Fenghour et al., 1998)
- Finite Volume Discretization; Variable switching for changes in fluid phase Structured/Unstructured grids
- Reactive transport modeling, including CO<sub>2</sub>-mineral reactions with many degrees of freedom

### **PFLOTRAN Scaling on Yellowstone**

Yellowstone is a 1.5petaflops supercomputer with 72,288 processor cores & 144.6 TB of memory. <u>http://www2.cisl.ucar.e</u> <u>du/resources/yellowsto</u> ne

1-unit model (3.2M): \* 20 yr CO<sub>2</sub> injection + 2000 yr monitoring \* 2048 cores: 9 hours



**1-unit model (25 M):** CO<sub>2</sub> injection w/ reactive chemistry

### Dissolved CO<sub>2</sub>



- Under both low and high variance conditions, the 1-unit model can reasonably capture the plume footprint of the FHM.
- Base on results of the upscaling study, the 8-unit and 3-unit models (simulations are ongoing) should yield more accurate dissolved CO<sub>2</sub> predictions than the 1-unit model. 12

## Design of Experiment (1-unit)

| ۹ 📃 💌 |         |            |               |       |          |             |             |            |            |             |             |  |
|-------|---------|------------|---------------|-------|----------|-------------|-------------|------------|------------|-------------|-------------|--|
|       | Pattern | T_Gradient | Brin_Salinity | K_Cap | Inj_Rate | SR_EOI      | SR_EOM      | CO2_LK_EOI | CO2_LK_EOM | Brin_LK_EOI | Brin_LK_EOM |  |
|       | ++00    | 1          | 1             | 0     | 0        | 0.10835058  | 0.296383093 | 2.45e-12   | 1.31e-10   | 0.000000678 | 0.0000361   |  |
| 2     | 0+0-    | 0          | 1             | 0     | -1       | 0.115352993 | 0.290742974 | 3.99e-12   | 9.84e-11   | 0.00000011  | 0.00000272  |  |
|       | 0000    | 0          | 0             | 0     | 0        | 0.142377741 | 0.538398262 | 1.7e-12    | 9.23e-11   | 0.000000466 | 0.00000253  |  |
| 4     | -00-    | -1         | 0             | 0     | -1       | 0.153843493 | 0.55349839  | 2.48e-12   | 6.18e-11   | 0.000000678 | 0.00000169  |  |
| 5     | 00      | 0          | -1            | -1    | 0        | 0.199386363 | 0.437694529 | 2.13e-11   | 9.38e-10   | 0.00000583  | 0.0000256   |  |
| 6     | 0+0+    | 0          | 1             | 0     | 1        | 0.100143073 | 0.284811298 | 7.88e-13   | 9.84e-11   | 0.000000218 | 0.00000272  |  |
| 7     | -0+0    | -1         | 0             | 1     | 0        | 0.138510286 | 0.687346742 | -1.77e-13  | 5.09e-12   | -4.84e-9    | 0.00000139  |  |
| 8     | 00      | -1         | -1            | 0     | 0        | 0.19363702  | 0.795624274 | 1.11e-12   | 6.17e-11   | 0.000000302 | 0.00000168  |  |
| 9     | +00-    | 1          | 0             | 0     | -1       | 0.152782269 | 0.552892793 | 4.92e-12   | 1.22e-10   | 0.00000135  | 0.0000336   |  |
| 10    | +0+0    | 1          | 0             | 1     | 0        | 0.138169204 | 0.672851482 | -3.45e-13  | 1.01e-11   | -9.48e-9    | 0.00000276  |  |
| 11    | 0-+0    | 0          | -1            | 1     | 0        | 0.187461554 | 0.930414357 | -2.96e-13  | 7.44e-12   | -8.08e-9    | 0.00000203  |  |
| 12    | +0-0    | 1          | 0             | -1    | 0        | 0.147983085 | 0.297567526 | 2.84e-11   | 1.25e-9    | 0.0000078   | 0.0000342   |  |
| 13    | +00+    | 1          | 0             | 0     | 1        | 0.13057312  | 0.514362598 | 9.6e-13    | 1.22e-10   | 0.000000264 | 0.0000336   |  |
| 14    | 00+-    | 0          | 0             | 1     | -1       | 0.149784337 | 0.703780661 | -2.01e-13  | 7.59e-12   | -5.52e-9    | 0.00000208  |  |
| 15    | 0+-0    | 0          | 1             | -1    | 0        | 0.113389545 | 0.221478542 | 2.28e-11   | 0.00000001 | 0.00000631  | 0.0000276   |  |
| 16    | 0000    | 0          | 0             | 0     | 0        | 0.142377741 | 0.538398262 | 1.7e-12    | 9.23e-11   | 0.000000466 | 0.00000253  |  |
| 17    | 00      | 0          | 0             | -1    | -1       | 0.156476955 | 0.282904316 | 4.28e-11   | 9.39e-10   | 0.00000117  | 0.0000257   |  |
| 18    | +-00    | 1          | -1            | 0     | 0        | 0.192891704 | 0.787607997 | 2.21e-12   | 1.22e-10   | 0.000000606 | 0.00000334  |  |
| 19    | 00-+    | 0          | 0             | -1    | 1        | 0.134054747 | 0.306602194 | 1.05e-11   | 9.39e-10   | 0.00000288  | 0.0000257   |  |
| 20    | 00++    | 0          | 0             | 1     | 1        | 0.126123131 | 0.665760497 | -2.57e-13  | 7.59e-12   | -7.04e-9    | 0.00000208  |  |
| 21    | 0-0+    | 0          | -1            | 0     | 1        | 0.176212246 | 0.794850016 | 6.9e-13    | 9.2e-11    | 0.000000189 | 0.00000251  |  |
| 22    | -+00    | -1         | 1             | 0     | 0        | 0.108954684 | 0.28941944  | 1.22e-12   | 6.59e-11   | 0.000000338 | 0.00000182  |  |
| 23    | -00+    | -1         | 0             | 0     | 1        | 0.131172432 | 0.515158456 | 4.69e-13   | 6.18e-11   | 0.000000128 | 0.00000169  |  |
| 24    | -0-0    | -1         | 0             | -1    | 0        | 0.148748884 | 0.296929128 | 1.42e-11   | 6.29e-10   | 0.00000389  | 0.0000172   |  |
| 25    | 0++0    | 0          | 1             | 1     | 0        | 0.106233377 | 0.319291597 | -2.51e-13  | 8.19e-12   | -6.94e-9    | 0.00000226  |  |
| 26    | 0000    | 0          | 0             | 0     | 0        | 0.142377741 | 0.538398262 | 1.7e-12    | 9.23e-11   | 0.000000466 | 0.00000253  |  |
| 27    | 0-0-    | 0          | -1            | 0     | -1       | 0.210735883 | 0.815954164 | 3.66e-12   | 9.2e-11    | 0.000000999 | 0.00000251  |  |

# Parameter Ranking (1-unit)

#### **Outcome: dissolved CO<sub>2</sub> at End of Monitoring**

| Response SR_EOM             |           |           |         |         |
|-----------------------------|-----------|-----------|---------|---------|
| ⊿ Sorted Parameter Estima   | tes       |           |         |         |
| Term                        | Estimate  | Std Error | t Ratio | Prob> t |
| Brin_Salinity               | -0.238335 | 0.009485  | -25.13  | <.0001* |
| K_Cap                       | 0.1780224 | 0.009485  | 18.77   | <.0001* |
| Brin_Salinity*K_Cap         | -0.098727 | 0.016428  | -6.01   | <.0001* |
| K_Cap*K_Cap                 | -0.054452 | 0.014227  | -3.83   | 0.0024* |
| Inj_Rate                    | -0.009852 | 0.009485  | -1.04   | 0.3194  |
| K_Cap*Inj_Rate              | -0.01543  | 0.016428  | -0.94   | 0.3661  |
| Brin_Salinity*Inj_Rate      | 0.0037931 | 0.016428  | 0.23    | 0.8213  |
| T_Gradient*K_Cap            | -0.003783 | 0.016428  | -0.23   | 0.8217  |
| T_Gradient*Brin_Salinity    | 0.003745  | 0.016428  | 0.23    | 0.8235  |
| Inj_Rate*Inj_Rate           | 0.0028854 | 0.014227  | 0.20    | 0.8427  |
| T_Gradient                  | -0.001359 | 0.009485  | -0.14   | 0.8884  |
| Brin_Salinity*Brin_Salinity | 0.0007546 | 0.014227  | 0.05    | 0.9586  |
| T_Gradient*T_Gradient       | 0.0001758 | 0.014227  | 0.01    | 0.9903  |
| T_Gradient*Inj_Rate         | -4.757e-5 | 0.016428  | -0.00   | 0.9977  |

### **Mineral List**

| Mineral     | Formula   | Init VF (%) |
|-------------|---|-------------|
| Quartz      | SiO <sub>2</sub>  | 43.04213    |
| Calcite     | CaCO <sub>3</sub>   | 4.21872     |
| K-Feldspar  | KAISi <sub>3</sub> O <sub>8</sub>   | 15.77216    |
| Kaolinite   | $AI_2Si_2O_5(OH)_4$   | 0           |
| Albite      | NaAlSi <sub>3</sub> O <sub>8</sub>  | 0           |
| Plagioclase | (Na <sub>0.75</sub> ,Ca <sub>0.25</sub> )(Al <sub>1.25</sub> ,Si <sub>2.75</sub> )O <sub>8</sub>  | 4.06691     |
| Illite      | $K_{0.6}(Mg_{0.25}, AI_{1.8})(AI_{0.5}, Si_{3.5})O_{10}(OH)_2$  | 4.01098     |
| Hematite    | Fe <sub>2</sub> O <sub>3</sub>  | 1.598       |
| Dawsonite   | NaAICO <sub>3</sub> (OH) <sub>2</sub>   | 0           |
| Chlorite    | (Mg <sub>2.5</sub> , Fe <sub>2.5</sub> , AI)(AI, Si <sub>3</sub> )O <sub>10</sub> (OH) <sub>8</sub>                                       | 7.191       |
| Siderite    | FeCO <sub>3</sub>   | 0           |
| Ankerite    | Ca(Mg <sub>1.3</sub> , Fe <sub>0.7</sub> )(CO <sub>3</sub> ) <sub>2</sub>   | 0           |
| Magnesite   | MgCO <sub>3</sub>   | 0           |
| Na-Smectite | $Na_{0.290}(Mg_{0.26}, AI_{1.74})(AI_{0.03}, Si_{3.97})O_{10}(OH)_2$  | 0           |
| Ca-Smectite | Ca <sub>0.145</sub> (Mg <sub>0.26</sub> , Al <sub>1.74</sub> )(Al <sub>0.03</sub> , Si <sub>3.97</sub> )O <sub>10</sub> (OH) <sub>2</sub> | 0           |
| Dolomite    | $(CaMg)(CO_3)_2$  | 0           |

# CO<sub>2</sub> Simulation: Mineral Trapping

- Chlorite can provide cations such as Mg<sup>2+</sup> and Fe<sup>2+</sup>, which are essential chemical components for forming carbonate precipitates.
- The reactions between cations and CO<sub>2</sub> forms carbonate minerals (e.g., siderite, magnesite and ankerite) to trap CO<sub>2</sub> as precipitates.

### Changes in Volume Fraction: Chlorite after 2000 years



var(Ink) = 0.1

## Changes Volume Fraction: Siderite after 2000 years



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# Changes Volume Fraction: Magnesite after 2000 years



# Changes Volume Fraction: without Chlorite after 2000 years



### Summary

- Global upscaling computes equivalent ks for the geologic models with decreasing k resolution; for increasing reservoir ln(k) variances (0.1, 1.0, 4.5), FHM pressure and flow rate are captured well by the geologic models, but errors increase with variance.
- When the variance of ln(k) is low, the 1-unit model yields similar dissolution fingering as the FHM. When the variance of ln(k) is high, the 1-unit predicts more dissolution fingering per unit time (more optimistic dissolution storage estimate).
- Experimental design analysis suggests that brine salinity is the single most influential factor impacting CO<sub>2</sub> dissolution storage.
- Reactions between cations and CO<sub>2</sub> forms carbonate mineral precipitates (i.e., Siderite and Magnesite), leading to mineral storage. But, high degree of uncertainty exists in its prediction.
- Next step: For low and high variance systems, complete the DoE and RS analysis for all models with reactions to compare their parameter sensitivity & prediction uncertainty.

## Appendix

These slides will not be discussed during the presentation, but are mandatory

## **Organization Chart**



### Gantt Chart

| Budget Period | Task Name                                 | Duration | Start        | Finish       | Travel     | PI + Co-PI  | Postdoc     | Personnel+Tr |     | Qtr 4   | l, 2013 |         | Qtr 1,   | 2014     |            | Qtr 2,   | 2014  |         | Qtr 3, | 2014     |         | Qtr 4, 2 | 014     |      |
|---------------|---|----------|--------------|--------------|------------|-------------|-------------|--------------|-----|---------|---------|---------|----------|----------|------------|----------|-------|---------|--------|----------|---------|----------|---------|------|
| 2             |   |          |              |              |            |             |             |              | Sep | Oct     | Nov     | Dec     | Jan      | Feb      | Mar        | Apr      | May   | / Jun   | Jul    | Aug      | Sep     | Oct      | Nov     | Dec  |
| FOUR          | Uncertainty Analysis Fast reacting gec    | 129 days | Fri 11/1/13  | Wed 4/30/14  | \$0.00     | \$15,000.00 | \$36,450.00 | \$51,450.00  |     | FOUR    |         |         | :        |          |            | <u> </u> | \$51  | ,450.00 |        |          |         |          |         |      |
| visit UW      | Co-PI visit UW                            | 4 days   | Tue 11/19/13 | Fri 11/22/13 | \$1,500.00 | \$0.00      | \$0.00      | \$1,500.00   |     | visit   | UW 👔 🤅  | \$1,500 | .00      |          |            |          |       |         |        |          |         |          |         |      |
| AGU meeting   | Project team convene in AGU               | 5 days   | Mon 12/9/13  | Fri 12/13/13 | \$3,600.00 | \$0.00      | \$0.00      | \$3,600.00   |     | AGU ı   | meeting | j 🍵 \$3 | 3,600.00 | )        |            |          |       |         |        |          |         |          |         |      |
| 4.2           | Conduct a screening DoE analysis          | 66 days  | Fri 11/1/13  | Fri 1/31/14  | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     | 4.2     |         |         |          | h        |            |          |       |         |        |          |         |          |         |      |
| 4.3           | Conduct a RS analysis, verification, MC : | 63 days  | Mon 2/3/14   | Wed 4/30/14  | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         | 4.3      | <u> </u> |            | -        |       |         |        |          |         |          |         |      |
| DOE meeting   | PI and/or Co-PI visit DOE NETL            | 3 days?  | Mon 3/10/14  | Wed 3/12/14  | \$1,200.00 | \$0.00      | \$0.00      | \$1,200.00   |     |         |         |         | DOE m    | eeting   | <b>\$1</b> | ,200.00  | )     |         |        |          |         |          |         |      |
| FIVE          | Uncertainty Analysis slow reacting gec    | 132 days | Thu 5/1/14   | Fri 10/31/14 | \$0.00     | \$33,345.55 | \$36,450.00 | \$69,795.55  |     |         |         |         |          |          |            | FIVE     |       |         |        |          |         |          | \$69,79 | 5.55 |
| 5.2           | Conduct a screening DoE analysis          | 66 days  | Thu 5/1/14   | Thu 7/31/14  | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         |          |          |            | 5.2      |       |         |        | ի        |         |          |         |      |
| visit LANL    | PI & postdoc visiting Co-PI               | 5 days?  | Mon 6/30/14  | Fri 7/4/14   | \$3,000.00 | \$0.00      | \$0.00      | \$3,000.00   |     |         |         |         |          |          |            |          | visit | LANL    | \$3,0  | 00.00    |         |          |         |      |
| 5.3           | Conduct a RS analysis, verification, MC : | 66 days  | Fri 8/1/14   | Fri 10/31/14 | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         |          |          |            |          |       |         | 5.3    | <u> </u> |         |          |         |      |
|               |   |          |              |              |            |             |             |              |     |         |         |         |          |          |            |          |       |         |        |          |         |          |         |      |
| Milestone 10  | Updated PMP                               | 2 days   | Fri 11/29/13 | Mon 12/2/13  | \$0.00     | \$0.00      | \$0.00      | \$0.00       | N   | lilesto | one 10  | 0       |          |          |            |          |       |         |        |          |         |          |         |      |
| Milestone 11  | Completion of Task 4.2                    | 2 days   | Fri 1/31/14  | Mon 2/3/14   | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         | N       | lilesto | ne 11    | 0        |            |          |       |         |        |          |         |          |         |      |
| Milestone 12  | Completion of Task 4.3                    | 2 days   | Wed 4/30/14  | Thu 5/1/14   | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         |          | Mi       | ilesto     | ne 12    |       |         |        |          |         |          |         |      |
| Milestone 13  | Updated PMP                               | 2 days   | Fri 5/30/14  | Mon 6/2/14   | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         |          |          | N          | lilestor | ne 13 | 0       |        |          |         |          |         |      |
| Milestone 14  | Completion of Task 5.2                    | 2 days   | Thu 7/31/14  | Fri 8/1/14   | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         |          |          |            |          |       | Milesto | ne 14  | L        |         |          |         |      |
| Milestone 15  | Completion of Task 5.3                    | 2 days   | Fri 10/31/14 | Mon 11/3/14  | \$0.00     | \$0.00      | \$0.00      | \$0.00       |     |         |         |         |          |          |            |          |       |         |        | M        | ileston | ie 15 🔋  |         |      |

### FHM v. 1-Unit Model: $\sigma_{lnk}^2=0.1$



Dept. of Geology & Geophysics, University of Wyoming

An *example 1-Unit model* run for CO2 storage modeling simulated on the Yellowstone supercomputer. The problem domain is 7000 m x 7000 m x 250 m. Shown at 100 years for an isosurface of 0.0125 (mole fraction) of dissolved CO2. CO2 is injected at a depth of 50 m below the top at the center of the xy-domain for 20 years. The grid is 160 x 160 x 25 =0.64 million cells.



### FHM v. 1-Unit Model: $\sigma_{lnk}^2$ =4.5



Dept. of Geology & Geophysics, University of Wyoming

# **PFLOTRAN Scaling on Yellowstone**



### **PFLOTRAN** formulations

To model GCS, the following mass and energy conservation equations are solved:

$$\frac{\partial}{\partial t} \left[ \varphi \sum_{\alpha} (\rho_{\alpha} s_{\alpha} X_{i}^{\alpha}) \right] + \nabla \cdot \sum_{\alpha} (\rho_{\alpha} X_{i}^{\alpha} \vec{q}_{\alpha} - \varphi \rho_{\alpha} s_{\alpha} \tau_{\alpha} D_{\alpha} \nabla X_{i}^{\alpha}) = S_{i}$$
(1)

$$\frac{\partial}{\partial t} \left[ \varphi \sum_{\alpha} (\rho_{\alpha} s_{\alpha} U_{\alpha}) + (1 - \varphi) \rho_{r} C_{p,r} T \right] + \nabla \cdot \left[ \sum_{\alpha} (\vec{q}_{\alpha} \rho_{\alpha} H_{\alpha}) - \lambda \nabla T \right] = Q$$
(2)

 $\varphi$  denotes porosity, and  $\rho_{\alpha}$ ,  $s_{\alpha}$ ,  $\tau_{\alpha}$ ,  $D_{\alpha}$ ,  $U_{\alpha}$ ,  $H_{\alpha}$  refer to the density, saturation, tortuosity, diffusion coefficient, internal energy, and enthalpy of fluid phase  $\alpha$ , respectively. Two fluid phases (CO<sub>2</sub>, brine) will be modeled. The quantities  $X_i^{\alpha}$  denote the mole fraction of component *i* in phase  $\alpha$ . The quantities  $C_{p,r}$  and  $\lambda$  denote the rock heat capacity and conductivity, respectively. The summation is carried out over all fluid phases present in the system. The system is assumed locally to be in thermodynamic equilibrium with temperature  $T(\vec{x}; t)$  at position  $\vec{x}$  and time *t*. The quantity Q denotes an energy source/sink term.

The quantity *Si* denotes a source/sink term for the *i*th primary species describing reaction with minerals given by  $S_i = -\sum_m v_{im} I_m$ , with stoichiometric reaction coefficients  $v_{im}$  and kinetic rate  $I_m$  for the *m*th mineral, taken as positive for precipitation and negative for dissolution.

The flow rate  $\vec{q}_{\alpha}$  of fluid phase  $\alpha$  is given by the extended Darcy's law:  $\vec{q}_{\alpha} = -\frac{\bar{k}k_{\alpha}}{\mu_{\alpha}}(\nabla p_{\alpha} - \rho_{\alpha}gz)$ , with intrinsic permeability  $\bar{k}$ , relative permeability  $k_{\alpha}$ , fluid viscosity  $\mu_{\alpha}$ , and pressure  $p_{\alpha}$  of phase  $\alpha$ .