Implementation and Refinement of a Comprehensive Model for Dense Granular Flows

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Introduction

Motivation:

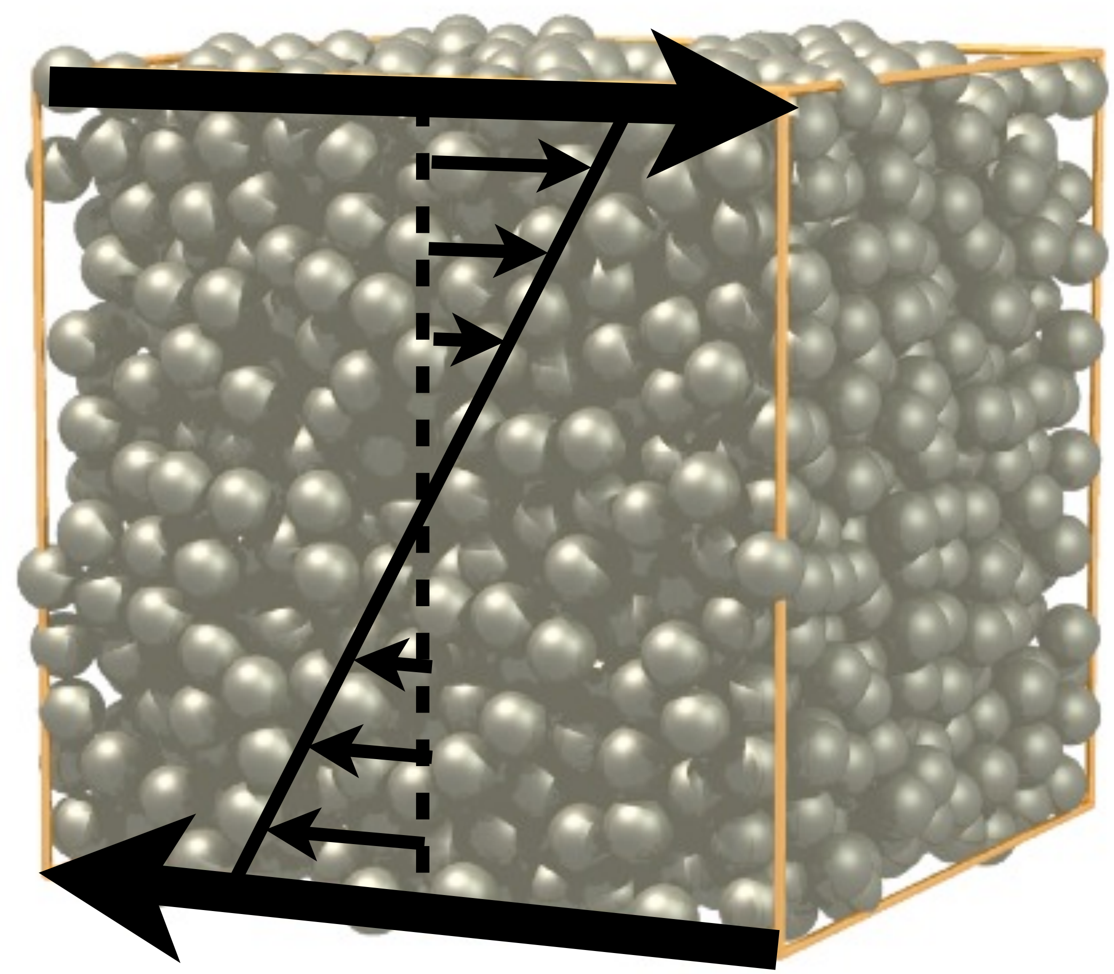
Granular flows exhibit multiple flow regimes characterized by different flow behavior

Goals:

- Develop rheological model spanning three regimes of *dense* granular flow
- Propose modified kinetic-theory model to bridge *dense and dilute* flow behavior
- Develop boundary-condition model for dense flows

Computational methodology

- Simulate particle dynamics of homogeneous assemblies under simple shear using discrete element method (DEM).
  - Linear spring-dashpot with frictional slider.
  - 3D periodic domain without gravity
  - Lees-Edwards boundary conditions or moving walls
- Extract stress and structural information by averaging.

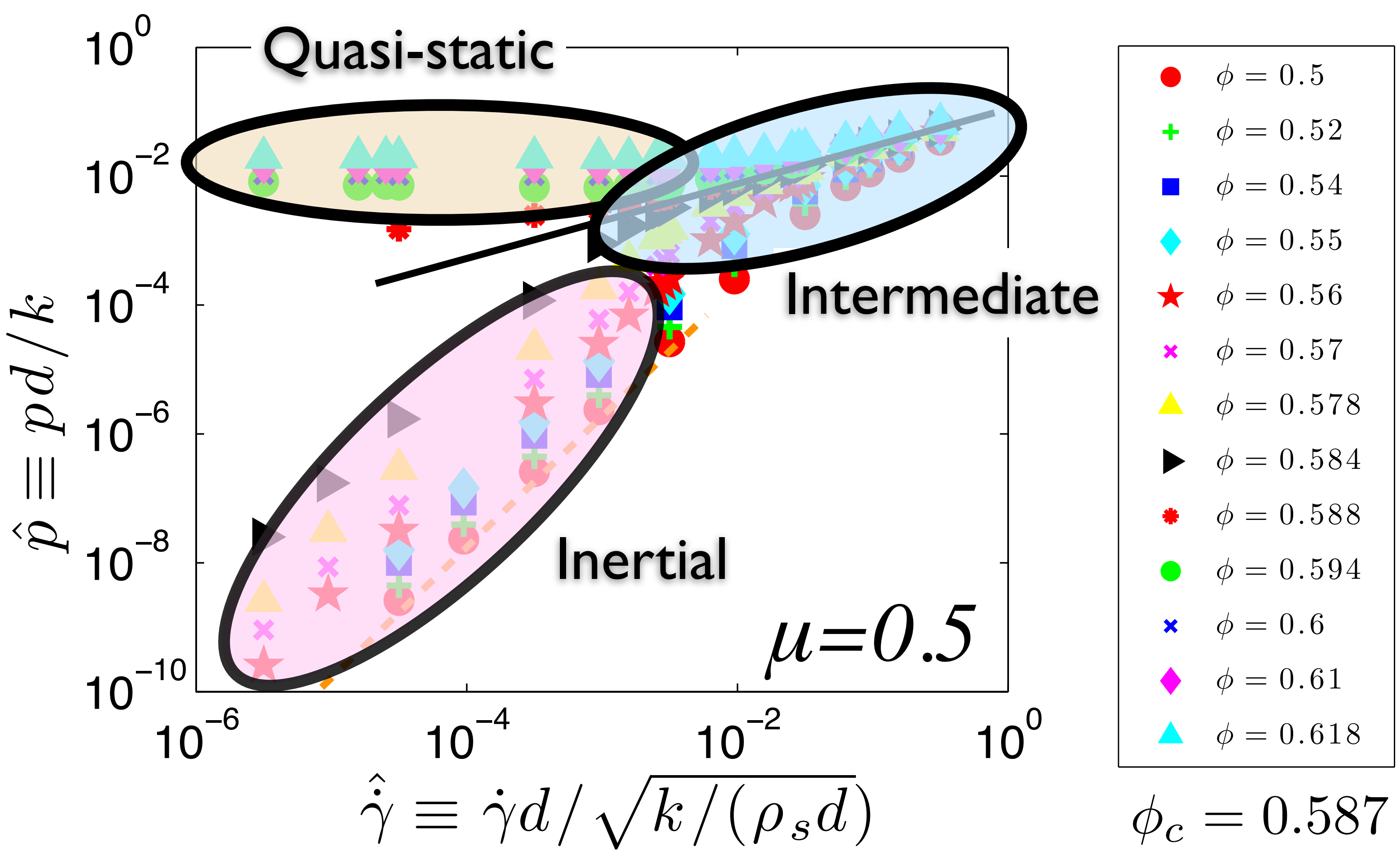


LAMMPS code. http://lammps.sandia.gov S. J. Plimpton. J Comp Phys, 117, 1-19 (1995)

Part I

Development of a rheological model for dense granular flows

Flow map



$\phi_c = 0.587$

Previous studies

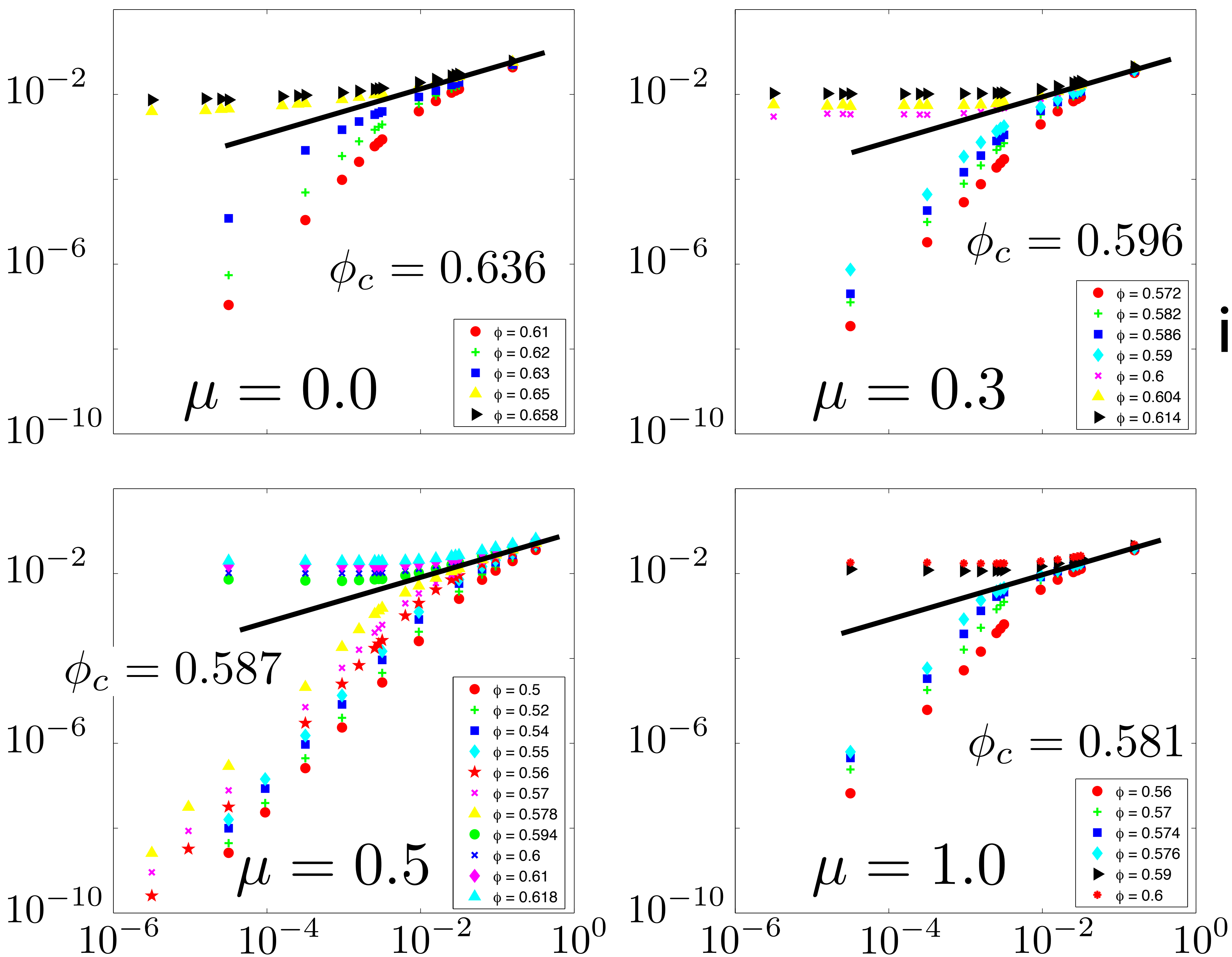
- Computational
  - C. S. Campbell, J. Fluid Mech. 465, 261 (2002).
  - T. Hatano, J. Phys. Soc. Japan 77, 123002 (2008).
- Experimental
  - K. N. Nordstrom *et al.* Phys. Rev. Lett. 105, 175701 (2010).

- Critical volume fraction ϕ_c and its flow curve $\hat{p} = \alpha \dot{\hat{\gamma}}^m$ distinguish the three flow regimes.
- Role of particle softness:
  - Large k $\Longrightarrow$ quasi-static or inertial regime
  - Small k $\Longrightarrow$ intermediate regime

S. Chialvo et al. PRE 85, 021305 (2012).

Effect of μ on pressure

$\hat{p}$ vs. $\hat{\dot{\gamma}}$



Intermediate asymptote

$$pd/k = \alpha \hat{\dot{\gamma}}^{1/2}$$

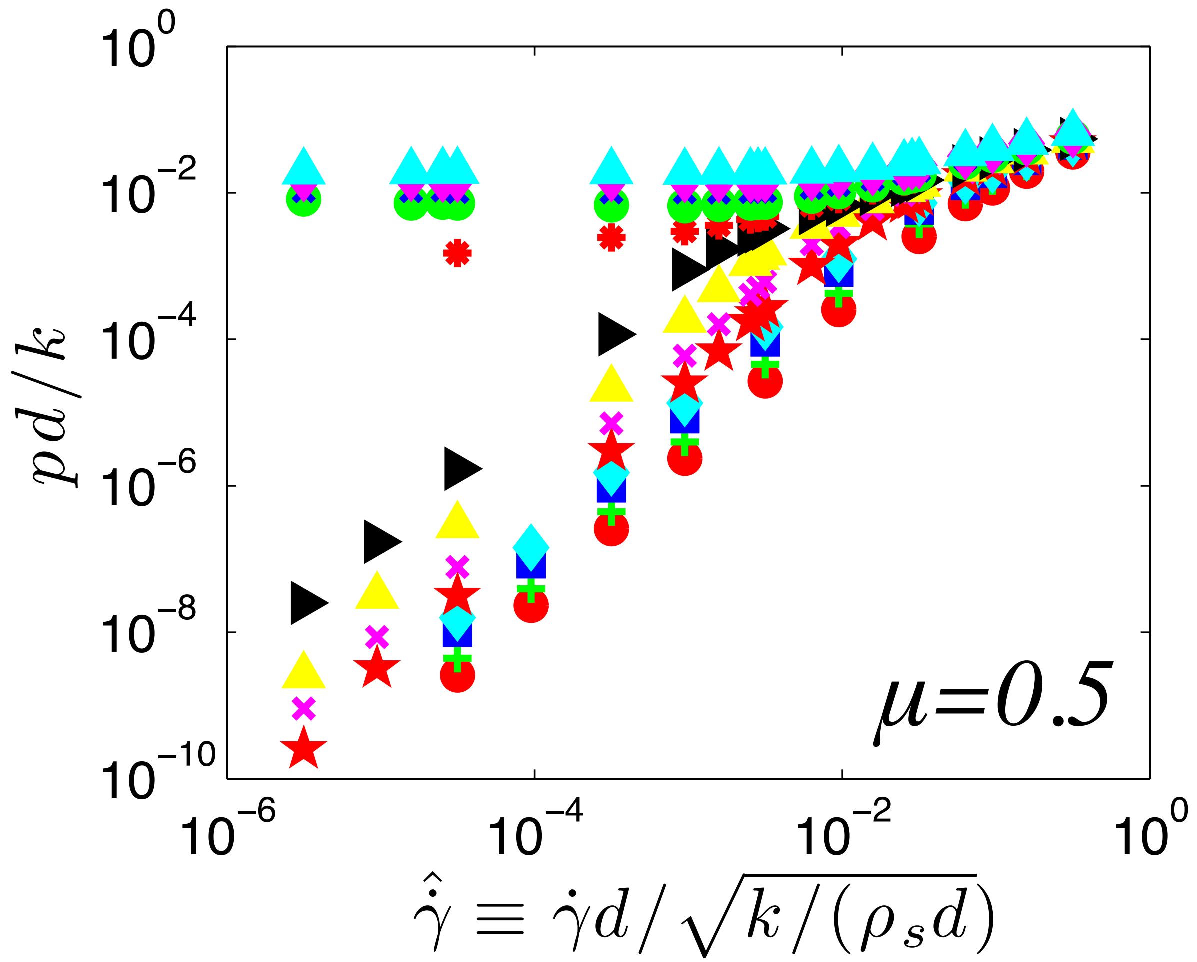
independent of μ

Critical volume fraction

$$\phi_c = \phi_c(\mu)$$

S. Chialvo et al. PRE 85, 021305 (2012).

Pressure scalings for frictional particles



Scaled pressure and shear rate†:

$$p^* = \hat{p}/|\phi - \phi_c|^a$$

$$\dot{\gamma}^* = \hat{\dot{\gamma}}/|\phi - \phi_c|^b$$

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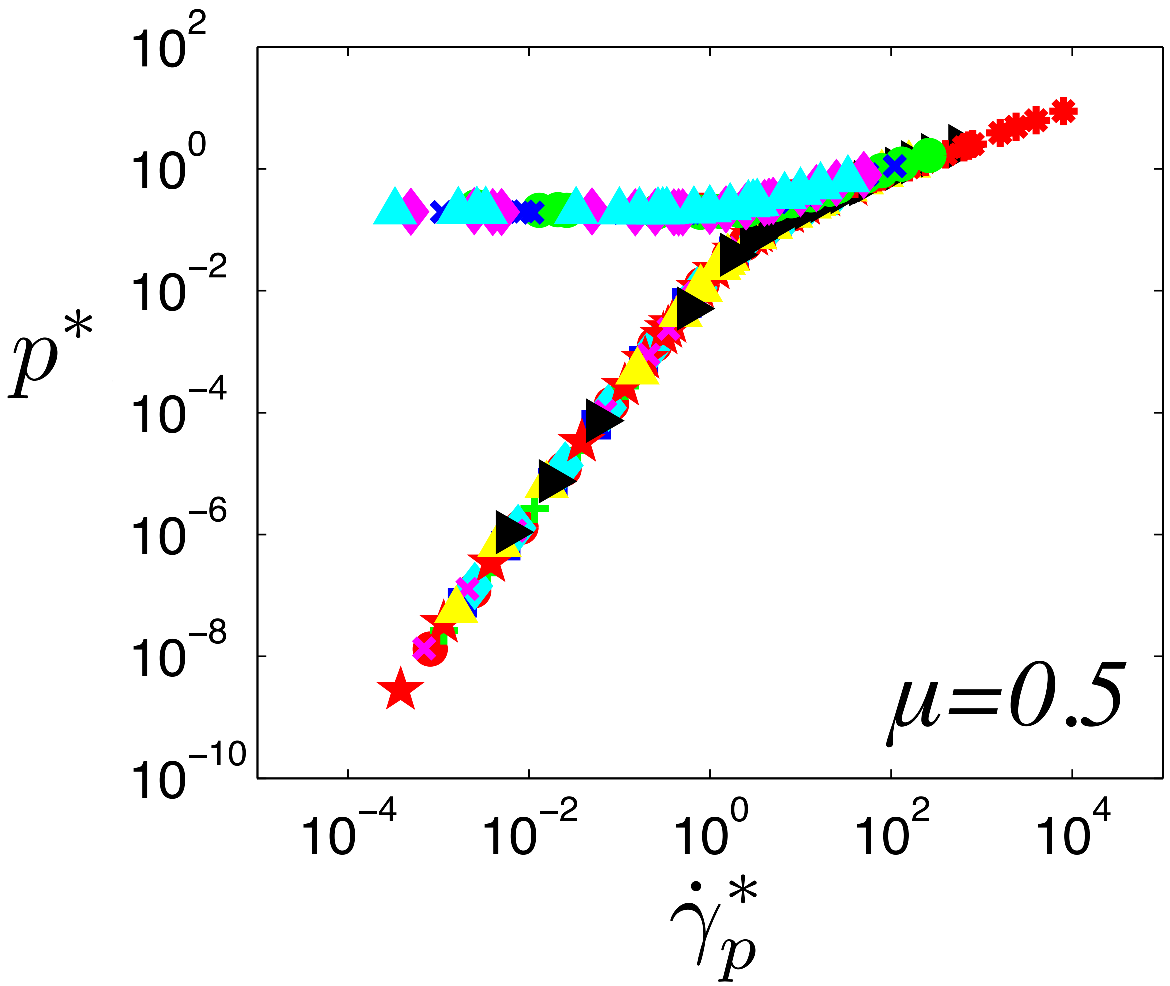

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$$p^* = \hat{p}/|\phi - \phi_c|^a$$

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Choose exponents:

$$a = 2/3$$

$$b = 4/3$$

} Independent of μ

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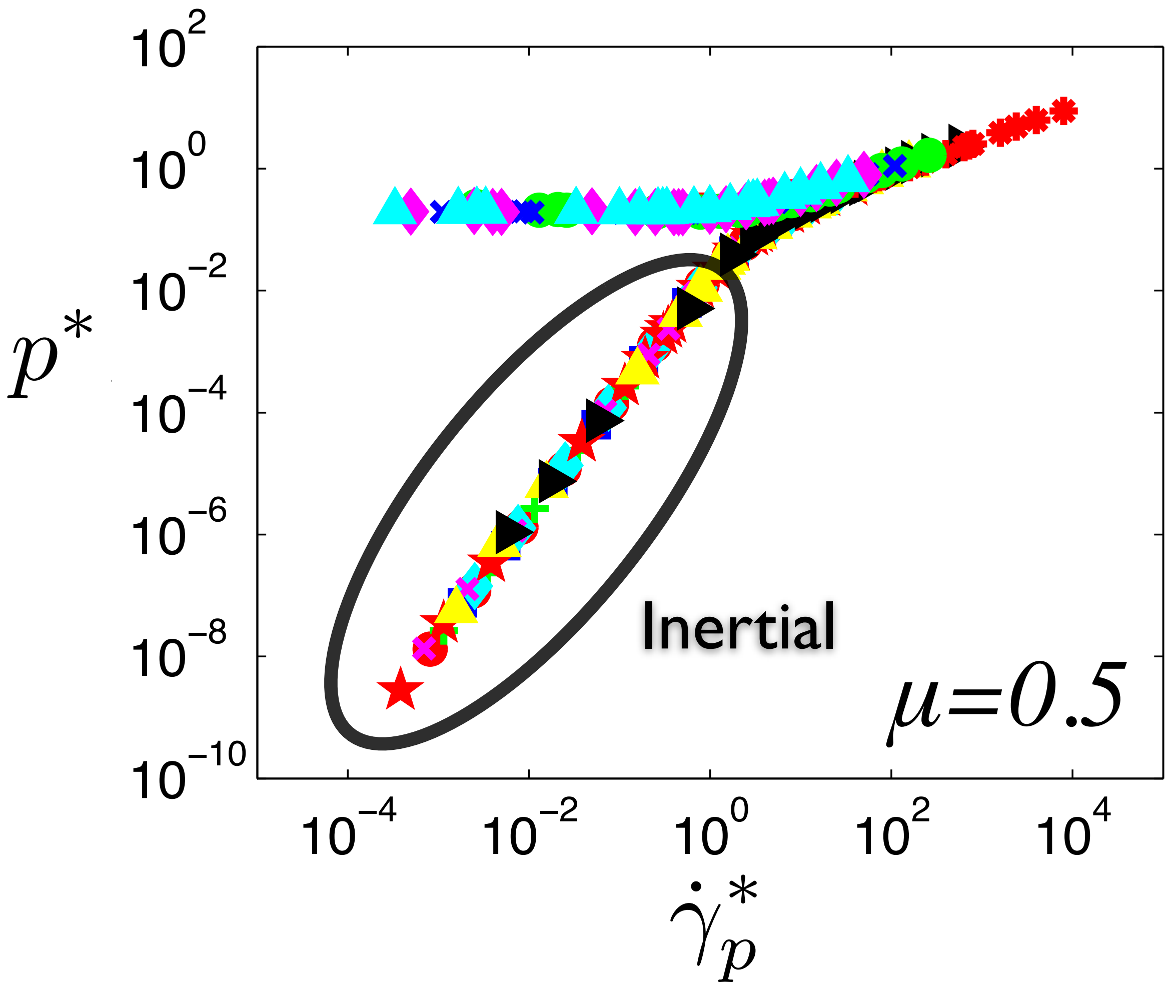
Pressure scalings for frictional particles



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Choose exponents:

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} Independent of μ

Inertial-regime pressure: $$p \sim \frac{\rho_s(\dot{\gamma}d)^2}{(\phi_c - \phi)^2}$$

T. Hatano, J. Phys. Soc. Jpn. 77, 123002 (2008).
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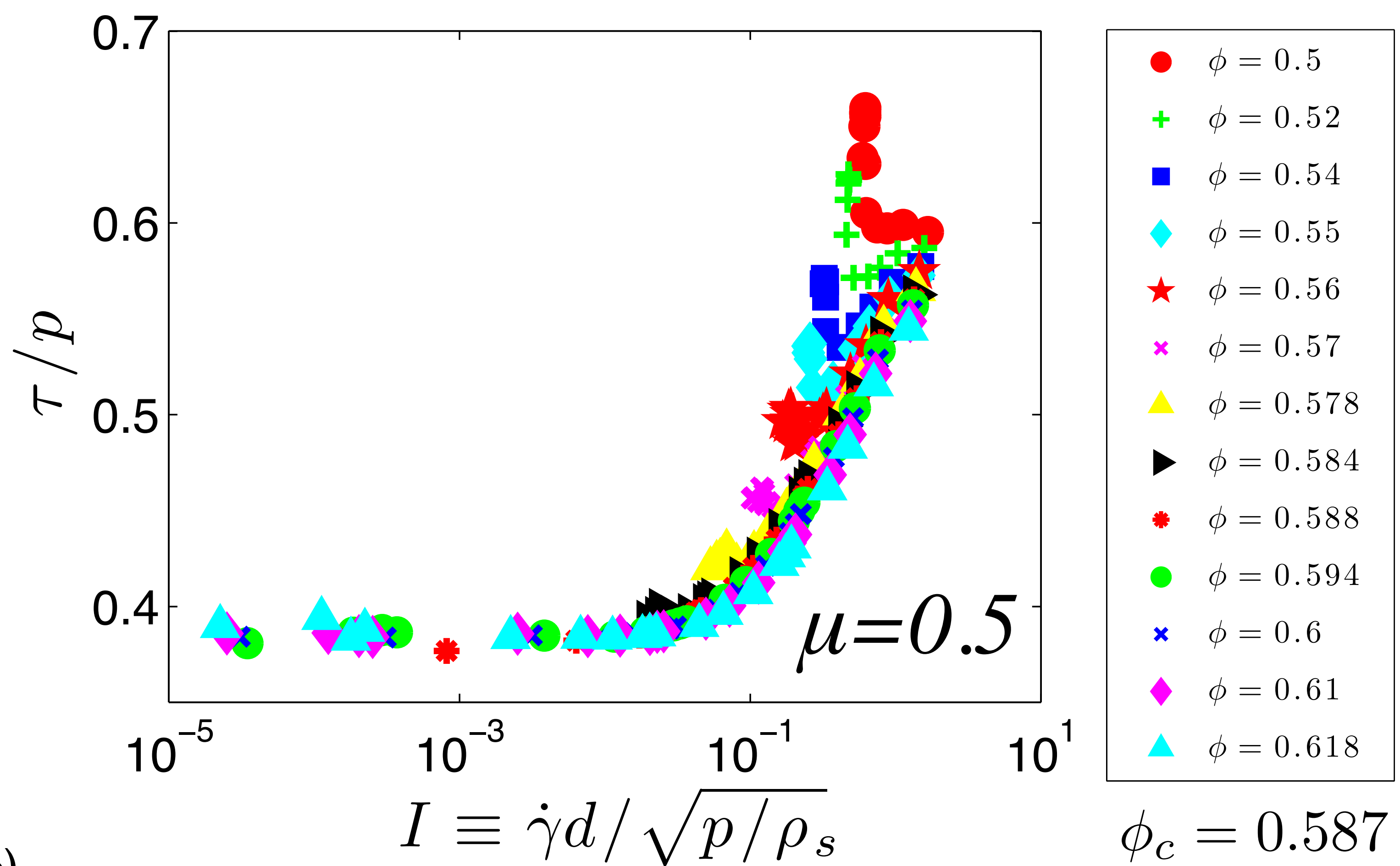
Shear stress ratio

- Definition: $\eta \equiv \tau / p$
- Inertial number†: $I \equiv \dfrac{\dot{\gamma} d}{\sqrt{p/\rho_s}} = \dfrac{\left(\text{timescale of particle rearrangement}\right)}{\left(\text{timescale of macroscopic deformation}\right)}$
- From dimensional analysis of hard spheres: $\eta = \eta(I)$
- Scatter at large I due to particle softness



$\phi_c = 0.587$

- P. Jop, Y. Forterre, and O. Pouliquen, Nature 441, 727 (2006).
- F. da Cruz et al, Phys. Rev. E 72, 021309 (2005).
- S. Chialvo et al. PRE 85, 021305 (2012).

Contributions to shear stress ratio

- Propose a model of the form:

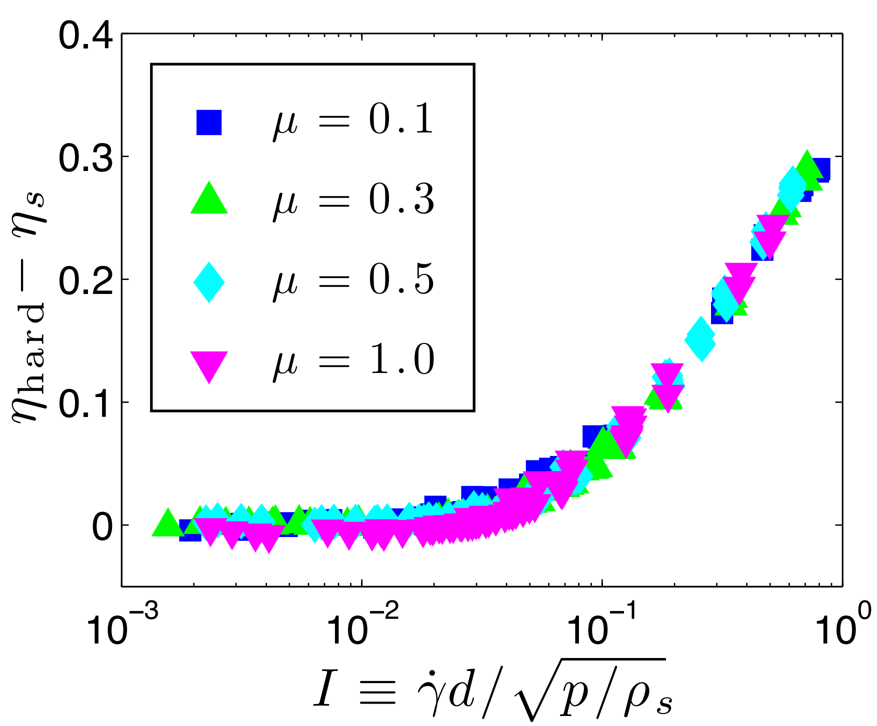
$$I \equiv \frac{\dot\gamma d}{\sqrt{p/\rho_s}}$$

$$\hat{\dot\gamma} \equiv \frac{\dot\gamma d}{\sqrt{k/(\rho_s d)}}$$

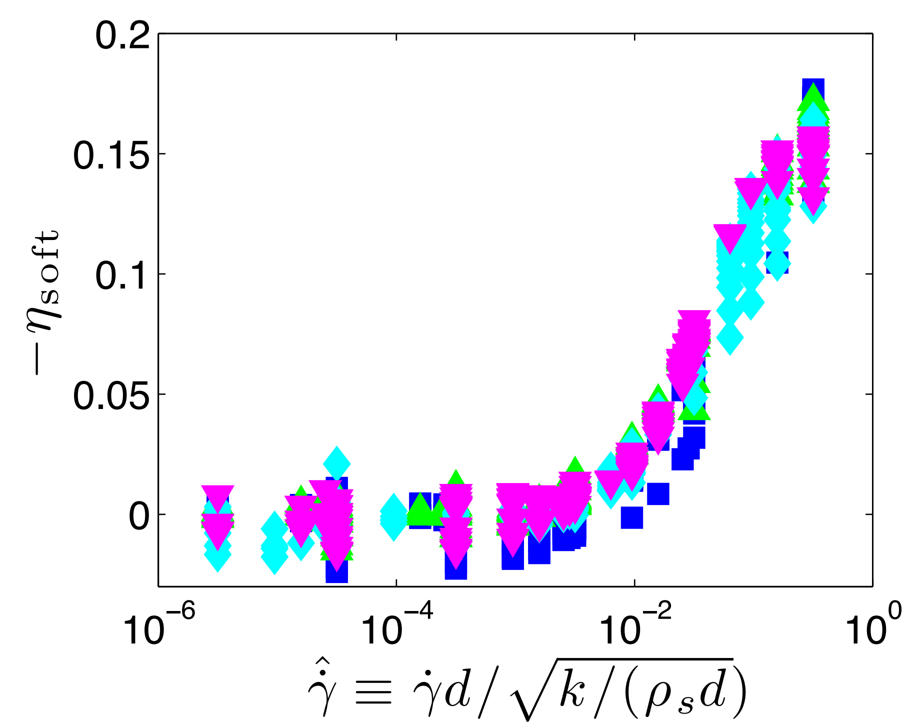
$$\eta(I, \hat{\dot\gamma}) = \eta_{\rm hard}(I) + \eta_{\rm soft}(\hat{\dot\gamma})$$

- Can fit each contribution to DEM data:

$\eta_{\rm hard}(I)$ (inertial and quasistatic)



$\eta_{\rm soft}(\hat{\dot\gamma})$ (intermediate)



- Simple linear model for inertial regime: $\eta = \eta_s + \alpha I$

S. Chialvo et al. PRE 85, 021305 (2012).

Part II

Modifications to a kinetic-theory model to bridge dense and dilute flow behavior

Kinetic-theory models

- Traditionally use kinetic-theory (KT) models for modeling inertial regime
- Most KT models designed for dilute flows of frictionless particles
- Can KT model be modified to capture dense-regime scalings?
- Seek modifications* to KT model of Garzó-Dufty (1999)†

*S. Chialvo, S. Sundaresan, accepted to Phys. Fluids.

†Garzó, V., Dufty, J.W. Phys. Rev. E 59, 5895 (1999).

Kinetic theory equations

Garzó-Dufty kinetic theory for simple shear flow

Pressure

$$p = \rho_s H(\phi, g_0(\phi))T$$

Energy dissipation rate

$$\Gamma = \frac{\rho_s}{d} K(\phi, e) T^{3/2}$$

Shear stress

$$\tau = \rho_s d \dot{\gamma} J(\phi) \sqrt{T}$$

Steady-state energy balance

$$\Gamma - \tau \dot{\gamma} = 0$$

Important quantities:

- Radial distribution function at contact $g_0 = g_0(\phi)$
  - Measure of packing
  - Diverges at random close packing
- Restitution coefficient e
  - Measure of dissipation
  - Has strong effect on temperature

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Modifications (in red)

$$p = \rho_s H(\phi, g_0(\phi, \phi_c(\mu)))T$$

$$\Gamma = \frac{\rho_s}{d} K(\phi, e_{\text{eff}}(e, \mu)) T^{3/2} \delta_\Gamma$$

$$\tau = \tau_s + \rho_s d \dot{\gamma} J(\phi) \sqrt{T} \delta_\tau$$

$$\Gamma - (\tau - \tau_s)\dot{\gamma} = 0$$

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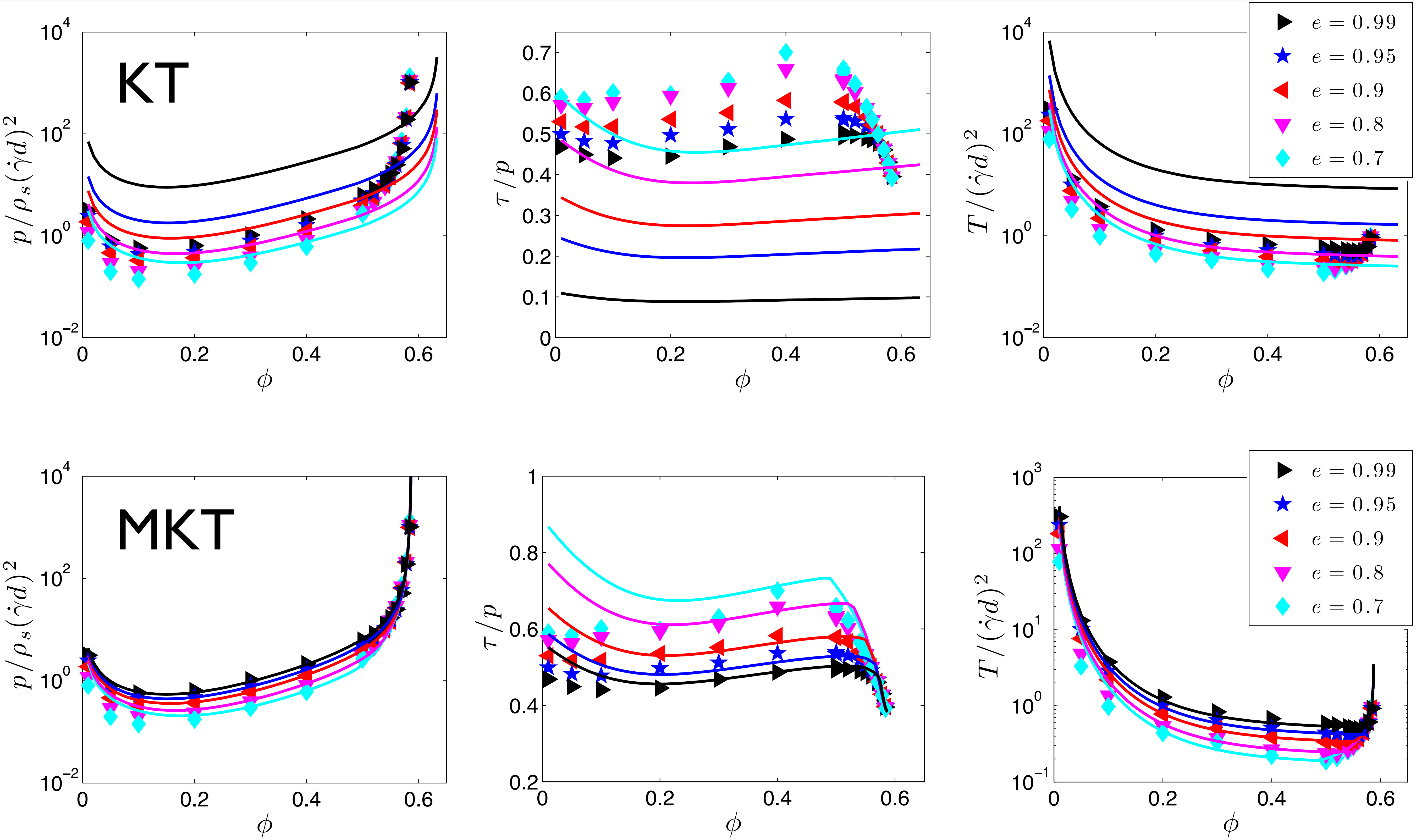
Garzó, V., Dufty, J.W. Phys. Rev. E 59, 5895 (1999).

Comparison of KT and MKT: $\mu = 0.5$



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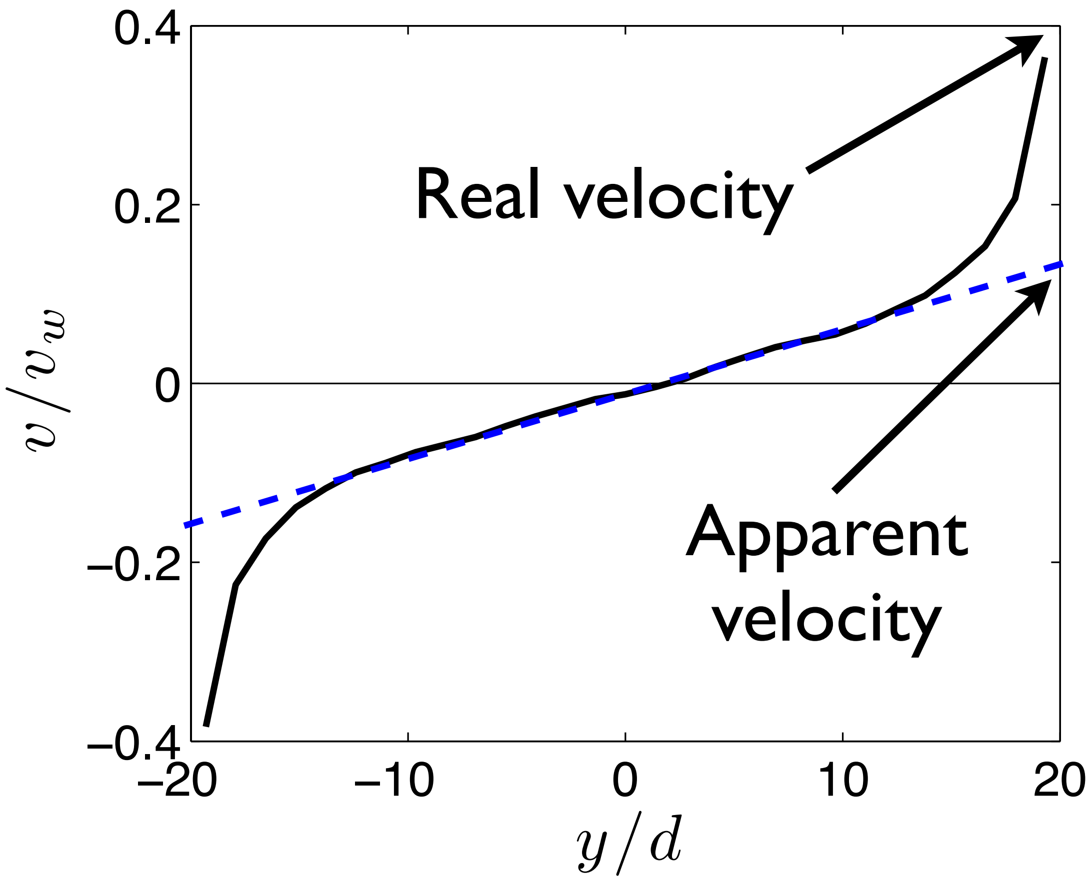
Part III

Development of a boundary-condition model
for dense granular flows

Boundary conditions for granular flows

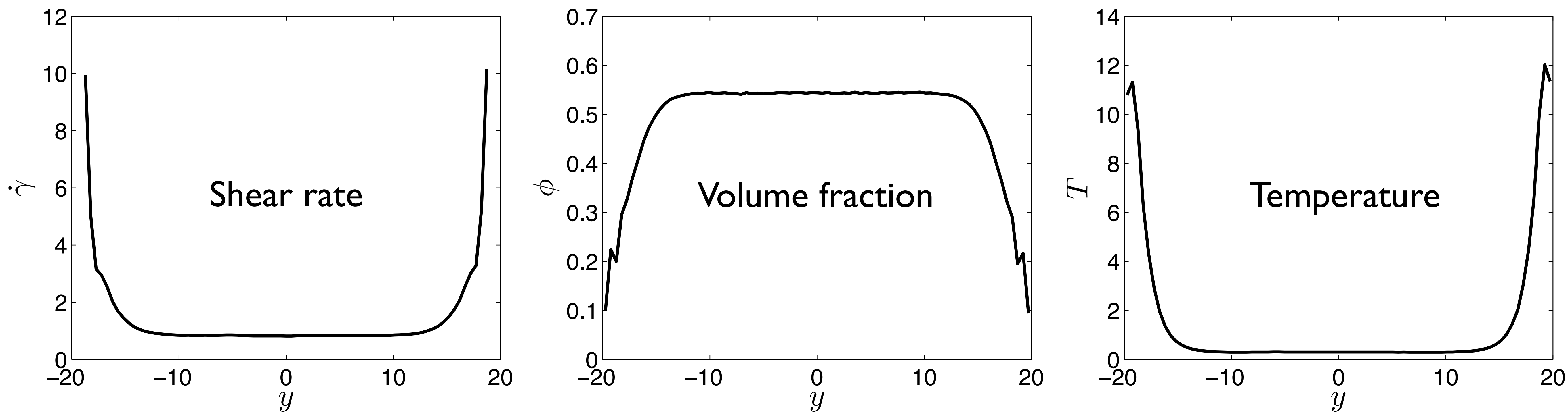
- Solution of continuum models (e.g. with CFD) requires knowing boundary conditions
- Unlike typical fluids that obey no-slip at walls, granular materials may exhibit no, partial, or full slip
- Slip velocity: $\boldsymbol{v}_{\text{slip}} = \boldsymbol{v} - \boldsymbol{v}_w$
- Boundary layer is generally small (~10 particles) and perhaps not worth resolving
- 'Apparent' slip velocity: extrapolate velocity $\boldsymbol{v}$ from core region to wall

$$\boldsymbol{v}_{\text{slip}}^{\text{app}} = \boldsymbol{v}^{\text{app}} - \boldsymbol{v}_w$$



Boundary conditions for granular flows

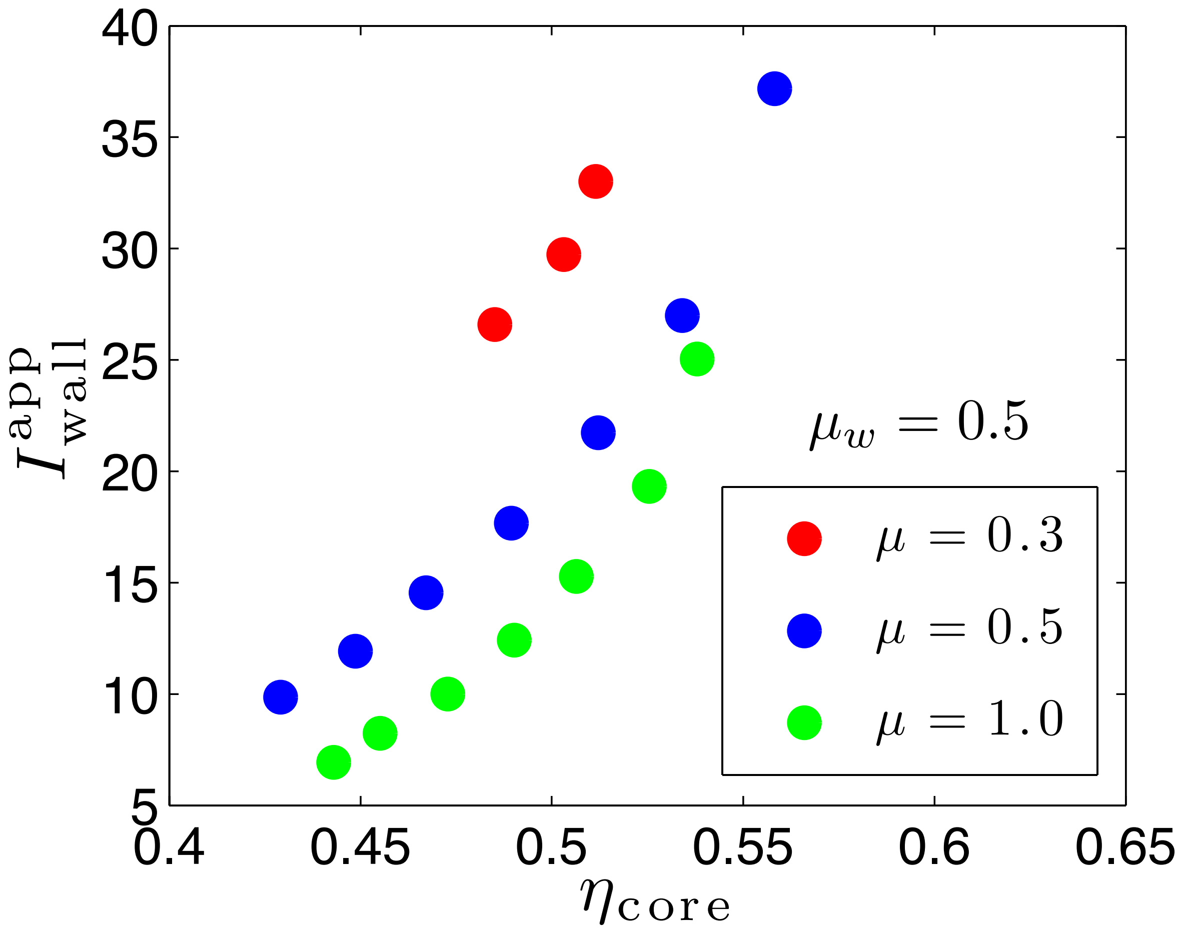
- Performed DEM simulations of wall-bounded simple shear flows
- Core region obeys dense-regime model for pressure and shear stress ratio



- Can we related slip behavior to core conditions?

Boundary conditions for granular flows

- Inertial numbers:
  - $$I_{\text{core}} = \frac{\dot{\gamma}_{\text{core}} d}{\sqrt{p_{\text{core}}/\rho_s}}$$
  - $$I_w^{\text{app}} = \frac{v_s^{\text{app}}}{\sqrt{p_{\text{core}}/\rho_s}}$$



- Possible model form:

$$I_w^{\text{app}} = I_0(\mu, \mu_w) + A(\mu, \mu_w)(\eta_{\text{core}} - \eta_s)^2$$

Summary and future work

- Developed rheological model spanning three regimes of *dense* granular flow*
- Proposed modified kinetic theory to capture rheological behavior for *dense and dilute* systems†
- Developing boundary-condition model for dense flows
- Will soon implement MKT model into MFIX continuum solver for testing on process-scale flow problems

*S. Chialvo et al. PRE 85, 021305 (2012).

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