

# High Throughput Computational Framework of Materials Properties for Extreme Environments

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# Team members

- **PI:** Zi-Kui Liu
  - In charge of the project
- **Co-PI:** Allison Beese
  - In charge of simulations based on finite element method (FEM)
- **Co-PI:** Shun-Li Shang
  - In charge of first-principles calculations based on density functional theory (DFT) and thermodynamic modeling based on the CALPHAD method
- **Senior Personnel:** Yi Wang
  - Development of DFTTK, calculations of thermodynamic properties
- **Graduate students:** Shipin Qin and John Shimanek
  - Perform both simulations and experiments

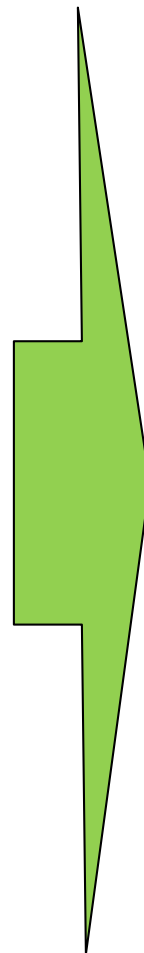
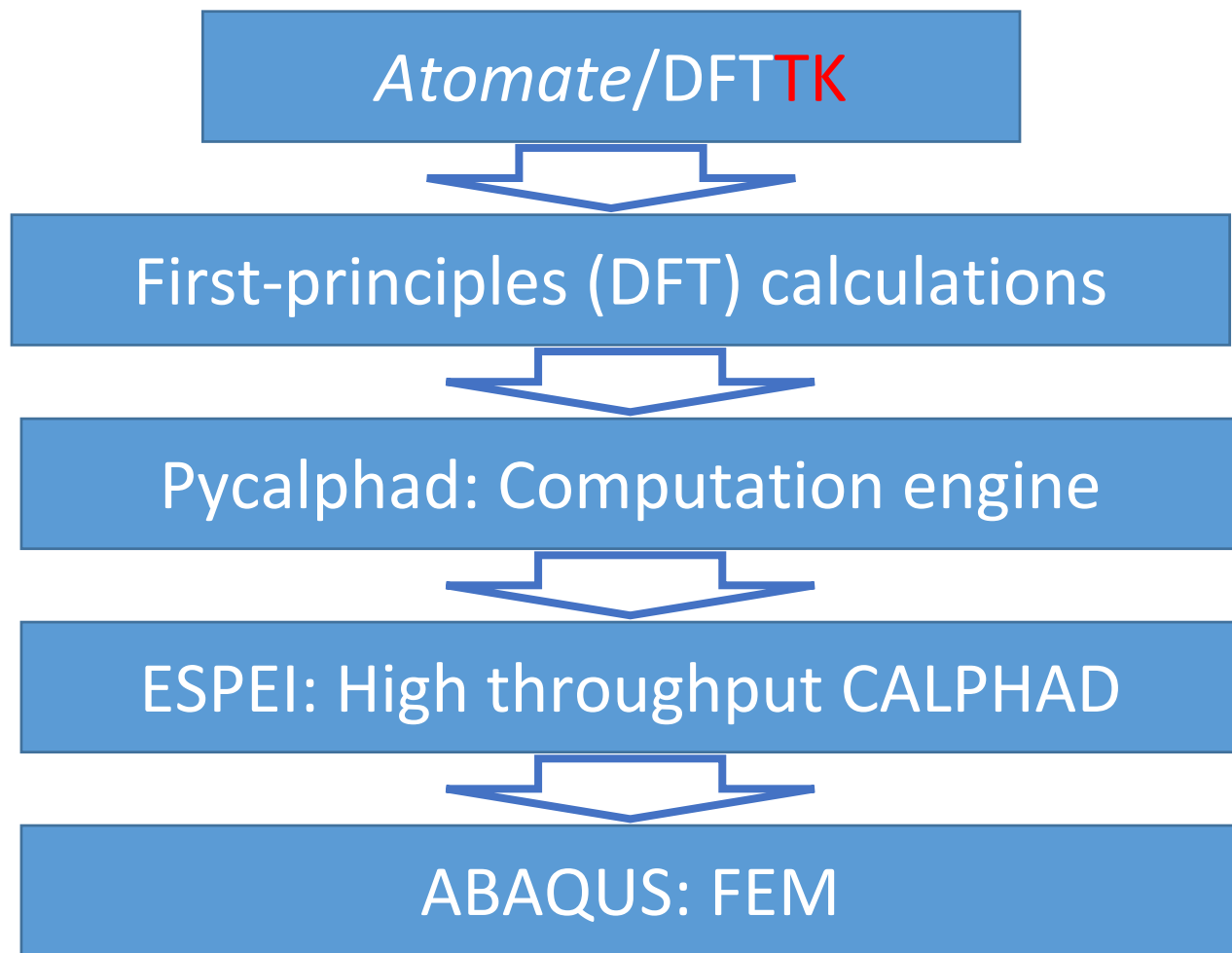
**Goal** ⇒ To establish a Python based open-source infrastructure with the following capabilities

- High throughput **first-principles calculations** for properties of interest, including such as phonon, thermodynamic, elastic, diffusion, vacancy formation, stacking and twin fault properties;
- High throughput **CALPHAD modeling** of the above properties using our unique capability based on *ESPEI* and *pycalphad*;
- New capabilities to predict the **stress-strain** behavior; and
- Phase-based new models for **tensile strength** prediction in common finite element method (FEM) analysis software.

# Outline

- Overview of the proposed tasks
- Computational approaches
  - First-principles calculations based on density functional theory (DFT)
  - Thermodynamic modeling using the CALPHAD technique
  - Simulations of strain-stress curve using finite element method (FEM)
- Results and discussion
  - DFT results of Ni and Ni<sub>3</sub>Al from pure shear deformation
  - FEM results of strain-stress curve for pure Ni single crystal
- Summary

# Overview of high throughput computational framework for materials properties under extreme environments

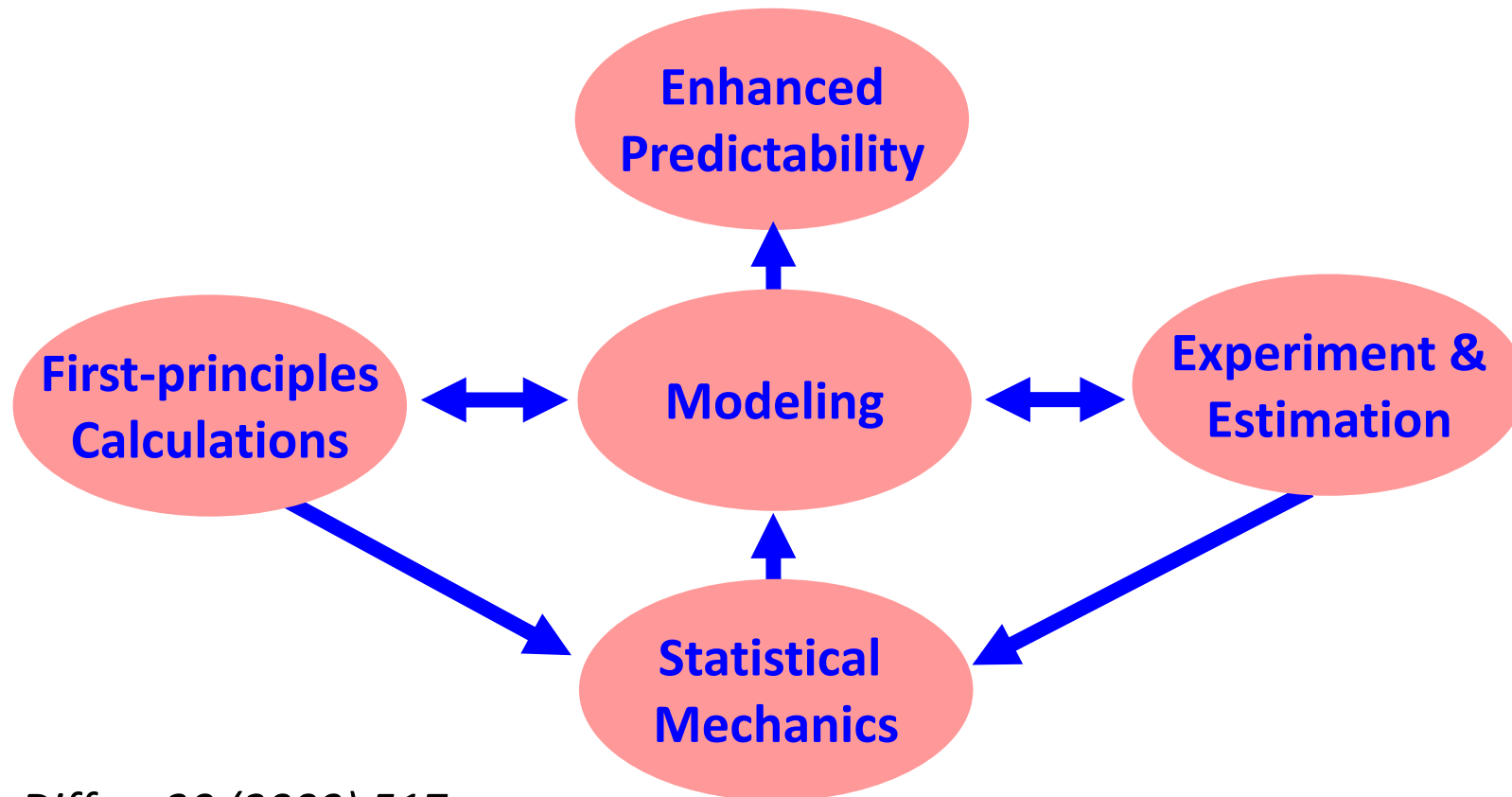


- ✓ Predict structure and thermodynamic properties using DFT and CALPHAD
- ✓ Use machine learning to reduce the amount of DFT calculations
- ✓ Apply FEM to simulate tensile strength
- ✓ Validate results and improve models

# DFT, thermodynamic modeling, FEM and experiments

## First-Principles Calculations and CALPHAD Modeling of Thermodynamics

Zi-Kui Liu

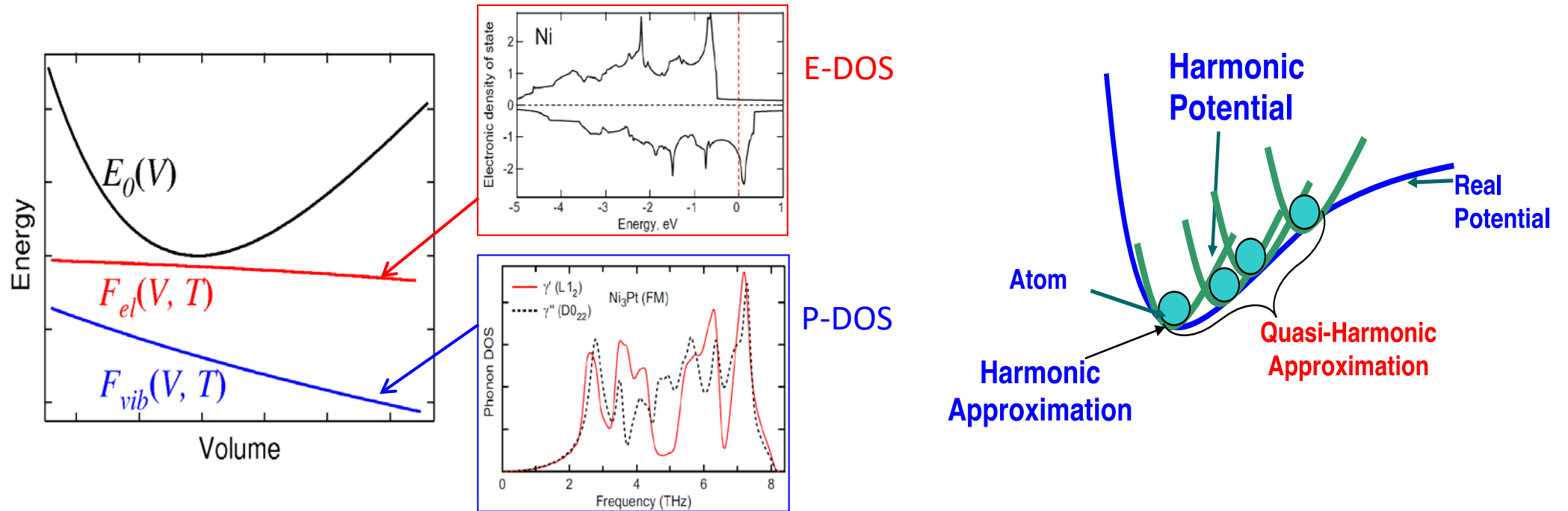


# DFT calculations of thermodynamic properties

## ■ Quasi-Harmonic approach

$$F(V, T) = E_0(V) + F_{vib}(V, T) + F_{el}(V, T)$$

- $E_0(V)$  Static energy at 0 K and volume  $V$ , i.e., EOS (by VASP)
- $F_{vib}(V, T)$  Vibrational contribution at  $V$  &  $T$  (Phonon or Debye model)
- $F_{el}(V, T)$  Thermal electronic contribution at  $V$  &  $T$  (by VASP)

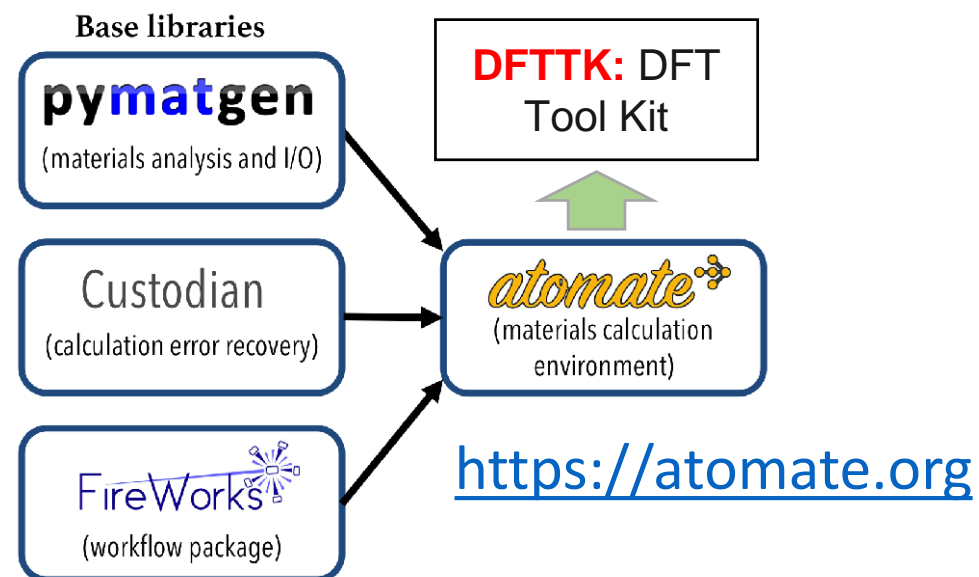


## Atomate: A high-level interface to generate, execute, and analyze computational materials science workflows



- High throughput DFT calculations
  - Allows large scale calculations in an efficient way
  - Allows greater control and automation of DFT (via e.g. the VASP code)
  - Allows for the introduction of machine learning
- DFTTK**: DFT calculations at high temperatures based on *atomate*, developed at Penn State

<https://github.com/PhasesResearchLab/dfttk>





# CALPHAD modeling

**Derivatives of Gibbs energy** →  
Thermochemical data: heat capacity,  
enthalpy, entropy, activity

$D_j d\xi_j = 0$  → Phase equilibrium data:  
phase stability, phase composition,  
phase boundary

**Gibbs Energy of Individual Phases**  
 $G^\phi(T, P, N_i, \xi_j)$

**Materials Design: Equilibrium, driving force,  
physical/chemical properties (1<sup>st</sup>, 2<sup>nd</sup> derivatives)**

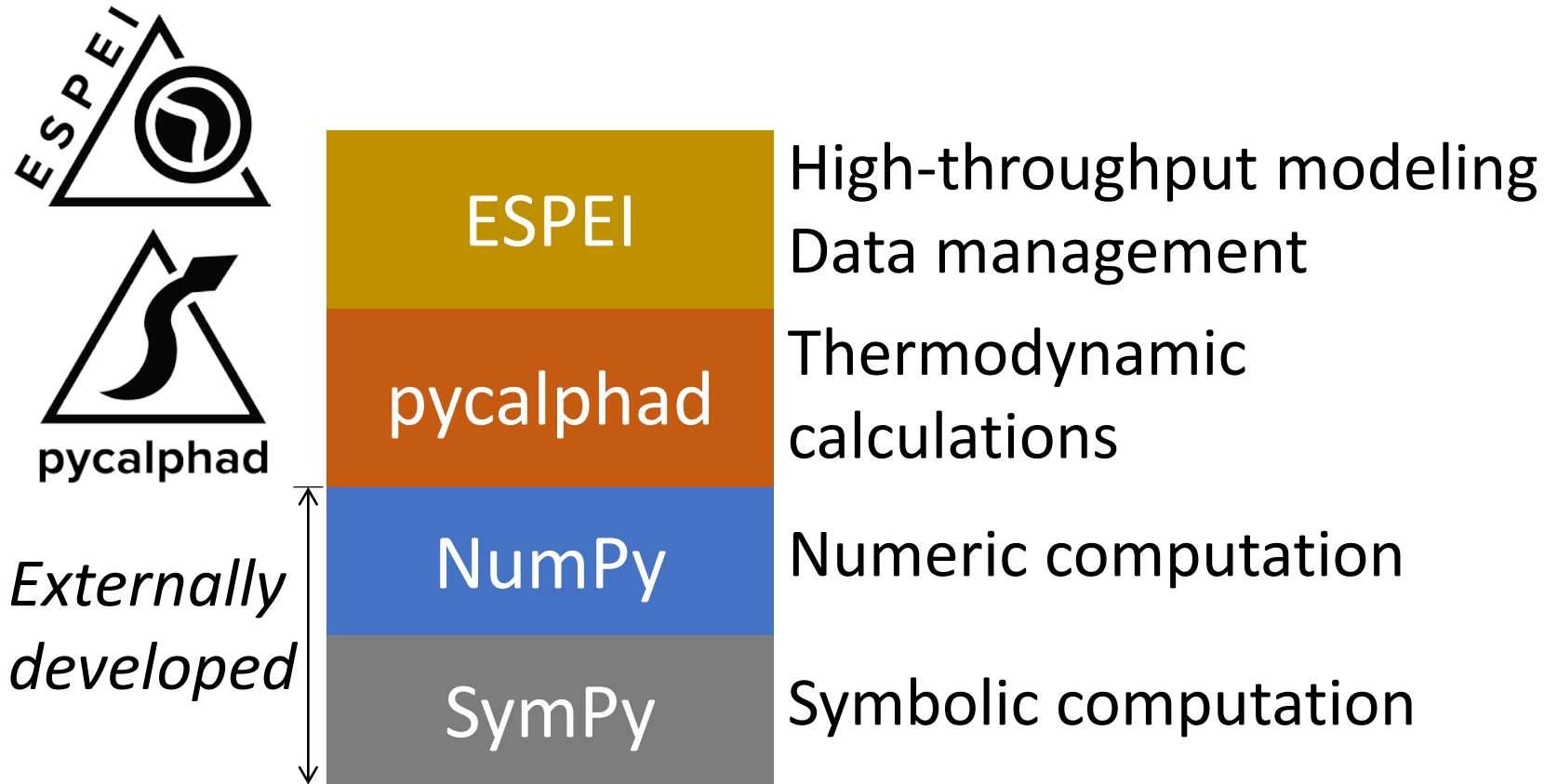
**Pure elements → Binary → Ternary → Multicomponent**



# CALPHAD Community

- Commercial Tools: ThermoCalc, Factsage, ComputTherm, JMatPro
  - Special issues in CALPHAD journal: volume 26 (2002) and 33 (2009)
- Open Source Codes: OpenCALPHAD, [pycalphad/ESPEI](#) (our Python-based codes)
- CALPHAD Organization, <http://www.calphad.org>
  - CALPHAD Journal: published by Elsevier since 1975
  - CALPHAD, Inc.: Private foundation for scholarships and awards
  - Annual conference: Gordon conference style

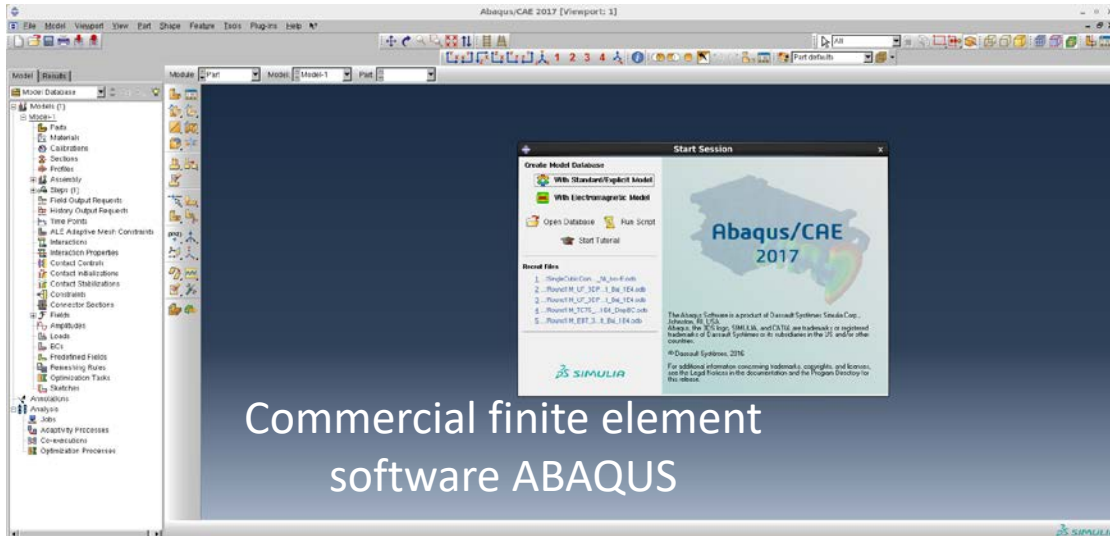
# ESPEI.org/pycalphad.org Software Stack



- **Model flexibility**
- **Model parameter evaluation**
- **Uncertainty quantification**

*JOM* **69** (2017) 886–892

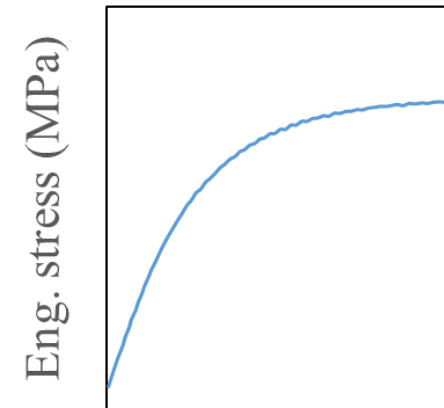
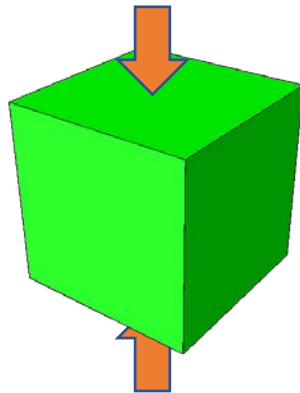
# Finite Element Method (FEM)



```

1 SUBROUTINE UMAT(stress,statev,ddsdde,sse,spd,scd,
2 1 rpl, ddsddt, drplde, drpldt,
3 2 stran,dstran,time,dtime,temp,dtemp,predef,dpred,cmname,
4 3 ndi,nshr,ntens,nstatv,props,nprops,coords,drot,pnewdt,
5 4 celent,dfgrd0,dfgrd1,noel,npt,layer,kspt,kstep,kinc)
6 c WRITE (6,*) '
7 c NOTE: MODIFICATIONS TO *UMAT FOR ABAQUS VERSION 5.3 (14 APR '94)
8 c
9 c (1) The list of variables above defining the *UMAT subroutine,
10 c and the first (standard) block of variables dimensioned below,
11 c have variable names added compared to earlier ABAQUS versions.
    
```

Material model: a crystal plasticity model. Implemented through ABAQUS subroutine UMAT (User defined MATERIAL)



Eng. strain

Engineering stress strain curves in macroscopic mechanical tests

Single element finite element model for macroscopic material behavior prediction

# Crystal plasticity FEM (CPFEM) framework:

## Phenomenological models and Physics/Dislocation-based models

### Phenomenological models

- Incorporate the concept of critical resolved shear stress (**CRSS**)
- Phenomenologically describe the evolution of CRSS with strain hardening and calibrate with 3 – 8 parameters

### Physics/Dislocation-based models

- Explicitly describe the evolution of **dislocation** densities, based on flow stress.
- Contain ~ 20 parameters or more, challenge for calibrations

# Crystal plasticity FEM (CPFEM) framework

Slipping rate in slip system  $\alpha$   
under a constant resolved  
shear stress  $\tau^\alpha$

Change of slipping resistance in  
slip system  $\alpha$  due to the  
slipping in all slip systems with  
 $n$  being the number of slip  
systems (12 for FCC)

$$\dot{\gamma}^\alpha = \dot{\gamma}_0 \left| \frac{\tau^\alpha}{\tau_c^\alpha} \right|^{\frac{1}{m}} \text{sgn}(\tau^\alpha)$$

$$\dot{\tau}_c^\alpha = \sum_{\beta=1}^n h_{\alpha\beta} |\dot{\gamma}^\beta|$$

$$h_{\alpha\beta} = q_{\alpha\beta} h_0 \text{sech}^2 \left| \frac{h_0 \gamma}{\tau_s - \tau_0} \right|$$

$$q_{\alpha\beta} = \begin{cases} 1, & \alpha = \beta \\ 1.4, & \alpha \neq \beta \end{cases}$$

- $\dot{\gamma}_0$  – Reference slipping rate (usually  $0.001 \text{ s}^{-1}$ )
- $\tau^\alpha$  – Applied resolved shear stress on slip system  $\alpha$
- $\tau_c^\alpha$  – Current strength of slip system  $\alpha$
- $m$  – hardening exponent (usually  $0.02 \sim 0.05$ )
- $h_0, \tau_s$  – hardening parameters
- $\tau_0$  – CRSS

The shear stress ( $\tau$ ) and shear  
strain ( $\gamma$ ) relationship on slip  
systems

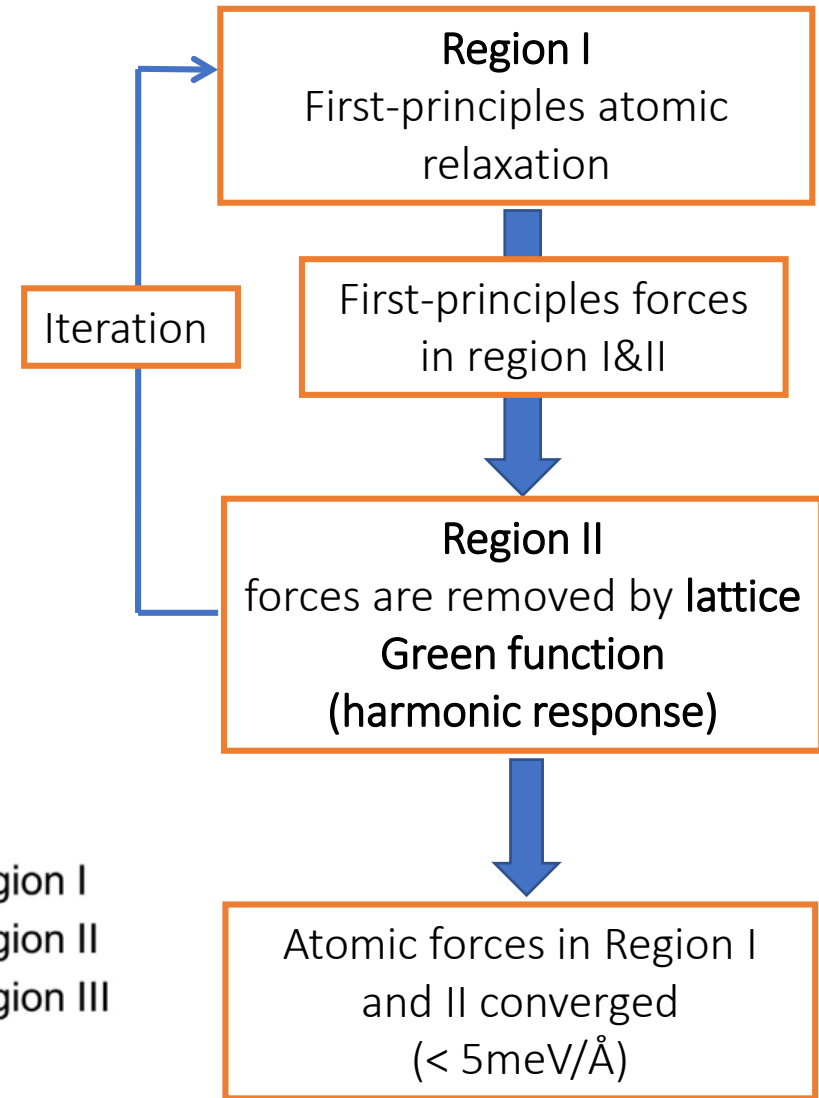
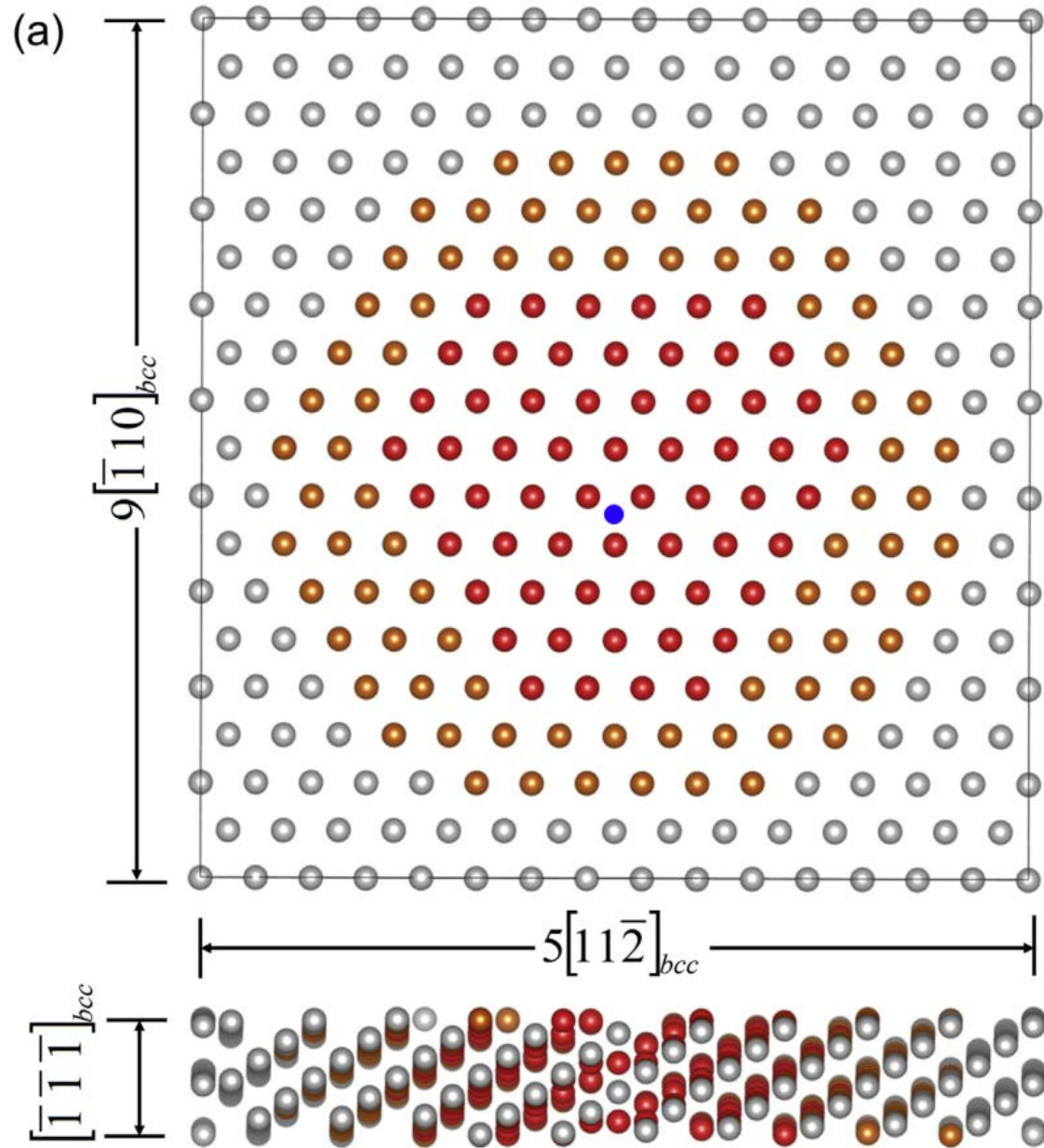


Determined from DFT  
calculations in the  
present study

Roters, Franz et al. *Acta Materialia*, 58, no. 4 (2010): 1152-1211.

Huang, Yonggang. *Harvard Univ.*, 1991.

# Flexible boundary condition method



[1] D.R. Trinkle, *Phys. Rev. B*, (2008)

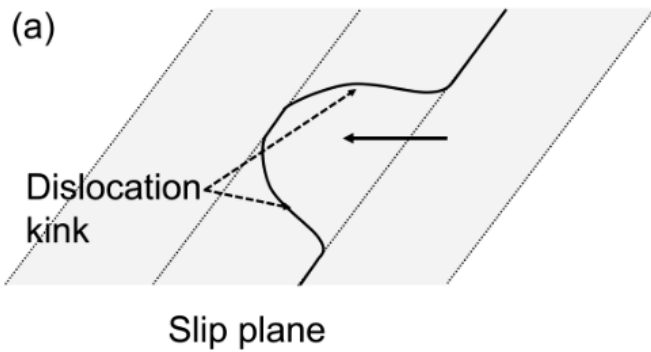
[2] J. Yasi, et al., *Phys. Rev. E*, (2012)

# Solid-solution plasticity model: *Acta Mat.* 141 (2017) 304

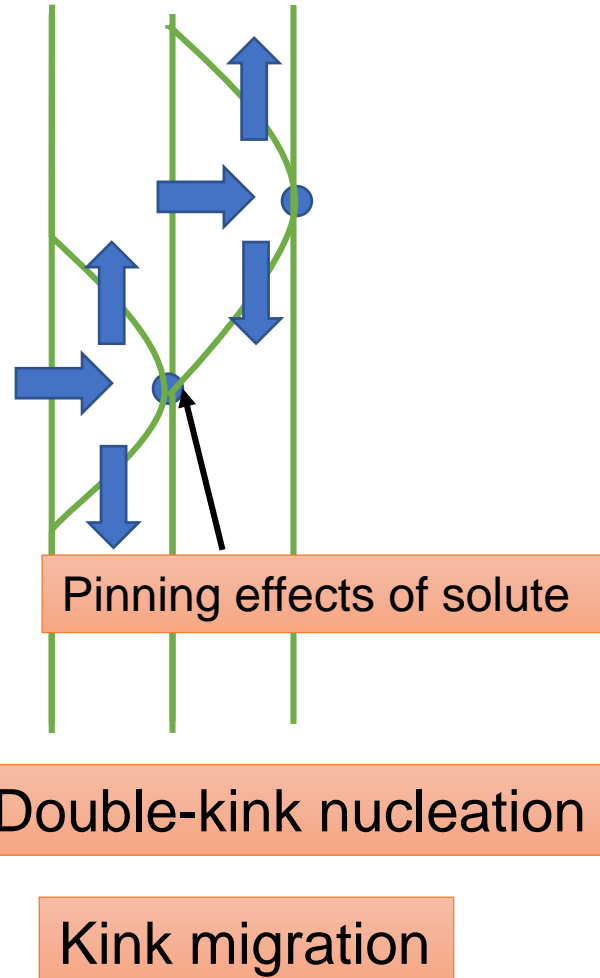
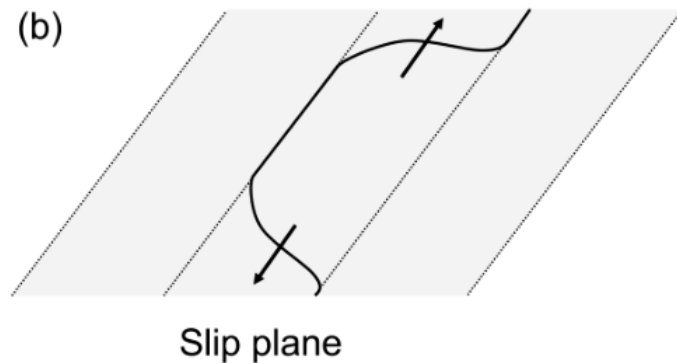
## Motion of $\frac{1}{2}\langle 111 \rangle \{110\}$ screw dislocation

## Thermally activated dislocation motion

### Step I: Double-kink nucleation



### Step II: Kink migration



### Nucleation Rate

$$v_{nucl.} \approx v_{dk} \exp\left(-\frac{\Delta H_{nucl.}(\sigma)}{k_B T}\right)$$

### Migration Rate

$$v_{mig.} \approx v_{km} \exp\left(-\frac{\Delta H_{mig.}(\sigma)}{k_B T}\right)$$

$$\Delta H(\sigma) = \Delta H^0 \left[1 - \left(\frac{\sigma}{\tau^0}\right)^p\right]^q$$

Solute-dislocation interaction

$E_{int}$   
Solute-dislocation  
Interaction energy

$\tau' = \tau^{-1} d\tau / dc$   
Peierls misfit



# Peierls-Nabarro (P-N) model & Peierls stress ( $\sigma_p$ )

- **P-N model: to describe dislocation**

- A hybrid atomistic-continuum approach, to bridge atomistic and continuum description of dislocations

- For example of Peierls stress  $\sigma_p$  (wide dislocation):

$\sigma_p$  or CRSS at 0 K  $\sigma_p = \frac{Kb}{a'} \exp(-2\pi\zeta / a')$

$b$ : Burgers vector;  $a'$ : atomic planes spacing

Half width of dislocation  $\zeta = \frac{Kb}{4\pi\tau_{\max}}$

$\tau_{\max}$ : ideal shear strength

Energy coefficient for isotropic polycrystal

$$K = \mu \left( \frac{\sin^2 \theta}{1 - \nu} + \cos^2 \theta \right)$$

$\mu$ : shear modulus;  $\nu$ : Poisson's ratio  
 $\theta$ : angle;  $0^\circ$  for screw &  $90^\circ$  edge dislocations

Energy coefficient for anisotropic single crystal, for example:

$$K_{ex} = (\bar{c}'_{11} + c'_{12}) \left[ \frac{c'_{66}(\bar{c}'_{11} - c'_{12})}{(\bar{c}'_{11} + c'_{12} + 2c'_{66})c'_{22}} \right]^{1/2}$$

$c'_{ij}$ : Translated elastic constants with "ex" of edge dislocation along x-direction

Joos and Duesbery *PRL* 78 (1997) 266  
 Hirth and Lothe, Theory of dislocation, 1982.

Two key input parameters:  
Elastic properties and ideal shear strength


# Data Repositories

- DFT-based first-principles calculations
  - Materials Project, <http://materialsproject.org/>
  - OQMD: An Open Quantum Materials Database, <http://oqmd.org>
  - AFLOW: Automatic Flow for Materials Discovery, <http://www.aflowlib.org>
  - In the process of establishing a data repository of free energy
- NIST
  - Materials Data Repository, <https://mgi.nist.gov/materials-data-repository>
  - Phase-Based Data Repository, <https://phasedata.nist.gov/>
  - Thermodynamic Research Center, <http://www.trc.nist.gov>
- Citrine Informatics, <https://citrine.io>

# Citrine Dataset Example: Cr-Fe Sigma Phase


- 192 endmembers with the remaining ~50 calculations to be uploaded soon
- Contains phase information and finite temperature properties
- Citrine Informatics. <https://doi.org/10.25920/YJRC-ZJ59>

$\text{Cr}_{12}\text{Fe}_{16}\text{Ni}_2$

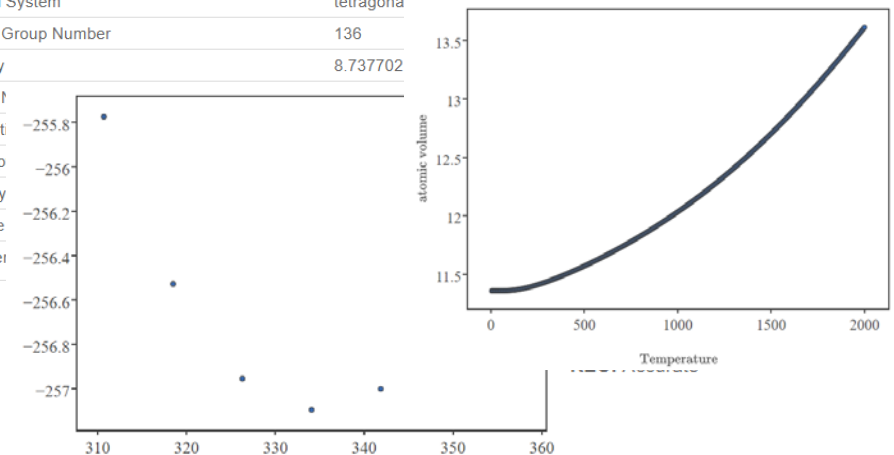


**Chemical formula:**  $\text{Cr}_{12}\text{Fe}_{16}\text{Ni}_2$   
**Materials Project ID:** tba

**Site Ratio:**



Property	Values
Space Group (Hermann-Mauguin Notation)	string
Space Group (Hall Notation)	-P4n2n
Point Group	4/mmm
Crystal System	tetragona
Space Group Number	136
Density	8.737702
Phase I	
Magnet	-255.8
Enthalp	-256
Entropy	-256.2
Volume	-256.4
Number	-256.4



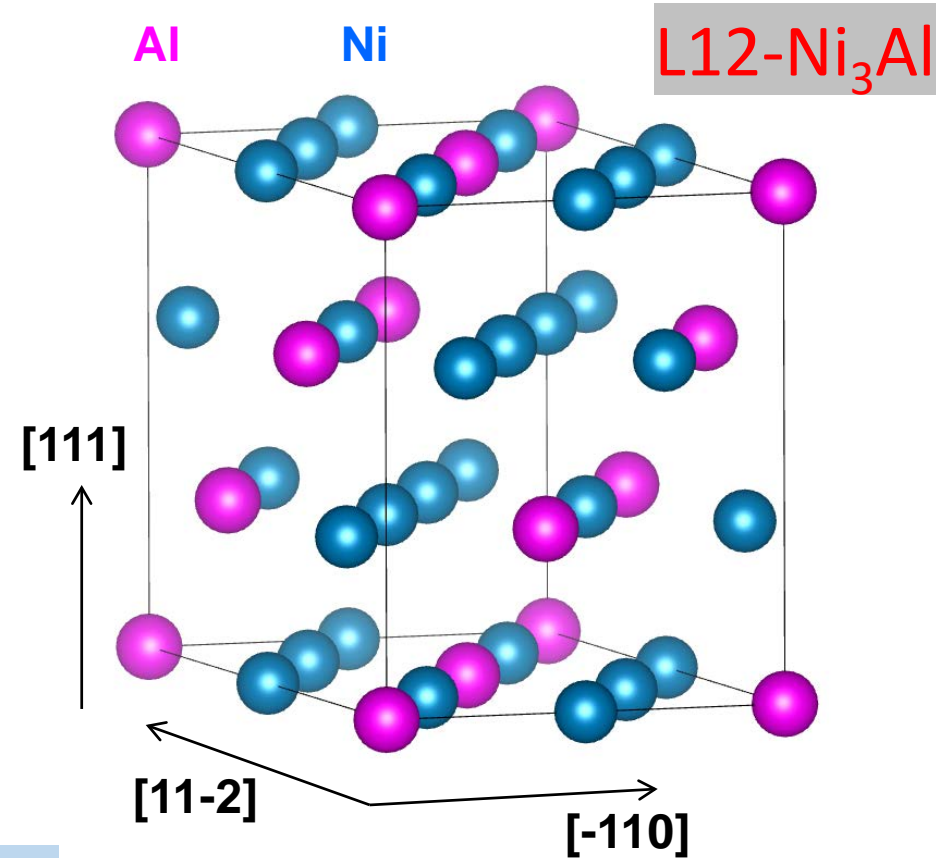
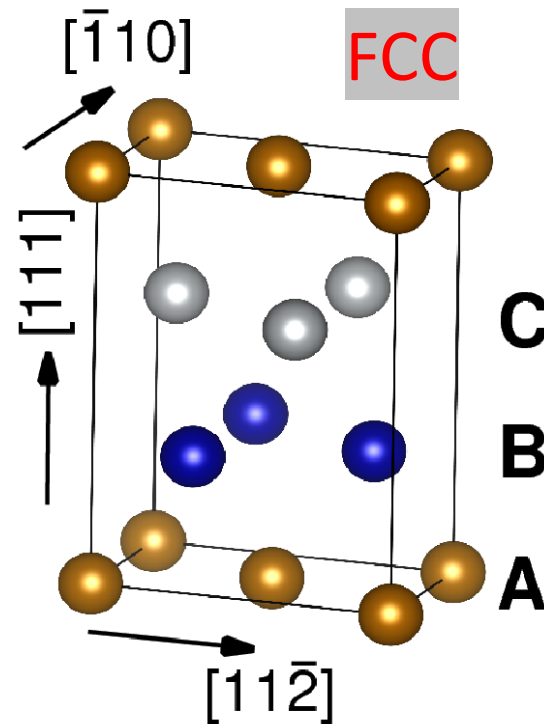
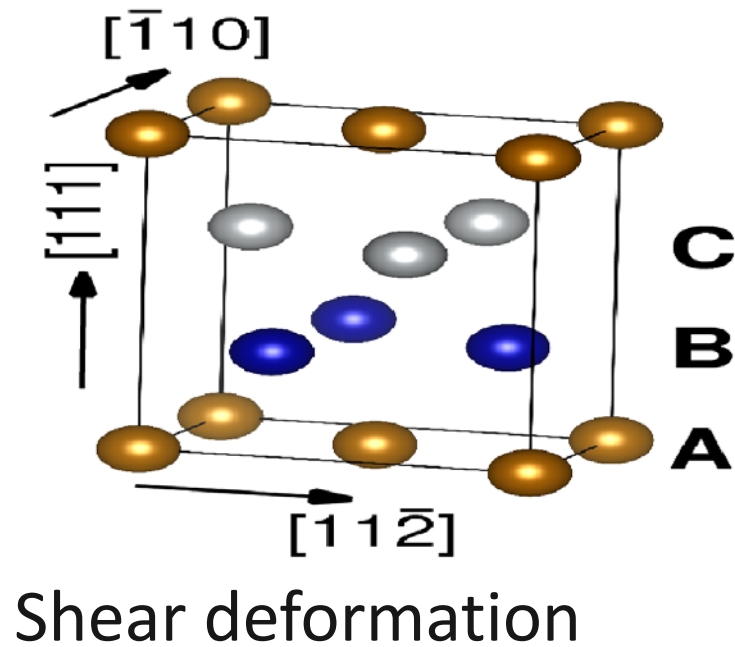
## Vasp Settings - incar

**NELM:** 100  
**LVHAR:** True  
**NEDOS:** 5001  
**LAECHG:** True  
**IRFAL:** False

Case study of Ni and Ni<sub>3</sub>Al:  
From DFT, FEM to mechanical  
properties

# DFT calculations of mechanical properties by shear deformation

- Calculations of ideal shear stress  $\tau_{IS}$ , stacking fault energies ( $\gamma_{SF}$ ,  $\gamma_{USF}$ ), CRSS using the Peierls-Nabarro model

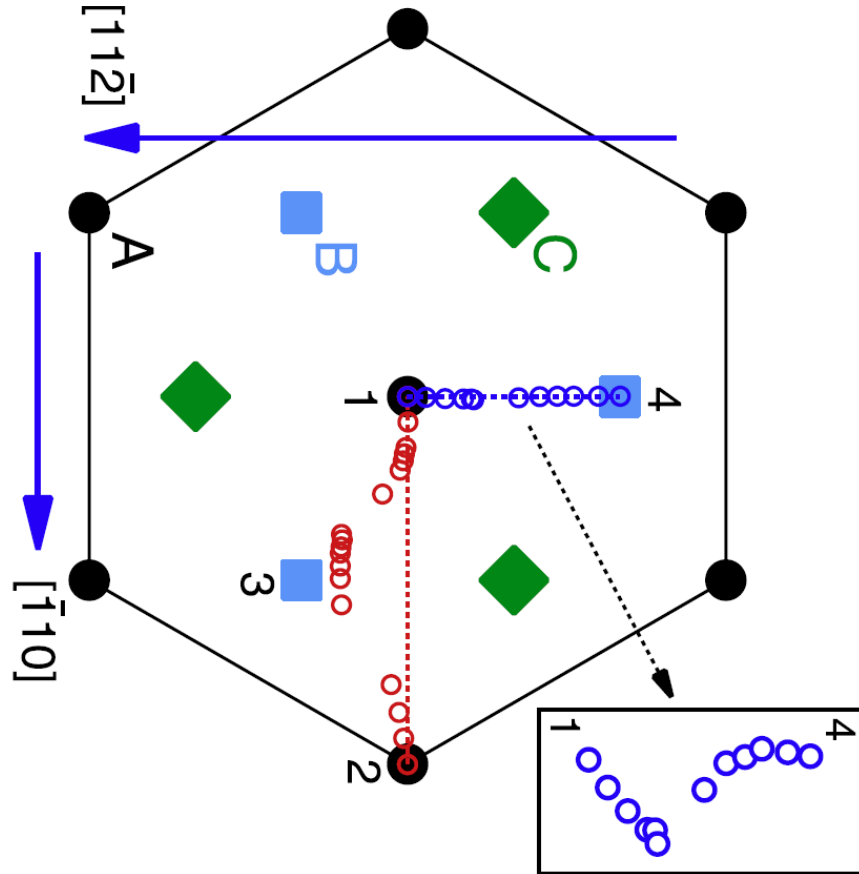


$n=1$ : Alias shear deformation (close to real case), we use this one  
 $n=\infty$ : Affine shear deformation (far from real case)

# Dislocation decomposition from pure shear deformation

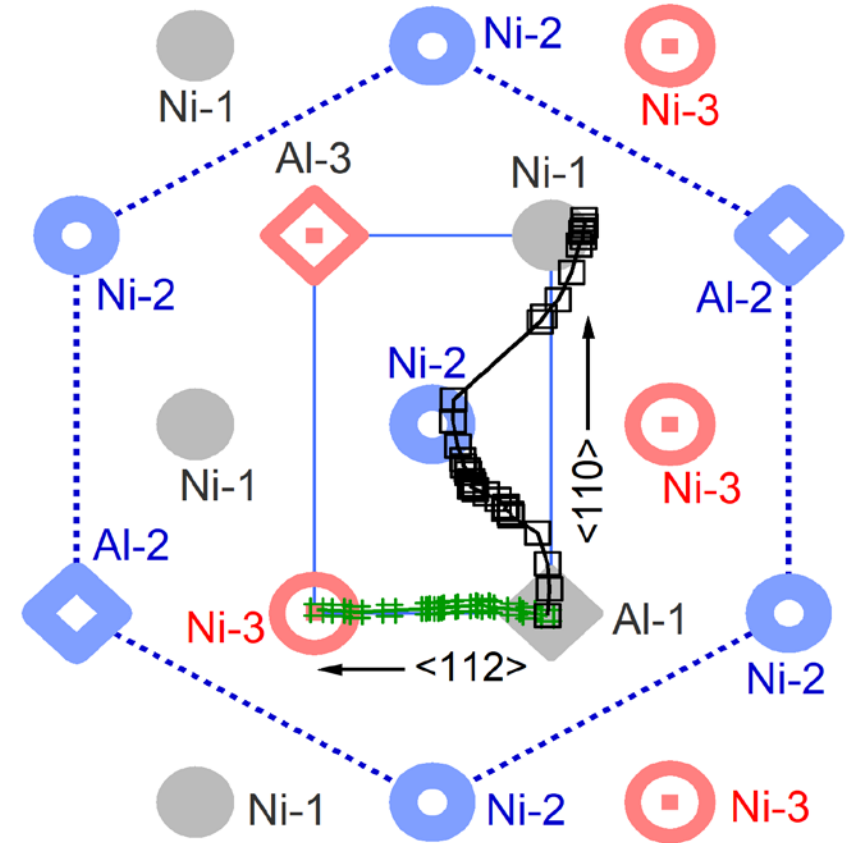
{111}<110> shear of Ni and Ni<sub>3</sub>Al along the <112> directions

fcc Ni



$$\langle 110 \rangle / 2 \rightarrow \langle 121 \rangle / 6 + \text{SF} + \langle 211 \rangle / 6$$

L1<sub>2</sub> Ni<sub>3</sub>Al

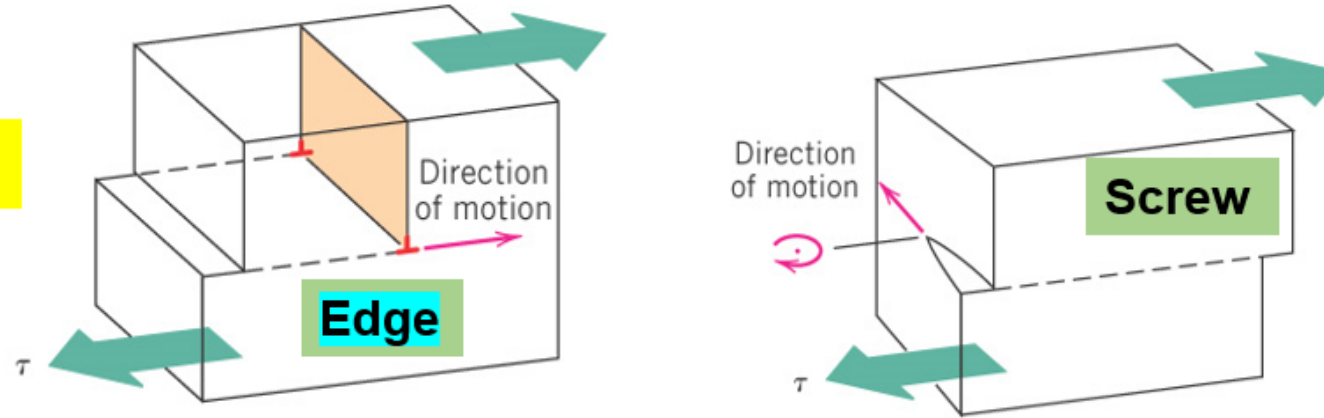


$$\langle 110 \rangle / 2 \rightarrow \langle 121 \rangle / 6 + \text{CSF} + \langle 211 \rangle / 6$$

# Dislocation decomposition from pure shear deformation

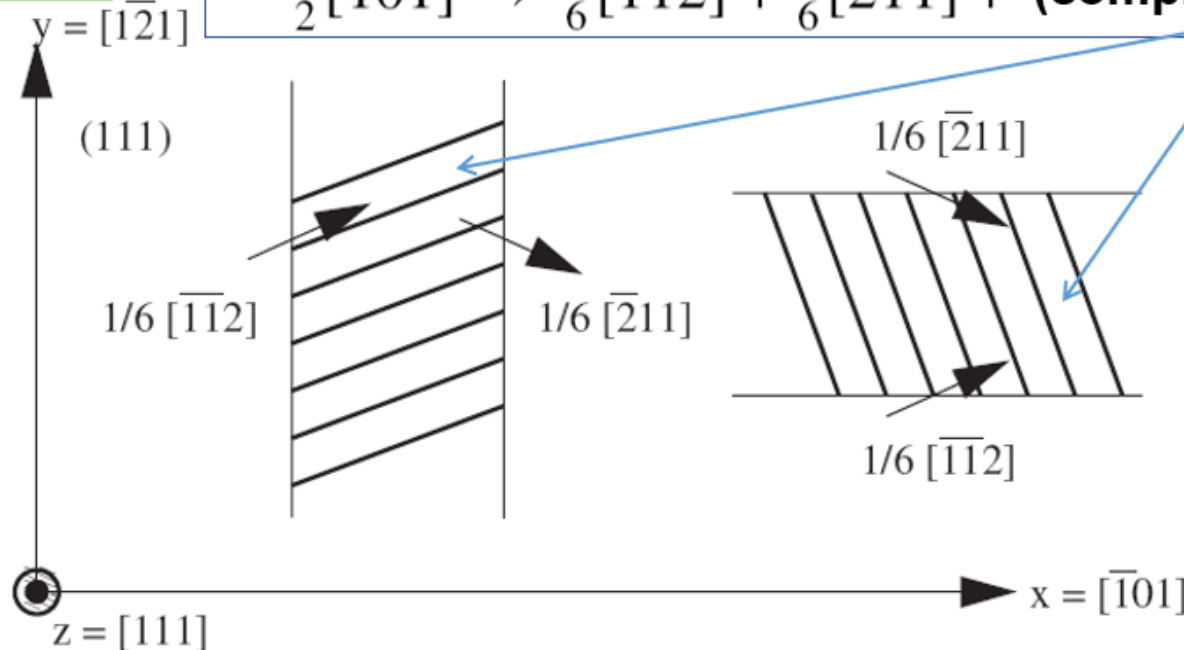
{111}<110> shear of Ni and Ni<sub>3</sub>Al along the <112> directions

## Summary



## Edge

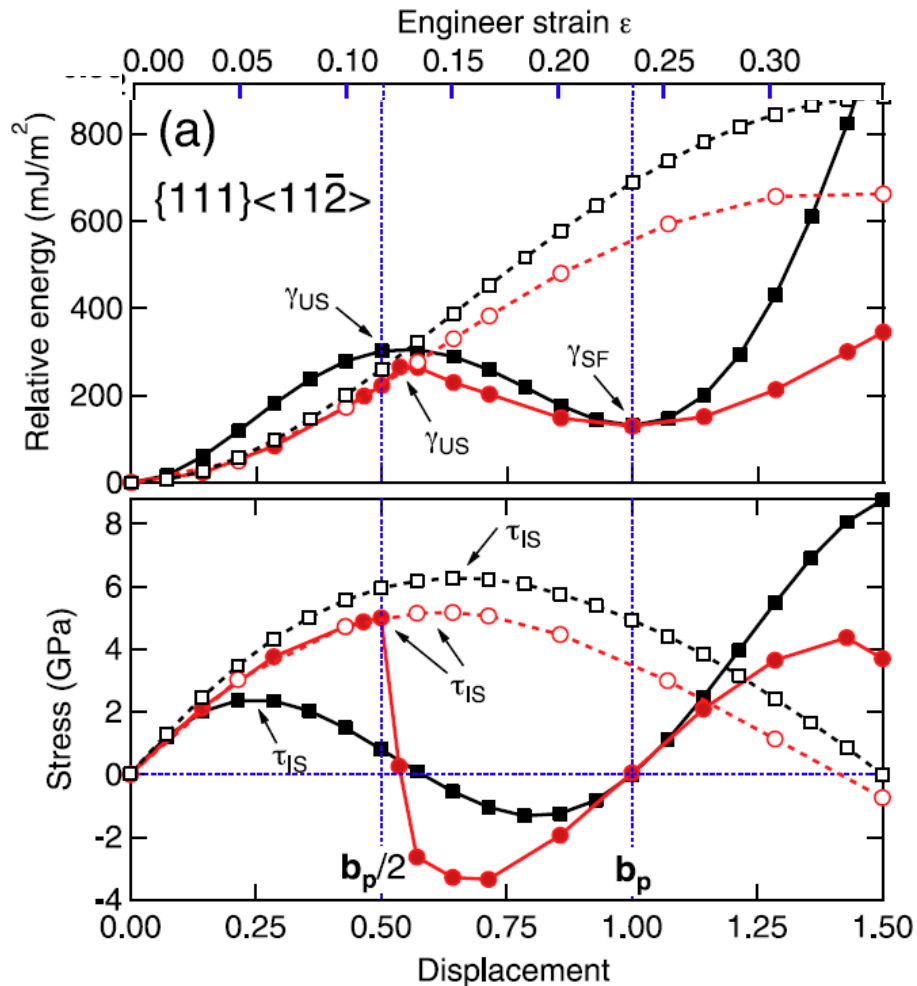
$$\frac{1}{2}[\bar{1}01] \rightarrow \frac{1}{6}[\bar{1}\bar{1}2] + \frac{1}{6}[\bar{2}11] + \text{(complex/)} \text{ stacking fault (C/SF)}$$



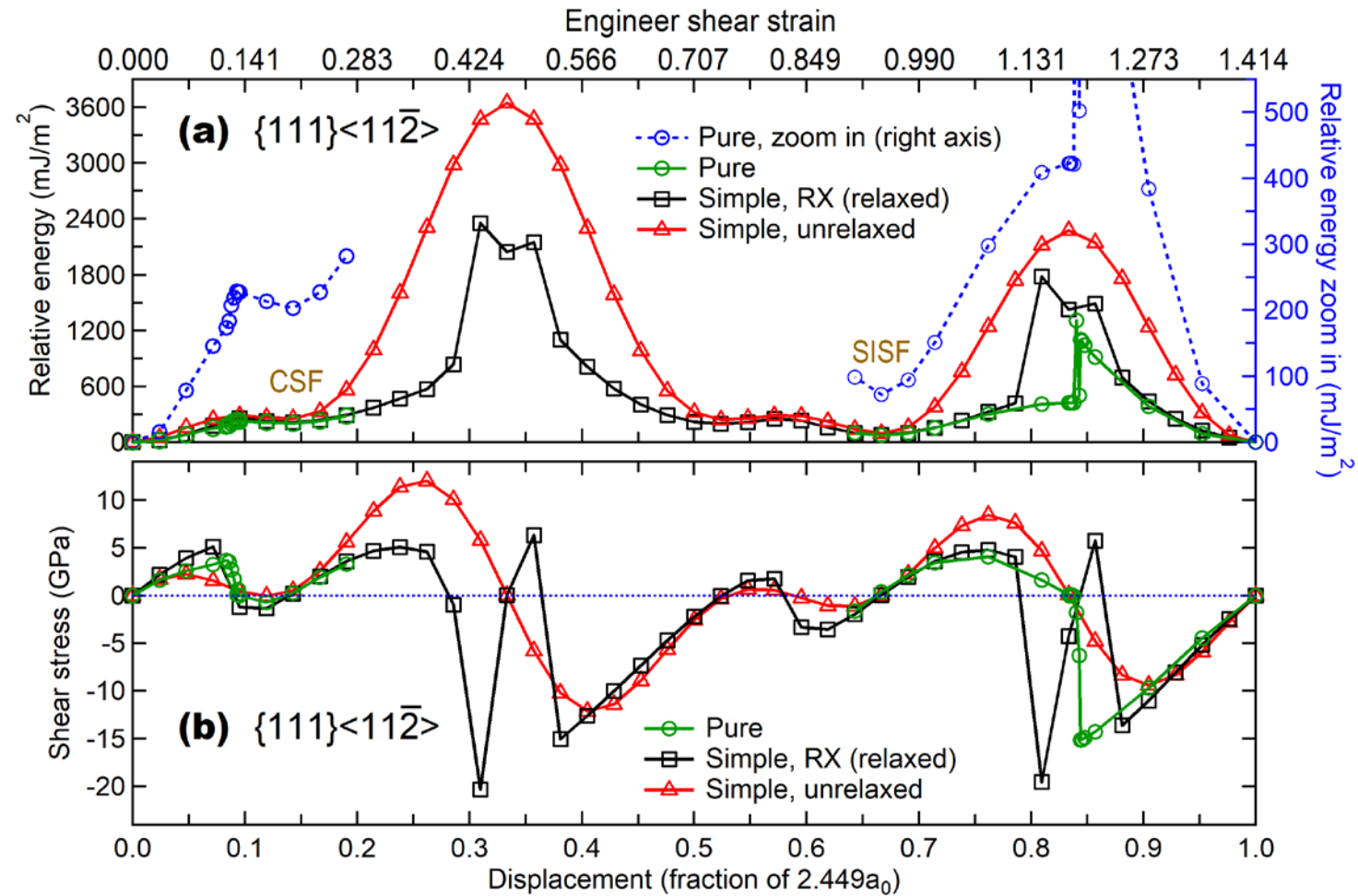
MSMSE 11, 2003, 883

# Shear deformation along $\{111\}\langle 11\bar{2}\rangle$ : fcc Ni, $L1_2$ Ni<sub>3</sub>Al

fcc Ni



$L1_2$  Ni<sub>3</sub>Al





# Stacking fault energy and ideal shear stress of Ni and Ni<sub>3</sub>Al (pure shear deformation based on the above slide)

Material	Slip system	Name	Stacking fault (mJ/m <sup>2</sup> )		Ideal shear stress $\tau_{\max}$ (GPa)
			Stable	Unstable	
Ni	{111}<112>	SF	127	263	5.1
Ni <sub>3</sub> Al	{111}<112>	CSF	203	229	3.7
Ni <sub>3</sub> Al	{111}<112>	SISF	<b>72</b>	1308	15.2
Ni <sub>3</sub> Al	{111}<110>	APB <sub>111</sub>	205	230	4.2

fcc Ni: **SF**: intrinsic stacking faults energy  
 Ni<sub>3</sub>Al: **CSF**: complex stacking fault  
 Ni<sub>3</sub>Al: **SISF**: Superlattice intrinsic stacking fault  
 Ni<sub>3</sub>Al: **APB**: antiphase boundary on {111}<110>  
 NOTE: **SF** and **CSF** are comparable

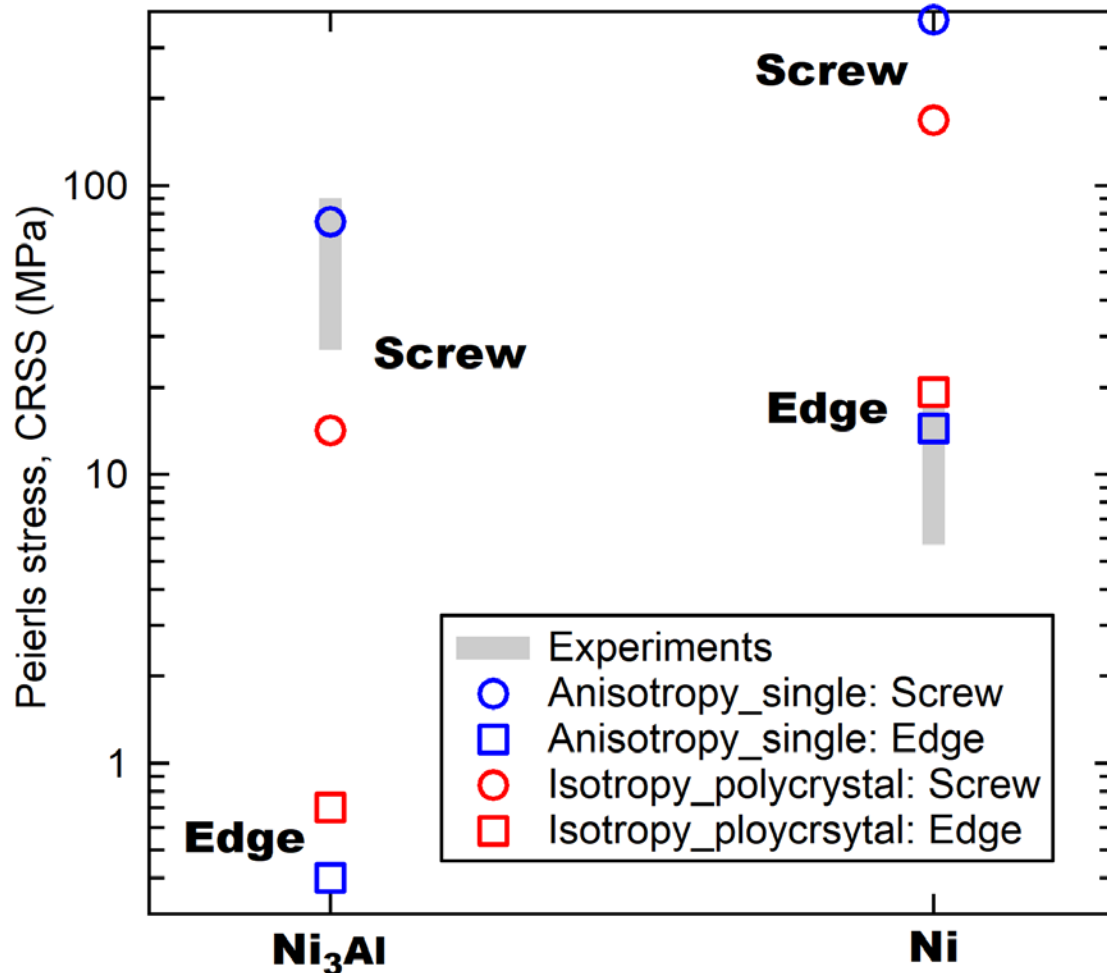
## Common belief:

$\tau_{\max}$  values of Ni and Ni<sub>3</sub>Al are similar (~ 8 GPa)

## This work:

$\tau_{\max}$  value of Ni<sub>3</sub>Al (3.7) is ~ 30% lower w.r.t. Ni (5.1)  
**Why?** (see the next slide)

# Predicted Peierls stress (CRSS at 0 K) of Ni and Ni<sub>3</sub>Al



- **fcc  $\gamma$ -Ni: more slip systems**

- **This work:** edge dislocations are dominant
- **Expt:<sup>a</sup>** Edge dislocation dipoles are dominant in the first stage of fcc single crystal deformation

- **L1<sub>2</sub>  $\gamma'$ -Ni<sub>3</sub>Al: few slip systems**

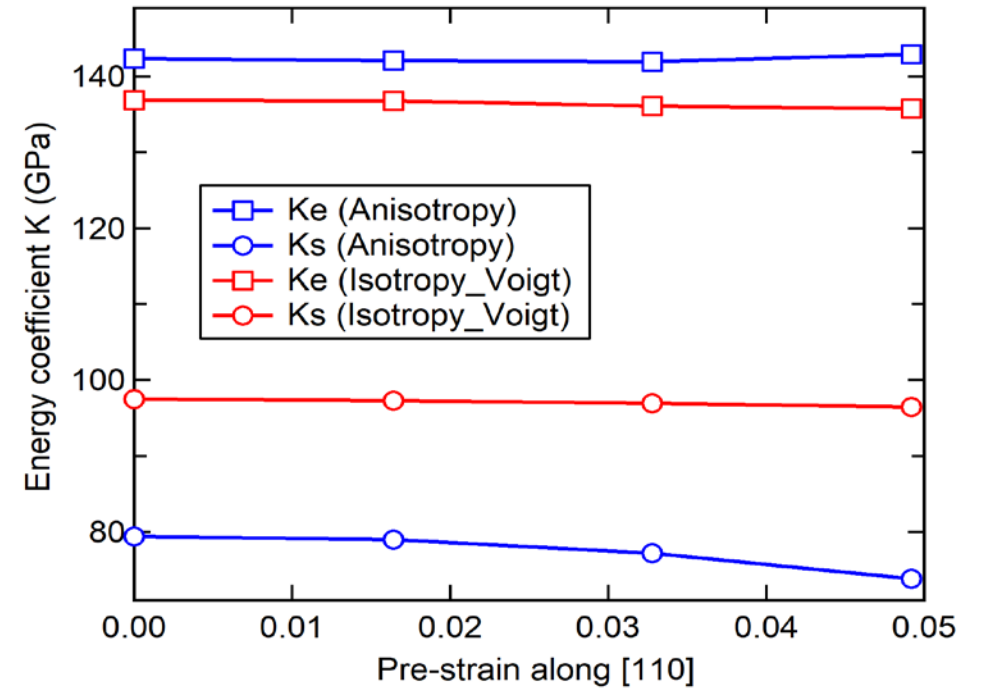
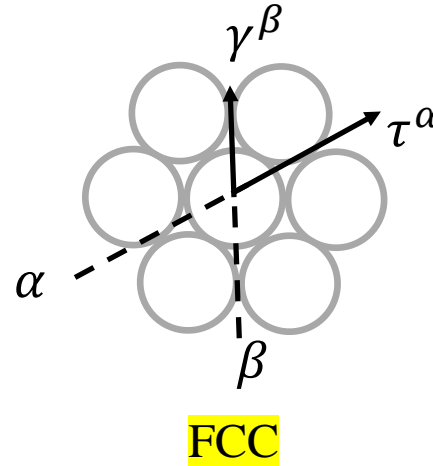
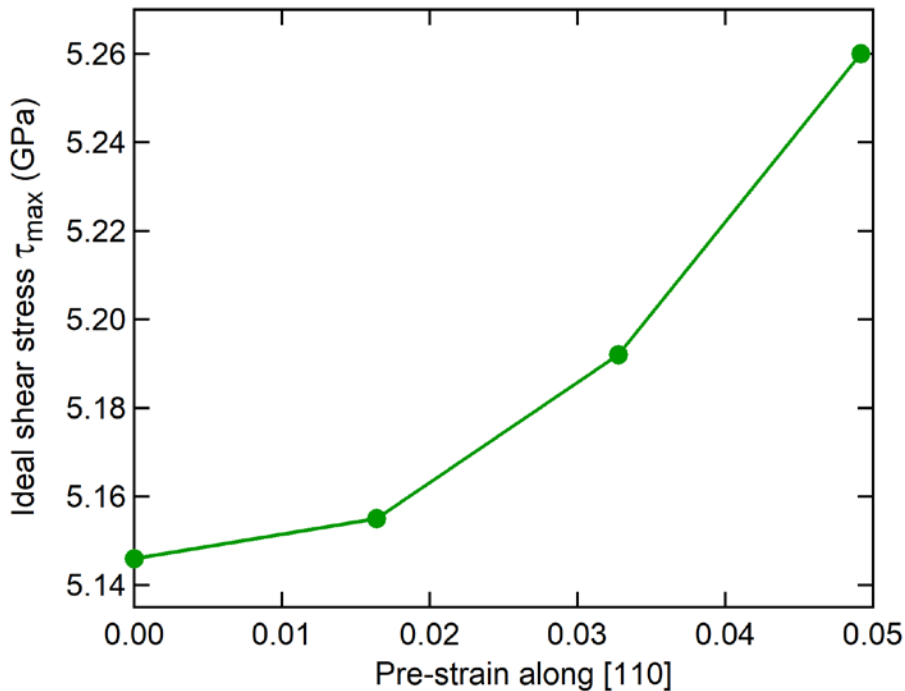
- **This work:** screw dislocations are dominant
- **Expt:<sup>b</sup>** Mainly screw dislocations, edge dislocations mainly act as links between screw dislocations. Cross-slip of screw dislocations, adopting the Kear-Wiltsdorf (**K-W**) locking configurations, will result in yield strength anomaly (**YSA**).

<sup>a</sup>Nöhring and Curtin, Acta Mater 128 (2017) 135-148

<sup>b</sup>Wang-Koh, Mater Sci Technol 33 (2017) 934-943.

# {111}<112> pure shear deformation of fcc Ni

Ideal shear stress  $\tau_{\max}$  and energy coefficient  $K$  along  $\alpha$ -direction as a function of pre-strain along  $\beta$ -direction:



$\tau_{\max}$  slightly increases, hence, CRSS will slightly increase

Ke (edge) almost constant  
Ks (screw) constant or slightly decreasing

# Crystal plasticity FEM (CPFEM) predictions

**CPFEM**

$$\dot{\gamma}^\alpha = \dot{\gamma}_0 \left| \frac{\tau^\alpha}{\tau_c^\alpha} \right|^{\frac{1}{m}} \text{sgn}(\tau^\alpha)$$

$$\dot{\tau}_c^\alpha = \sum_{\beta=1}^n h_{\alpha\beta} |\dot{\gamma}^\beta|$$

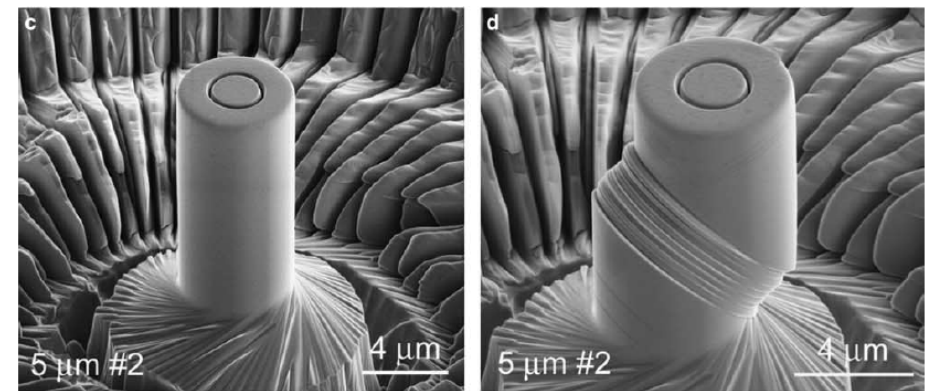
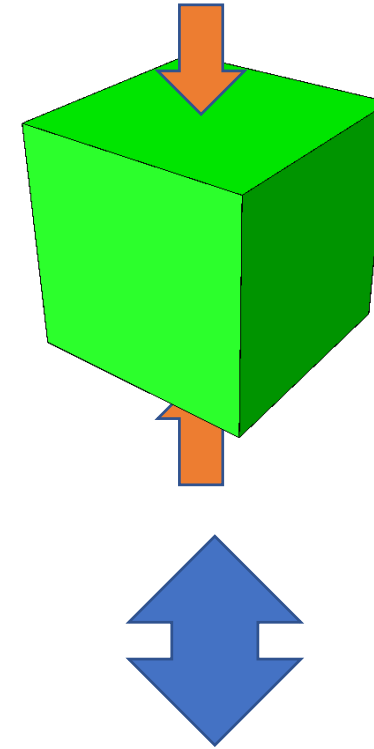
$$h_{\alpha\beta} = q_{\alpha\beta} h_0 \text{sech}^2 \left| \frac{h_0 \gamma}{\tau_s - \tau_0} \right|$$

$$q_{\alpha\beta} = \begin{cases} 1, & \alpha = \beta \\ 1.4, & \alpha \neq \beta \end{cases}$$

$$\tau_c = \sigma_P$$

**DFT**

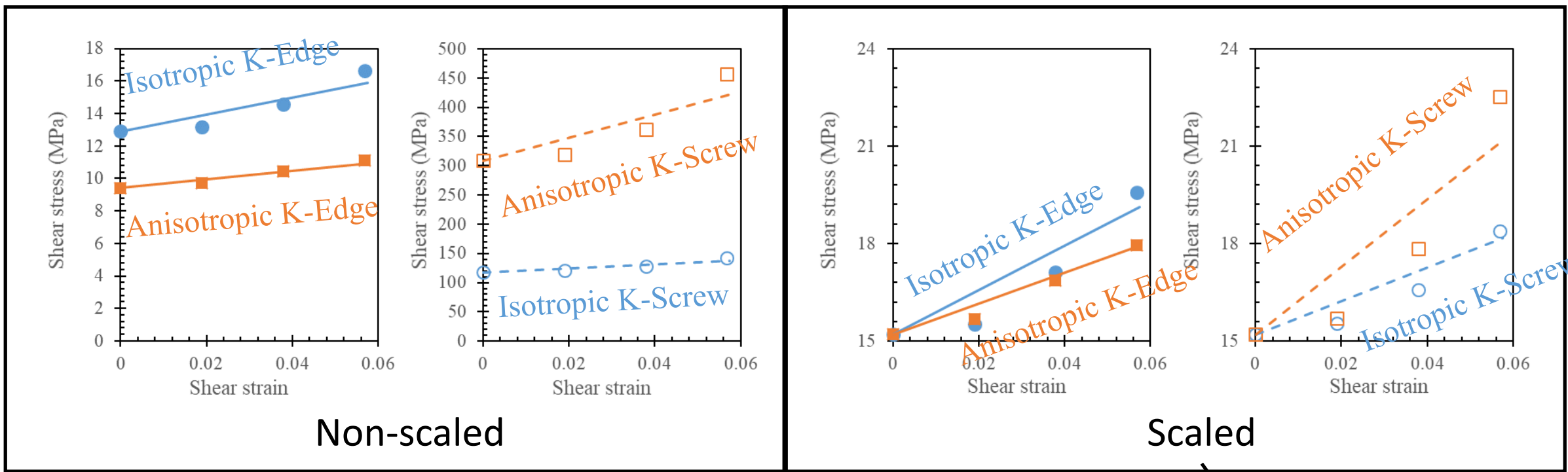
$$\sigma_p = \frac{Kb}{a'} \exp(-2\pi\zeta/a')$$



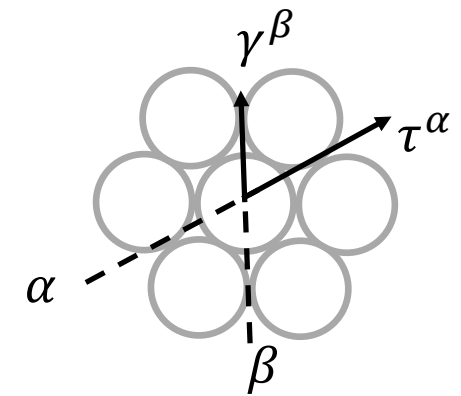
*Dimiduk et al., Acta Mater 53(2005) 4065.*

# Crystal plasticity model parameter calibrations

## Shear stress strain behavior on slip systems



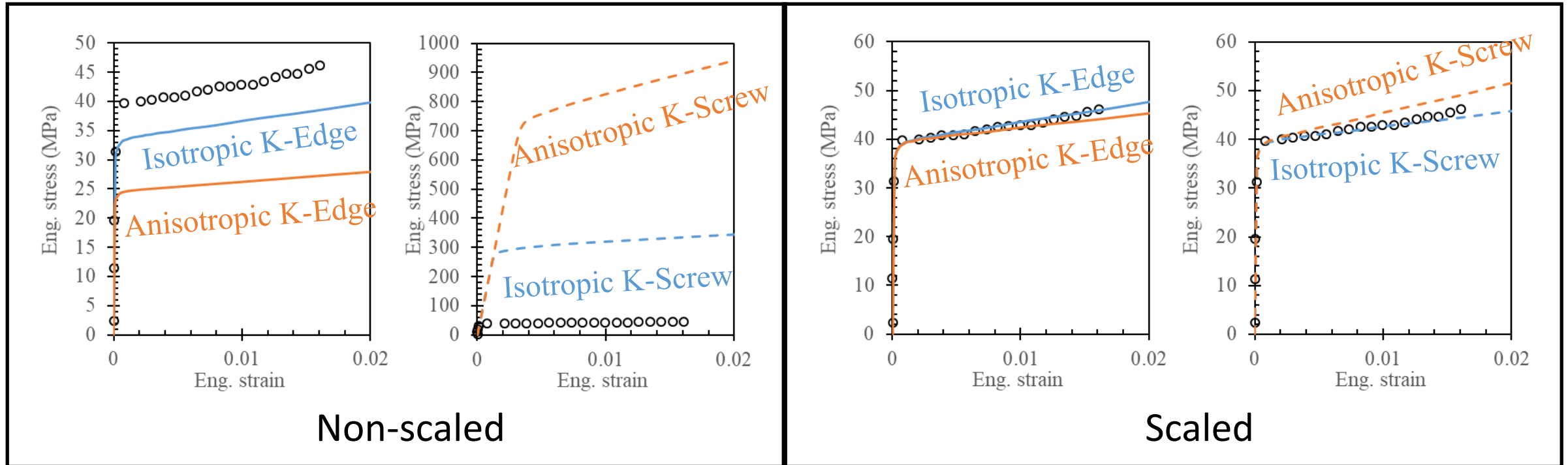
Symbols: DFT calculations  
 Lines: CPFEM calibration



Based on experimental initial CRSS value (15.2 MPa)

# Results of CPFEM predictions for single crystal of fcc Ni

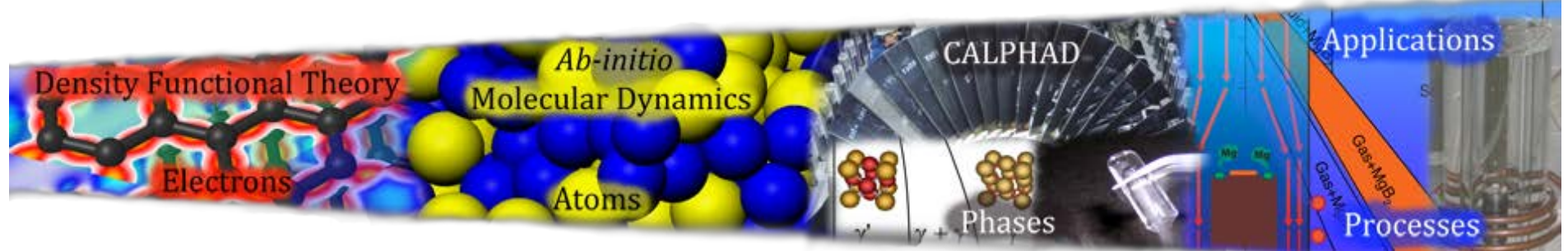
## Engineering stress strain behavior of a bulk single crystal under uniaxial compression



Symbols: Experiments (Dimiduk et al. Acta Mater 2005)

Lines: CPFEM results

# Summary



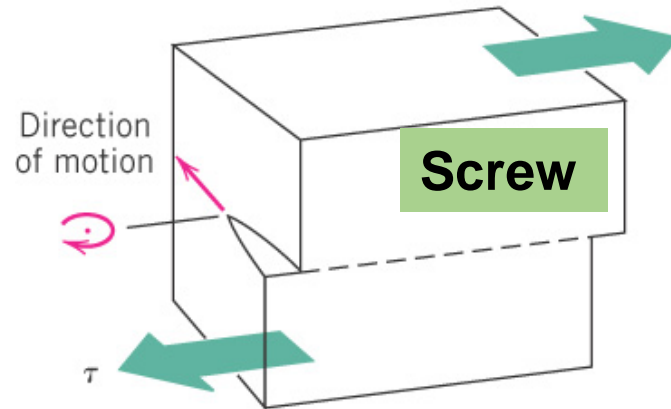
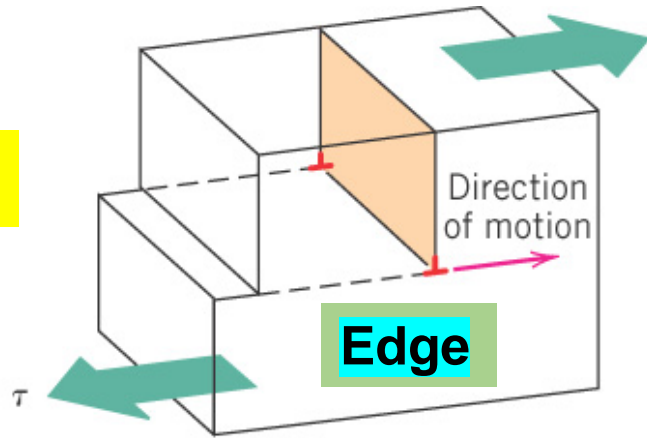
## Multiscale approach from electron, atoms, to phase, and applications

- High-throughput calculations for fast data-gain
  - DFT-based first-principles calculations of thermodynamic/mechanical properties
  - CALPHAD modeling to develop the databases of interest
- Web-accessible codes and databases for broader impact
  - Python-based open source **codes**: DFTTK, ESPEI, Pycalphad
  - **Databases**: via Citrine.io
- Understanding and prediction of materials properties for extreme environments
  - DFT calculations of shear deformation
  - Phase-based FEM simulations of mechanical properties
  - Case study of Ni and Ni<sub>3</sub>Al: CRSS, dislocation decomposition, strain-stress curves



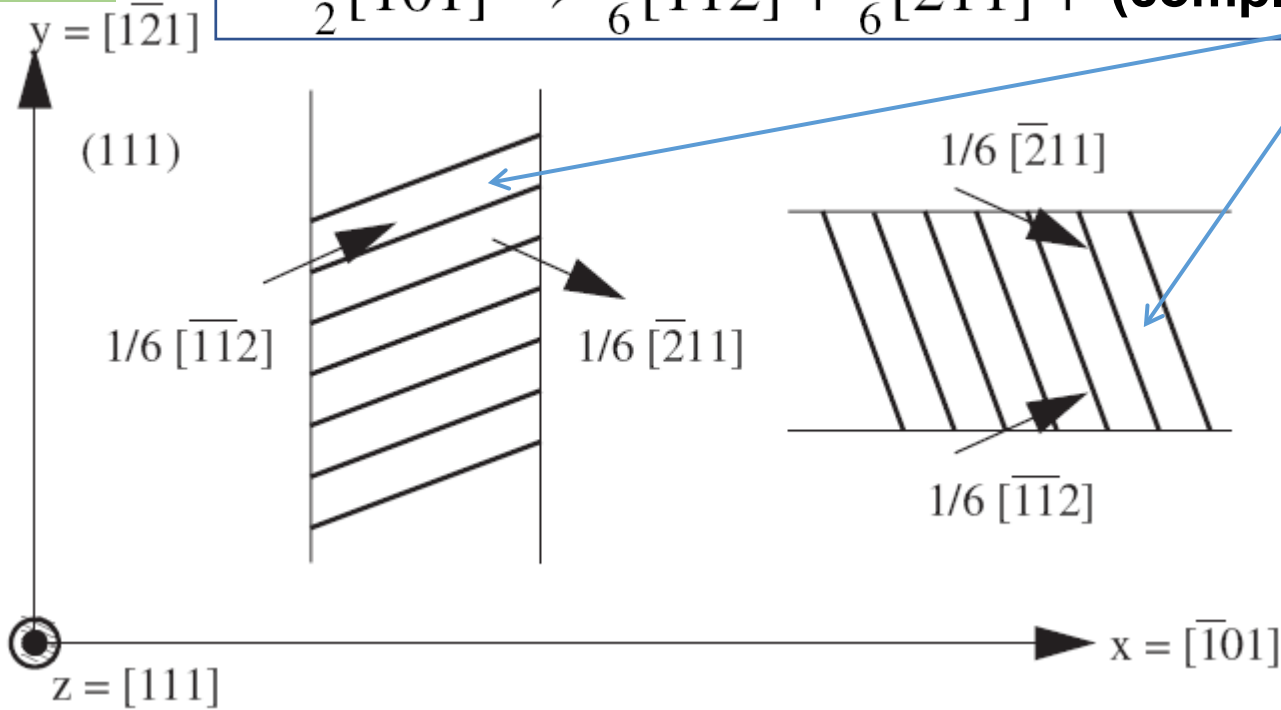


**Summary**



**Edge**

$$\frac{1}{2}[\bar{1}01] \rightarrow \frac{1}{6}[\bar{1}\bar{1}2] + \frac{1}{6}[\bar{2}11] + \text{(complex/)} \text{ stacking fault (C/SF)}$$



**Screw**

**MSMSE 11, 2003, 883**