Fluid-particle drag in polydisperse gas-solid suspensions

William Holloway, Xiaolong Yin, and Sankaran Sundaresan
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Morgantown, WV
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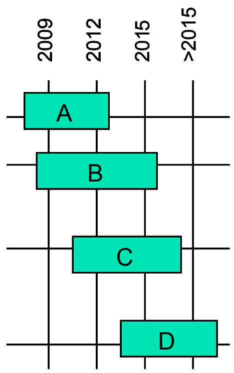
8:40-9:00 am

Outline

- Breakdown of Princeton Tasks
- Bidisperse drag formulation
- Simulation procedures
- Low Re results
- Moderate Re results
- Summary

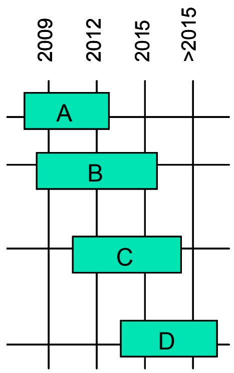
Connection to Roadmap

Princeton Tasks



Roadmap

Connection to Roadmap



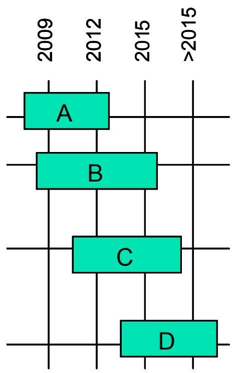
Princeton Tasks

Roadmap

Task 2.2:

LBM/DTIBM simulations of flow through assemblies of binary particle mixtures where the two types of particles have non-zero relative velocities.

Connection to Roadmap



Princeton Tasks

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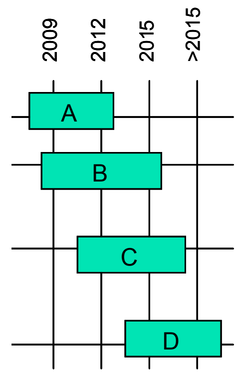
Roadmap

Near-term:

- Develop drag relations that can handle particle size and density distributions; applicable over the entire range of solids volume fraction.
- Development of constitutive relations for continuum models from discrete models such as DEM or LBM.

Connection to Roadmap



Princeton Tasks

Roadmap

Task 2.2:

LBM/DTIBM simulations of flow through assemblies of binary particle mixtures where the two types of particles have non-zero relative velocities.

Near-term:

- Develop drag relations that can handle particle size and density distributions; applicable over the entire range of solids volume fraction.
- Development of constitutive relations for continuum models from discrete models such as DEM or LBM.

Mid-term:

- Consider the effect of lubrication forces in particle-particle interactions.

Fluid-particle drag vs. total force

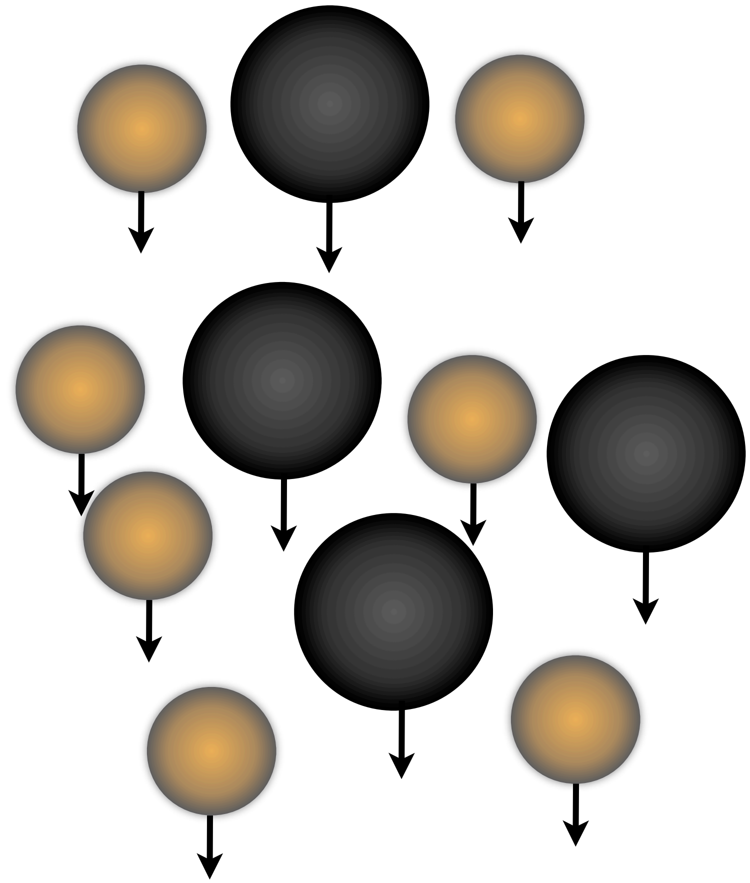
Pressure gradient

Particle-particle drag

$$f_i = -\phi_i \nabla p + f_{Di} - f_{ppdi}$$

Total force experienced by particle type *i*

Fluid-particle drag force



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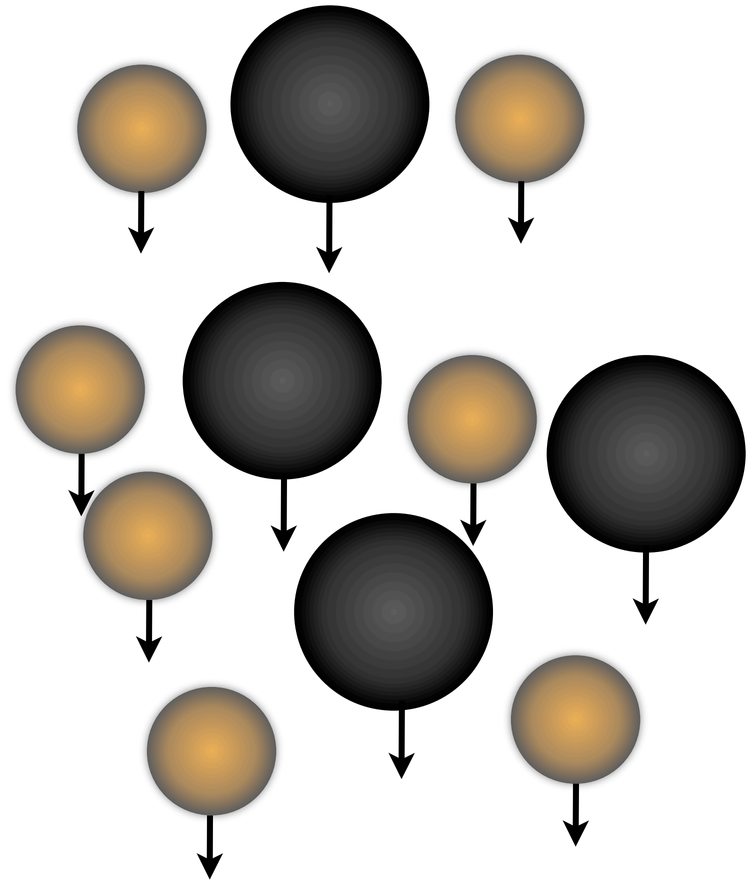

Bidisperse fluid-particle drag formulation:

$$f_{D1} = -\beta_{11}\Delta U_1 - \beta_{12}\Delta U_2$$
$$f_{D2} = -\beta_{21}\Delta U_1 - \beta_{22}\Delta U_2$$

Average fluid-particle drag per unit volume of suspension

Bidisperse drag formulation

$$f_{D1} = -\beta_{11}\Delta U_1 - \beta_{12}\Delta U_2$$
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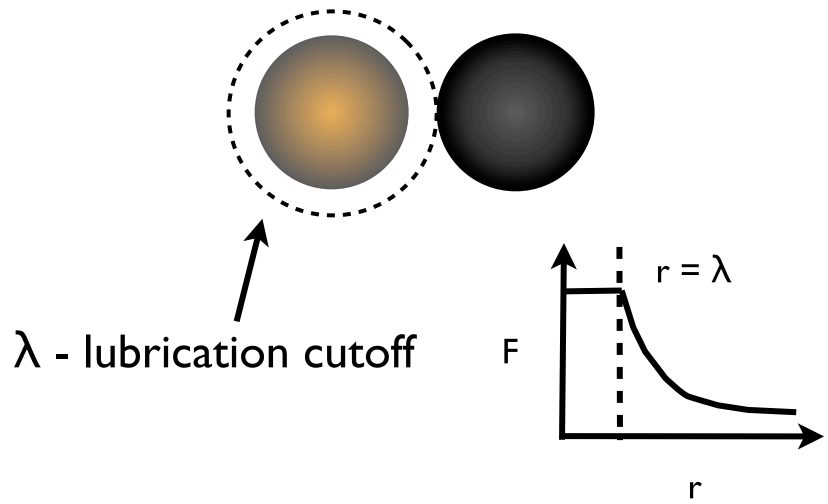
Volume specific friction coefficient



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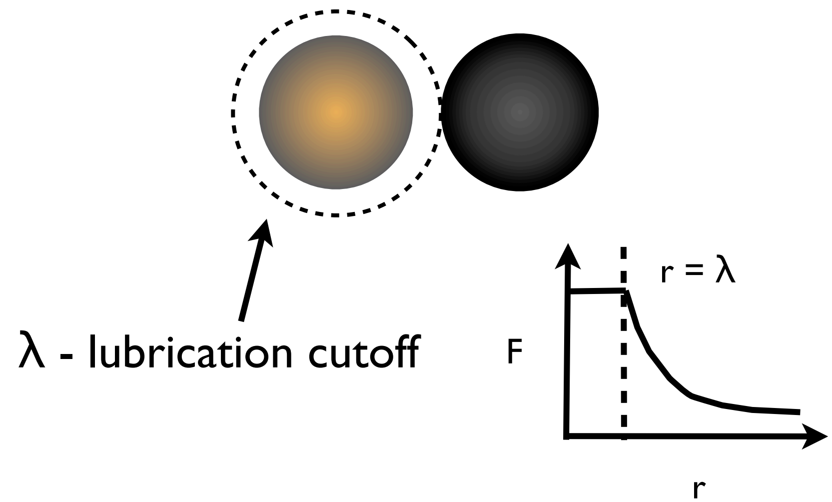


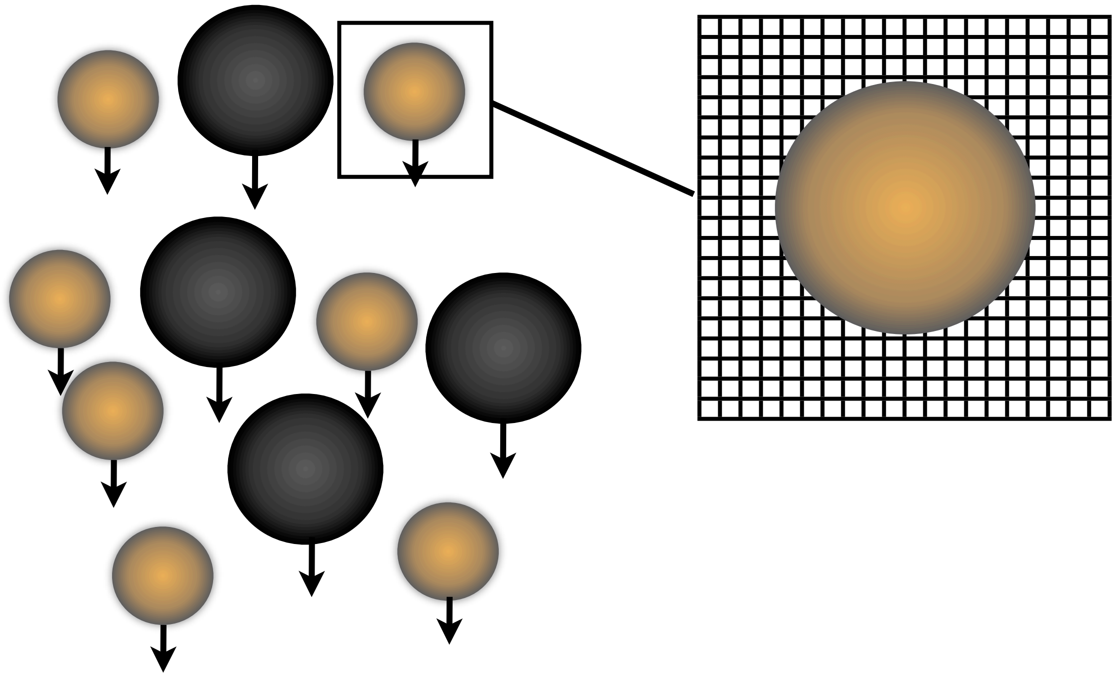
$$\beta_{ij} = \beta_{ij}(\phi_i, \phi_j, d_i, d_j, \Delta U_i, \Delta U_j, < u_i^2 >, < u_j^2 >, \lambda)$$

low Re → $\beta_{ij} = \beta_{ij}(\phi_i, \phi_j, d_i, d_j, \lambda)$

moderate Re → $\beta_{ij} = \beta_{ij}(\phi_i, \phi_j, d_i, d_j, \Delta U_i, \Delta U_j, \lambda)$

Fluctuating particle velocities found to be small contribution to the fluid-particle drag force (Wylie and Koch, *JFM* (2003), vol. 480, pp. 95-118)

Simulation Procedures



Numerical Method: Lattice Boltzmann

Fluid motion solved on a 3D cubic lattice with no slip boundary conditions

LBM References:
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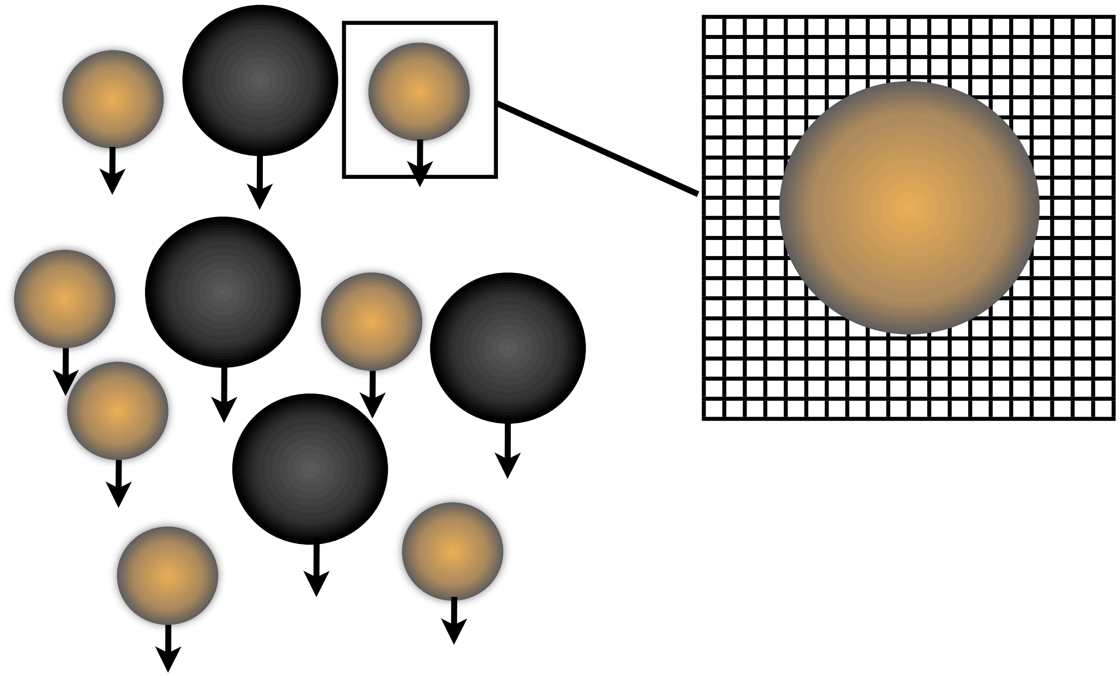
Yin and Sundaresan. *Ind. Eng. Chem. Res.* (2009), vol. 48, pp. 227-241

Simulation Procedures

- Generate initial configurations that satisfy binary hard sphere distribution.



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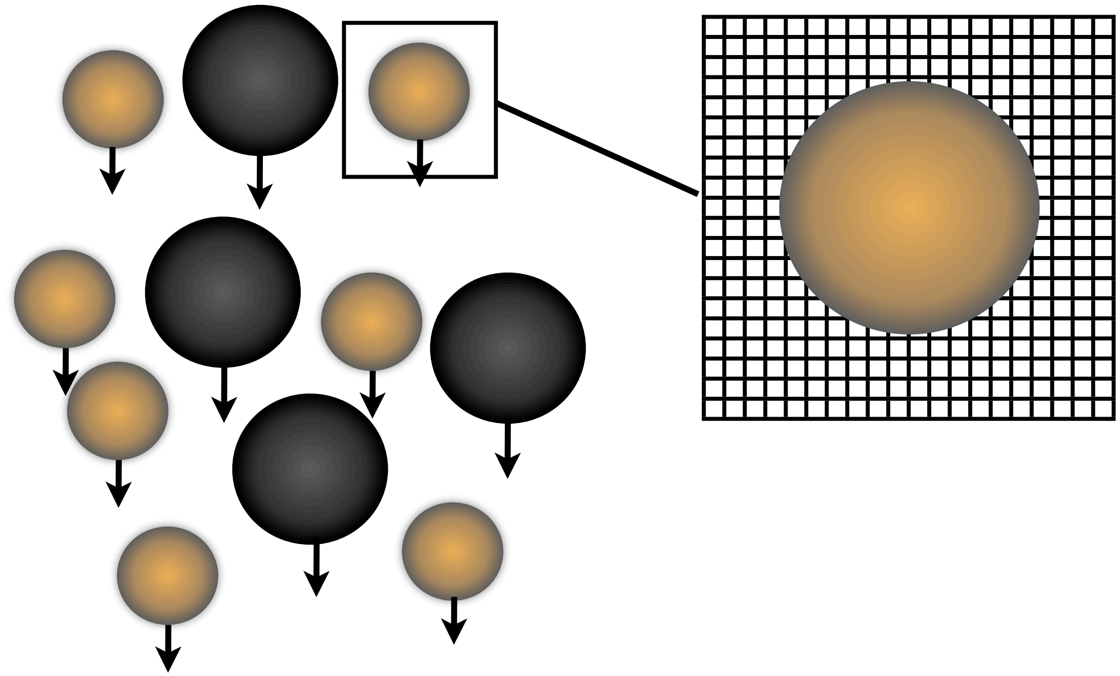
- Generate initial configurations that satisfy binary hard sphere distribution.
- Assign particles with velocities, but do not update particle positions. FROZEN SIMULATIONS (exact for Stokes flow, arguable for finite Re).



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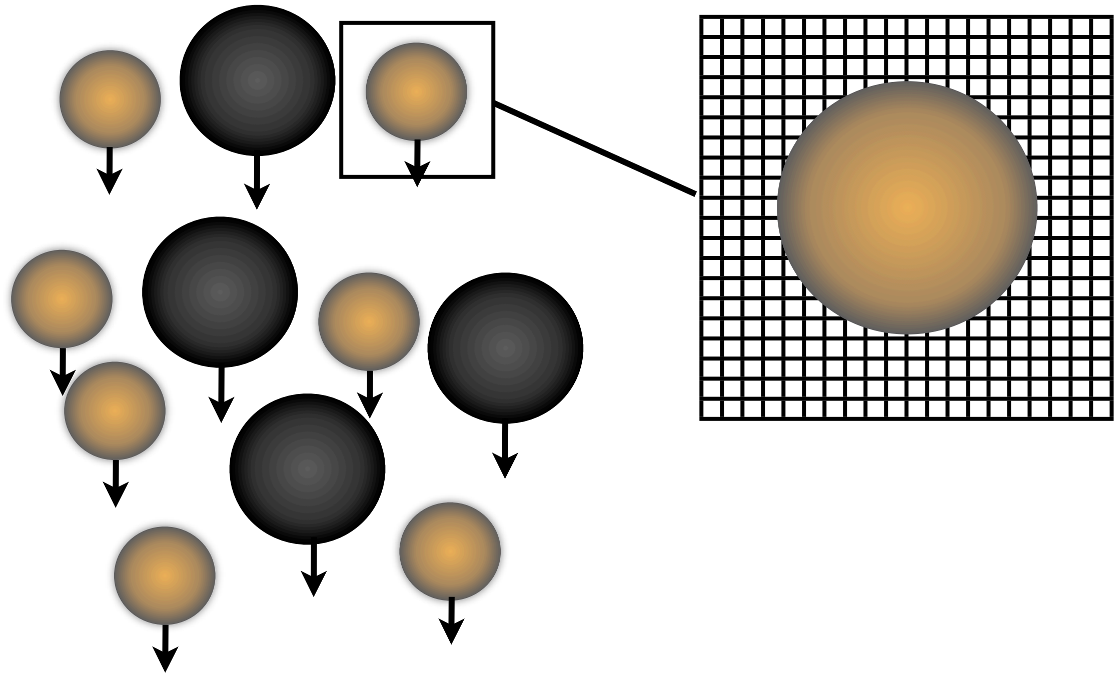

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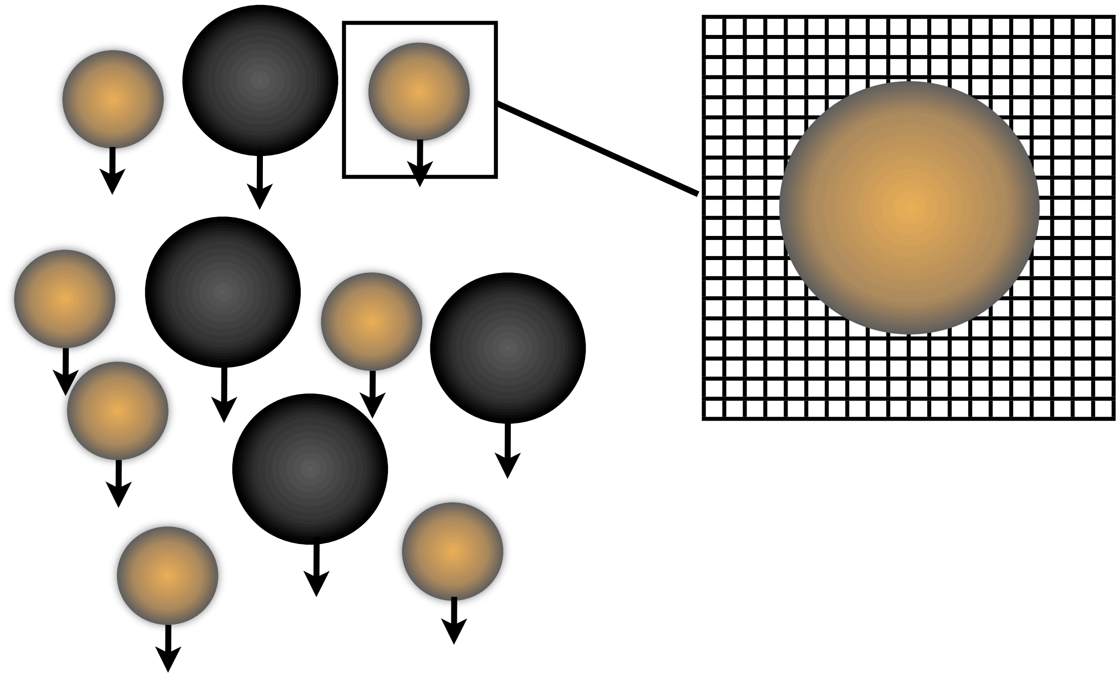
- Generate initial configurations that satisfy binary hard sphere distribution.
- Assign particles with velocities, but do not update particle positions. FROZEN SIMULATIONS (exact for Stokes flow, arguable for finite Re).
- Apply pressure gradient to enforce a net zero flow rate of fluid.
- Ensemble average multiple independent realizations.
- Solve for β_{ij}



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Low Re bidisperse systems

$$f_{D1} = -\beta_{11}\Delta U_1 - \beta_{12}\Delta U_2$$

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$$\beta_{11} + \beta_{12} = \beta_1 = -\frac{f_{D1-fixed}}{\Delta U}$$
$$\beta_{21} + \beta_{22} = \beta_2 = -\frac{f_{D2-fixed}}{\Delta U}$$

Recovery of fixed bed drag when $\Delta U_1 = \Delta U_2$

Yin and Sundaresan. *Ind. Eng. Chem. Res.* (2009), vol. 48, pp. 227-241

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Recovery of fixed bed drag when $\Delta U_1 = \Delta U_2$

$$\begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \begin{pmatrix} \beta_1 - \beta_{12} & \beta_{12} \\ \beta_{12} & \beta_2 - \beta_{12} \end{pmatrix}$$

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↓

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One free parameter

$\Delta U_1 = \Delta U_2$ ('fixed bed') simulations: Extract β_1 and β_2
$\Delta U_1 \neq \Delta U_2$ ('moving suspension') simulations: Extract β_{12}

Yin and Sundaresan. *Ind. Eng. Chem. Res.* (2009), vol. 48, pp. 227-241

Low Re Bidisperse fixed beds

Fixed bed friction coefficient

$$\beta_i = \frac{18\mu\phi_i(1-\phi)}{d_i^2} F^*_{Di-fixed}(\phi, y_i)$$

Dimensionless drag force on particle of type i in a bidisperse fixed bed

Dimensionless size ratio

$$y_i = \frac{d_i}{< d >}$$

Sauter mean diameter

$$< d > = \sum_{i=1}^{n} \frac{n_i d_i^3}{n_i d_i^2}$$

Definitions follow van der Hoef *et al.*, JFM (2005), vol. 528, pp. 233-258

Drag law for bidisperse fixed beds

Drag in a bidisperse fixed bed

Drag in a monodisperse fixed bed

$$F^*_{Di-fixed} = \frac{1}{1-\phi} + \left(F^*_{D-fixed} - \frac{1}{1-\phi} \right) (ay_i + (1-a)y_i^2)$$

$$a = 1 - 2.660\phi + 9.096\phi^2 - 11.338\phi^3$$

$$F^*_{D-fixed} = \frac{10\phi}{(1-\phi)^2} + (1-\phi)^2(1+1.5\sqrt{\phi})$$

Refinement of original correction proposed by van der Hoef *et al.*

Yin and Sundaresan, *AIChE J.*, accepted (2009)
van der Hoef *et. al.*, *JFM*, (2005), vol. 528, pp. 233-254

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$$\frac{dP}{dx} = \frac{18\phi\mu\Delta U}{<d>^2} \left(F^*_{D-fixed} + \frac{1}{1-\phi} \left(\frac{\sigma_I \sigma_{III}}{\sigma_{II}^2} - 1 \right) \right)$$

Integrating over a continuous size distribution we can obtain the pressure drop through a polydisperse fixed bed

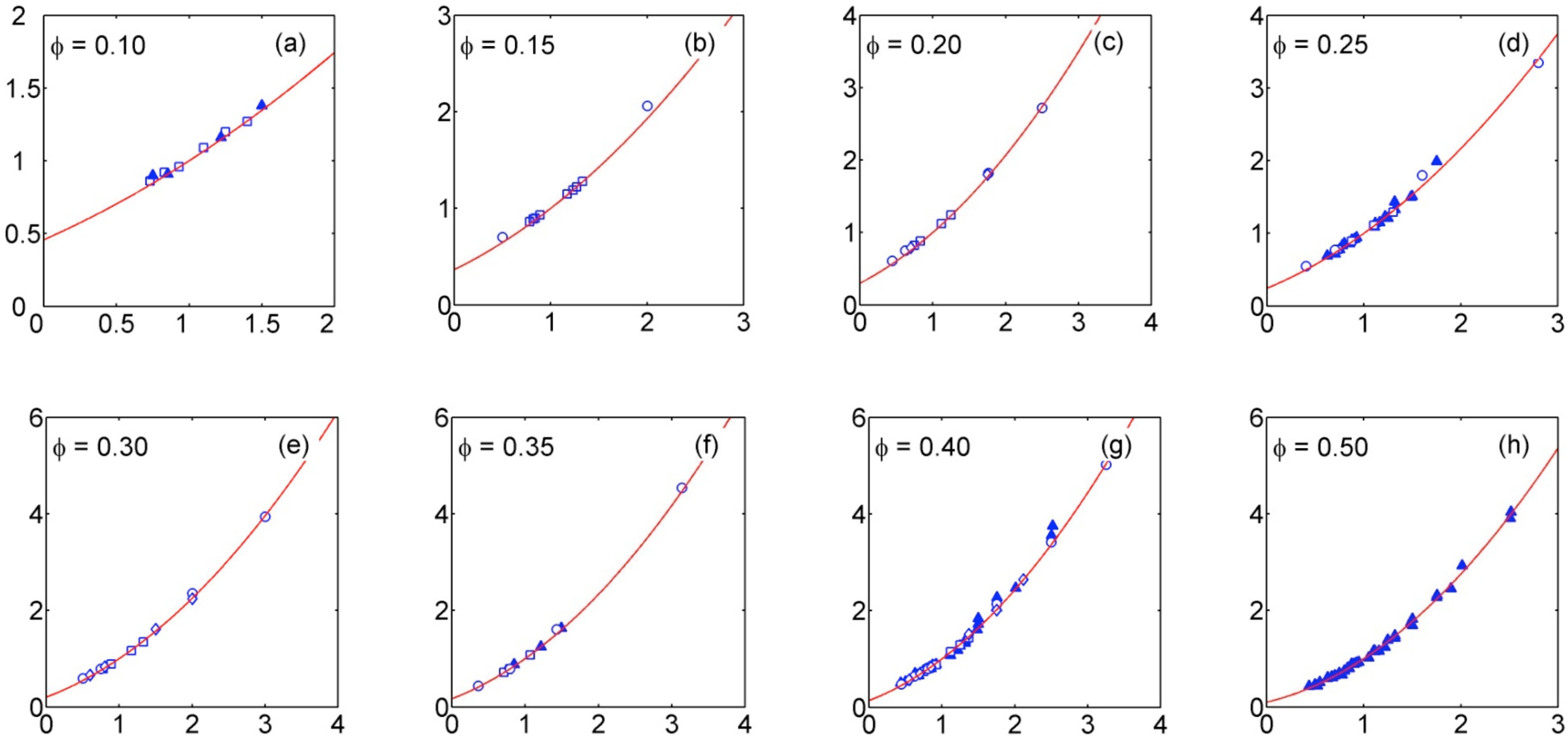
σ_I, σ_{II}, and σ_{III} are first, second, and third order moments of a particle size distribution

Low Re Bidisperse Fixed beds

Horizontal Axis: y_i

Vertical Axis: $\dfrac{F^*_{Di-fixed}}{F^*_{D-fixed}}$

Average error: 3.9%
Max error: 9.4%



Low Re bidisperse suspensions

$$f_{D1} = -\beta_1 \Delta U_1 - \beta_{12} \left(\Delta U_2 - \Delta U_1 \right)$$
$$f_{D2} = -\beta_2 \Delta U_2 - \beta_{12} \left(\Delta U_1 - \Delta U_2 \right)$$

$$\frac{\beta_{12}}{\phi_1 \phi_2} = -2\alpha \left(\frac{\frac{\beta_1}{\phi_1} \frac{\beta_2}{\phi_2}}{\frac{\beta_1}{\phi_1} + \frac{\beta_2}{\phi_2}} \right)$$

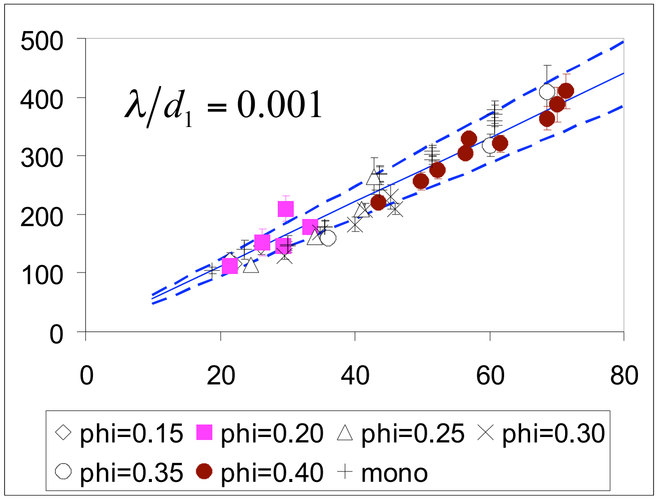
← Harmonic mean

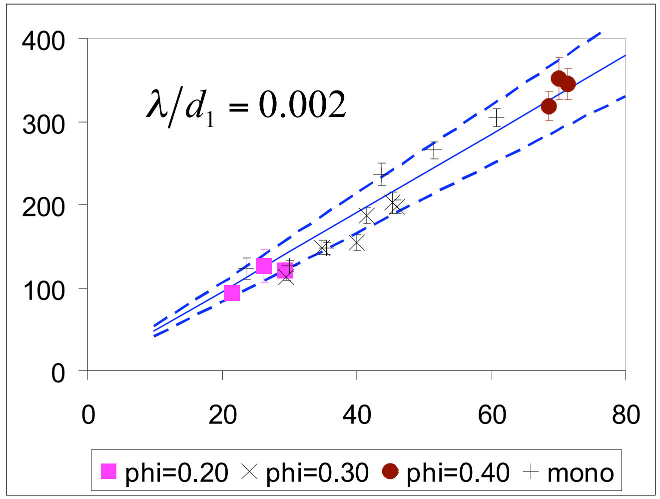
Particle-particle interaction proportional to the probability of mutual contact

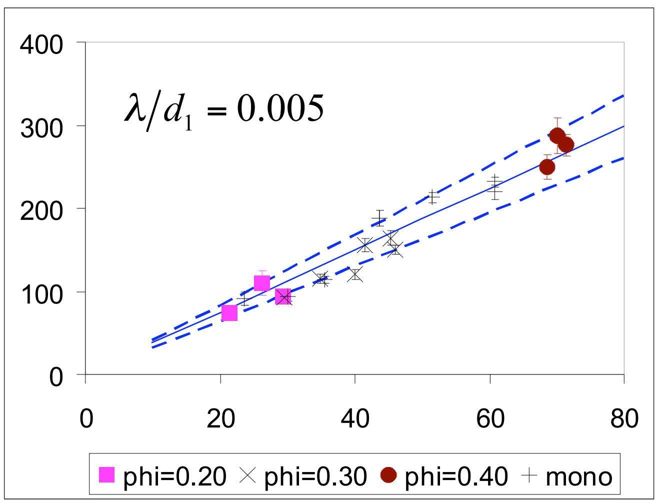
Yin and Sundaresan, *AIChE J.*, accepted (2009)

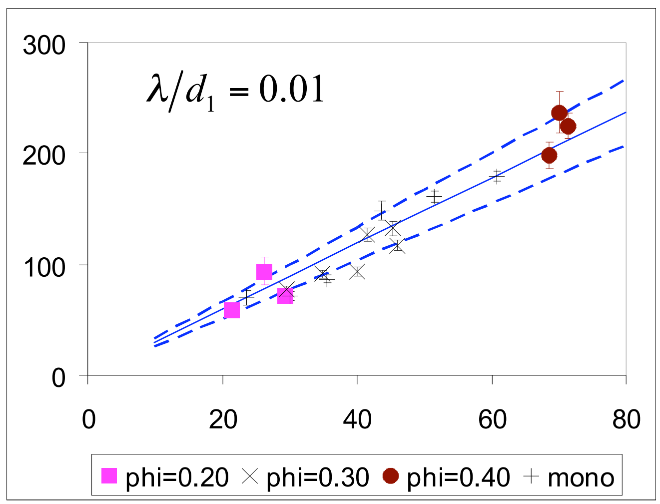
Low Re Bidisperse suspensions

β_{12} is a linear function of the harmonic mean of $\dfrac{\beta_1}{\phi_1}$ and $\dfrac{\beta_2}{\phi_2}$









Horizontal Axis:

$$\left(\frac{\frac{\beta_1^*}{\phi_1} \frac{\beta_2^*}{\phi_2}}{\frac{\beta_1^*}{\phi_1} + \frac{\beta_2^*}{\phi_2}} \right)$$

Vertical Axis:

$$-\frac{\beta_{12}^*}{\phi_1 \phi_2}$$

$$\beta_1^* = \frac{\beta_1 < d >^2}{\mu} \qquad \beta_2^* = \frac{\beta_2 < d >^2}{\mu}$$

$$\beta_{12}^* = \frac{\beta_{12} < d >^2}{\mu}$$

$$\alpha \left(\frac{\lambda}{d_1} \right) = 1.313 log_{10} \left(\frac{d_1}{\lambda} \right) - 1.249$$

Yin and Sundaresan, *AIChE J.*, accepted (2009)

Simulations at finite Re

Frozen suspension at finite Re

Moving suspension at finite Re

- Inertial lag prevents fluid from adapting to particle motion instantaneously

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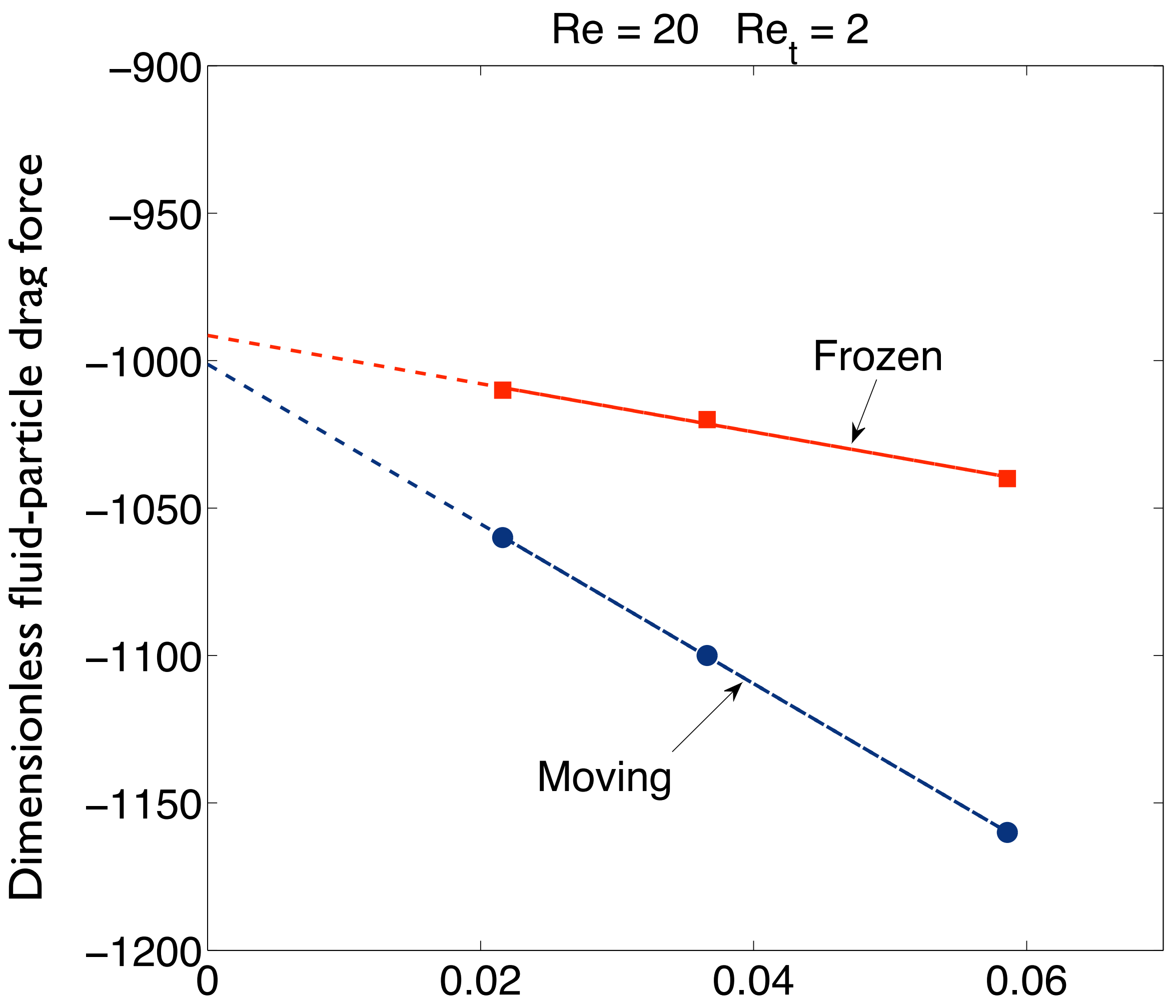
Simulation procedure at finite Re



$$\frac{1}{r_h^2}$$

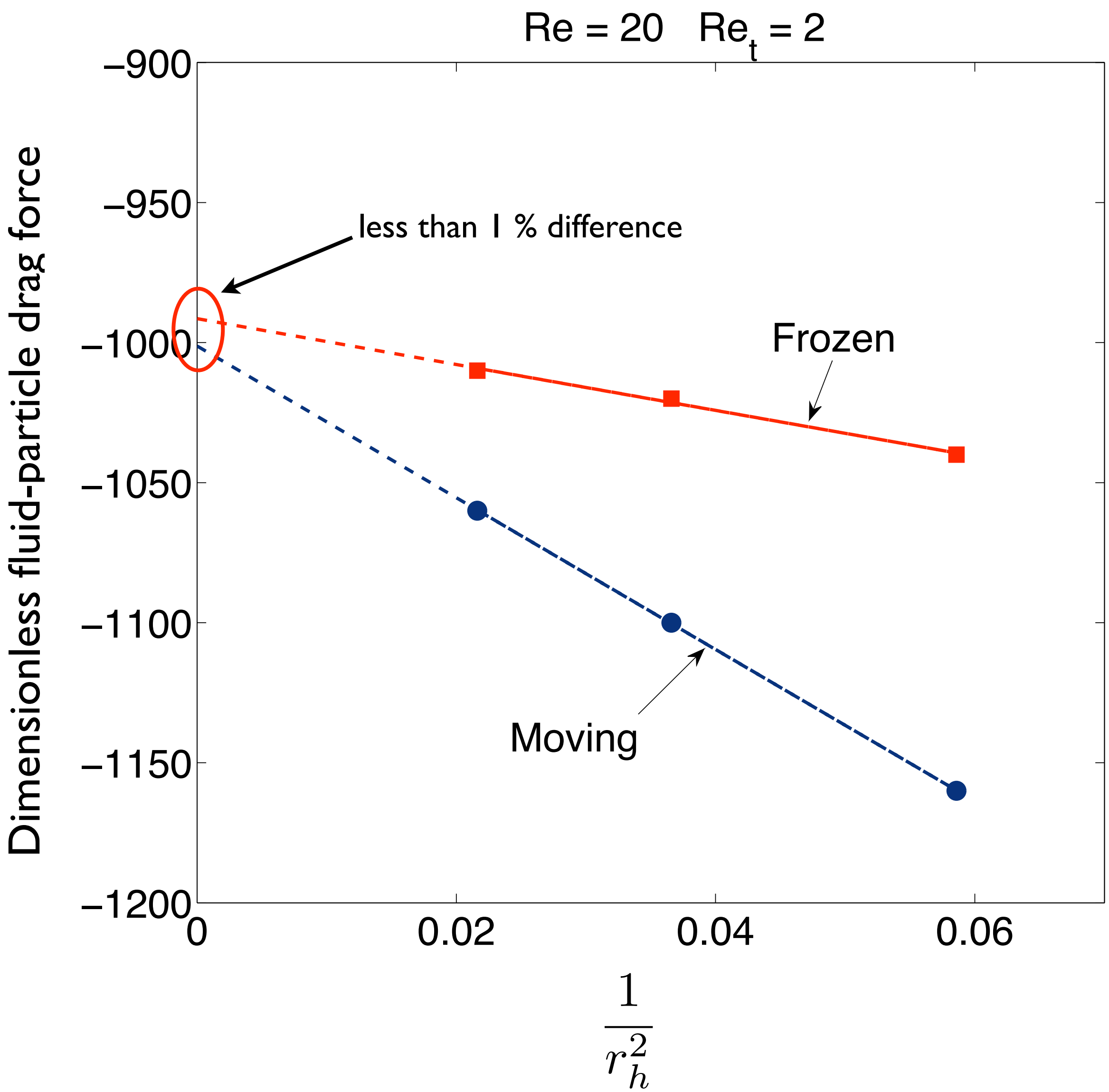
$$r_h = \frac{d(1-\phi)}{6\phi}$$

Average pore radius

van der Hoef *et al.*, JFM (2005), vol. 528, pp. 233-258 (extrapolation procedure)

Must extrapolate frozen simulations to infinite resolution to get an accurate measure of the drag force in a moving suspension.

Simulation procedure at finite Re



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Drag law for finite Re

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$$f_{D2} = -\beta_2 \Delta U_2 - \beta_{12} \left(\Delta U_1 - \Delta U_2 \right)$$

$$\beta_i = \frac{18(1-\phi)\phi_i \mu}{d_i^2} F^*_{Di-fixed} \qquad \frac{\beta_{12}}{\phi_1 \phi_2} = -2\alpha \left(\frac{\frac{\beta_1}{\phi_1} \frac{\beta_2}{\phi_2}}{\frac{\beta_1}{\phi_1} + \frac{\beta_2}{\phi_2}} \right) \qquad \alpha \left(\frac{\lambda}{d_1} \right) = 1.313 log_{10} \left(\frac{d_1}{\lambda} \right) - 1.249$$

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$$F^*_{Di-fixed} = \frac{1}{1-\phi} + \left(F^*_{D-fixed} - \frac{1}{1-\phi} \right) \left(a y_i + (1-a) y_i^2 \right)$$

$$F^*_{D-fixed} = \left(\frac{10\phi}{(1-\phi)^2} + (1-\phi)^2 (1 + 1.5\sqrt{\phi}) \right)$$

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$$F^*_{Di-fixed} = \frac{1}{1-\phi} + \left(F^*_{D-fixed} - \frac{1}{1-\phi}\right)\left(a y_i + (1-a) y_i^2\right)$$

$$F^*_{D-fixed} = \left(\frac{10\phi}{(1-\phi)^2} + (1-\phi)^2 (1 + 1.5\sqrt{\phi})\right)(1 + \chi_{BVK})$$

Beetstra, van der Hoef, and Kuipers., (2007) *AIChE J.*, vol. 53, pp. 489-501

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$$F^*_{D-fixed} = \left(\frac{10\phi}{(1-\phi)^2} + (1-\phi)^2(1+1.5\sqrt{\phi})\right)(1+\chi_{BVK})$$

$$\chi_{BVK} = \frac{0.413 Re_{mix}}{240\phi + 24(1-\phi)^4(1+1.5\sqrt{\phi})} \frac{(1-\phi)^{-1} + 3\phi(1-\phi) + 8.4 Re_{mix}^{-0.343}}{1 + 10^{3\phi} Re_{mix}^{\frac{-(1+4\phi)}{2}}}$$

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$$F^*_{Di-fixed} = \frac{1}{1-\phi} + \left(F^*_{D-fixed} - \frac{1}{1-\phi}\right)\left(a y_i + (1-a) y_i^2\right)$$

$$F^*_{D-fixed} = \left(\frac{10\phi}{(1-\phi)^2} + (1-\phi)^2 (1 + 1.5\sqrt{\phi})\right)(1 + \chi_{BVK})$$

$$\chi_{BVK} = \frac{0.413 Re_{mix}}{240\phi + 24(1-\phi)^4 (1 + 1.5\sqrt{\phi})} \frac{(1-\phi)^{-1} + 3\phi(1-\phi) + 8.4 Re_{mix}^{-0.343}}{1 + 10^{3\phi} Re_{mix}^{\frac{-(1+4\phi)}{2}}}$$

$$Re_{mix} = \frac{|\Delta U_{mix}| < d > (1-\phi)}{\nu}$$

Beetstra, van der Hoef, and Kuipers., (2007) *AIChE J.*, vol. 53, pp. 489-501

Drag law for finite Re

$$f_{D1} = -\beta_1 \Delta U_1 - \beta_{12} \left(\Delta U_2 - \Delta U_1 \right)$$

$$f_{D2} = -\beta_2 \Delta U_2 - \beta_{12} \left(\Delta U_1 - \Delta U_2 \right)$$

$$\beta_i = \frac{18(1-\phi)\phi_i \mu}{d_i^2} F^*_{Di-fixed} \qquad \frac{\beta_{12}}{\phi_1 \phi_2} = -2\alpha \left(\frac{\frac{\beta_1}{\phi_1} \frac{\beta_2}{\phi_2}}{\frac{\beta_1}{\phi_1} + \frac{\beta_2}{\phi_2}} \right) \qquad \alpha\left(\frac{\lambda}{d_1}\right) = 1.313 log_{10}\left(\frac{d_1}{\lambda}\right) - 1.249$$

$$F^*_{Di-fixed} = \frac{1}{1-\phi} + \left(F^*_{D-fixed} - \frac{1}{1-\phi} \right) (a y_i + (1-a) y_i^2)$$

$$F^*_{D-fixed} = \left(\frac{10\phi}{(1-\phi)^2} + (1-\phi)^2 (1 + 1.5\sqrt{\phi}) \right) (1 + \chi_{BVK})$$

$$\chi_{BVK} = \frac{0.413 Re_{mix}}{240\phi + 24(1-\phi)^4 (1 + 1.5\sqrt{\phi})} \frac{(1-\phi)^{-1} + 3\phi(1-\phi) + 8.4 Re_{mix}^{-0.343}}{1 + 10^{3\phi} Re_{mix}^{\frac{-(1+4\phi)}{2}}}$$

$$Re_{mix} = \frac{|\Delta U_{mix}| < d > (1-\phi)}{\nu} \qquad \Delta U_{mix} = \frac{1}{\phi} \sum_{i=1}^{n} \phi_i \Delta U_i$$

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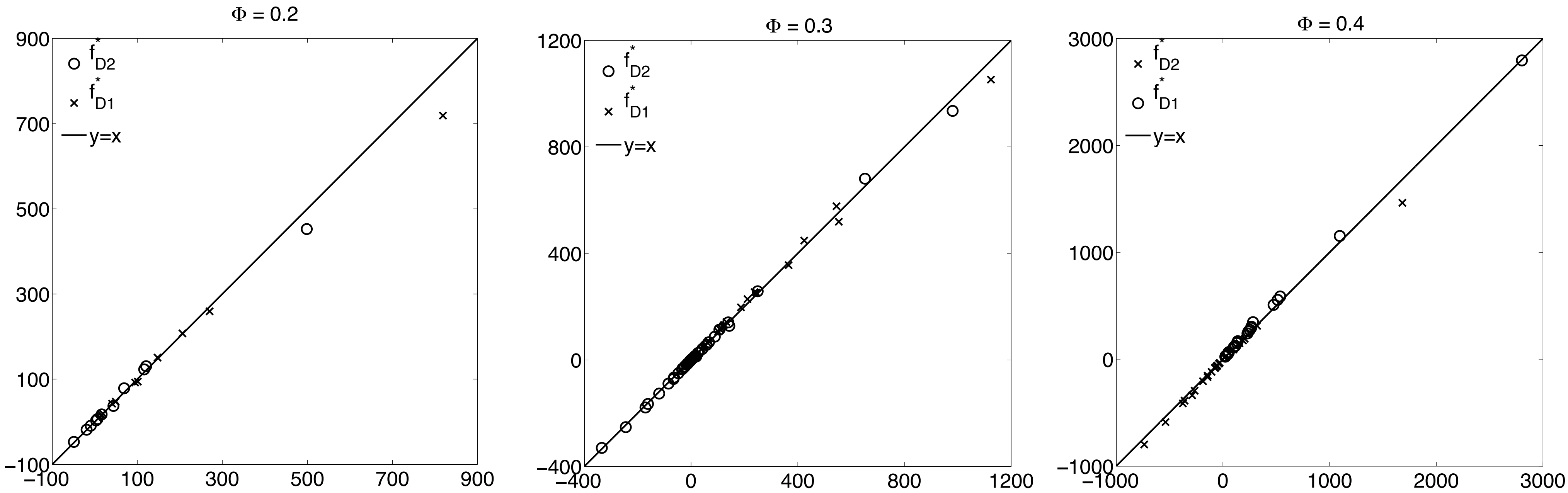
Finite Re bidisperse suspension data



Re$_{mix}$ range: 0-40
Φ_1:Φ_2 range: 1-3
d_1:d_2 range: 1-2
Re_1:Re_2 range: -1:3

Horizontal axis: Simulated f_{Di}^*
Vertical axis: Predicted f_{Di}^*

Average error: 5%
Max error: 25%

Holloway, Yin and Sundaresan, (in preparation)

Looking ahead

- Combine LBM results at moderate Re together with IBM results from Subramaniam group at higher Re.
- Perform freely evolving bidisperse simulations to investigate particle-particle collisional interactions in sedimenting systems.

Summary

- Fluid-particle drag relation developed that accurately predicts fluid-particle drag in Stokesian suspensions with particle-particle relative motion and size differences.

- Drag relation extended to account for moderate fluid inertia in bidisperse suspension flows.

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