2010 UTSR Workshop on Heat Transfer
Co-Organized by Penn State University and DoE NETL
19-21 October 2010

The Role of the Biot Number in Turbine-Cooling Design and Analysis

Tom Shih,[1,2] Kyle Chi,[1] Prashanth Ramachandran,[1] Robin Ames,[3] and Rich Dennis[3]

[1]School of Aeronautics and Astronautics, Purdue University

[2]Ames National Laboratory, U.S. Dept. of Energy

[3]National Energy Technology Laboratory, U.S. Dept. of Energy

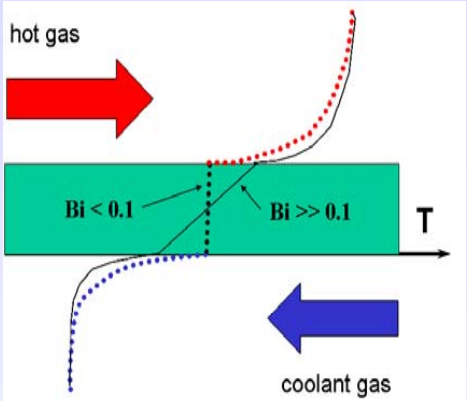
DoE – NETL & Ames Laboratory

PURDUE UNIVERSITY

What are the issues on Biot numbers?

Biot number is a dimensionless parameter that cuts across equations.

- gas phase: Re, Pr, Nu, Mach, …
- solid phase: Fourier number



Biot number is easy to define only in 0-D and 1-D.

$$q'' = \frac{T_h - T_c}{\frac{1}{h_h} + \frac{L_M}{k_M} + \frac{1}{h_c}}$$

If local T_{hot}&h_{hot} and $T_{coolant}$&$h_{coolant}$ vary along the material surface (2-D, 3-D), then definition of Bi is unclear.

$$Bi = \left(\frac{L_S}{k_S}\right) \Big/ \left(\frac{1}{h_h} + \frac{1}{h_c}\right)$$

Thus, need to understand meaning of Biot numbers and how it could be used in design of experiments

Previous Work

Though many have performed conjugate heat transfer studies (e.g., Bohn, Heidmann, Kassab, …), few have examined the meaning and effects of the Biot number with large variations in local h or its use in *design of experiments.*

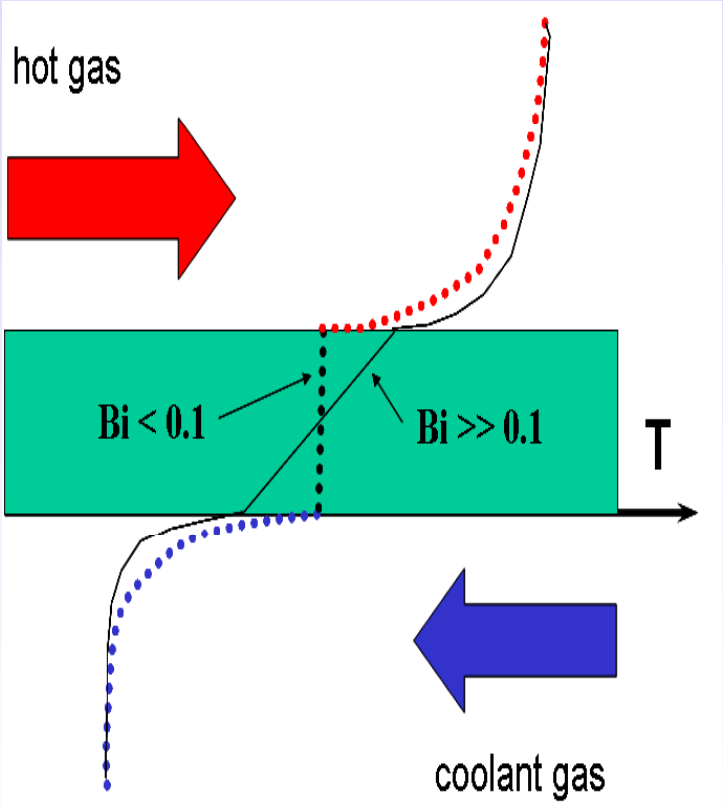
Experimental:

- Use 1-D conduction to get h (transient liquid crystal) and T_{aw} (infrared thermal imaging): Chyu, Hans, Ekkad, Bogard, Thole, Popp, Diller, Guo,Xu, Ding, …
- Maikell, Bogard, Piggush, &Kohli (IGTI 2009-60286): Performed a Biot number similarity study to get overall effectiveness.

Computational:

- Shih, Chi, Bryden, Alsup, & Dennis (IGTI 2009-59726): Performed a CFD study to understand the meaning of Biot number.

Objectives and Approach

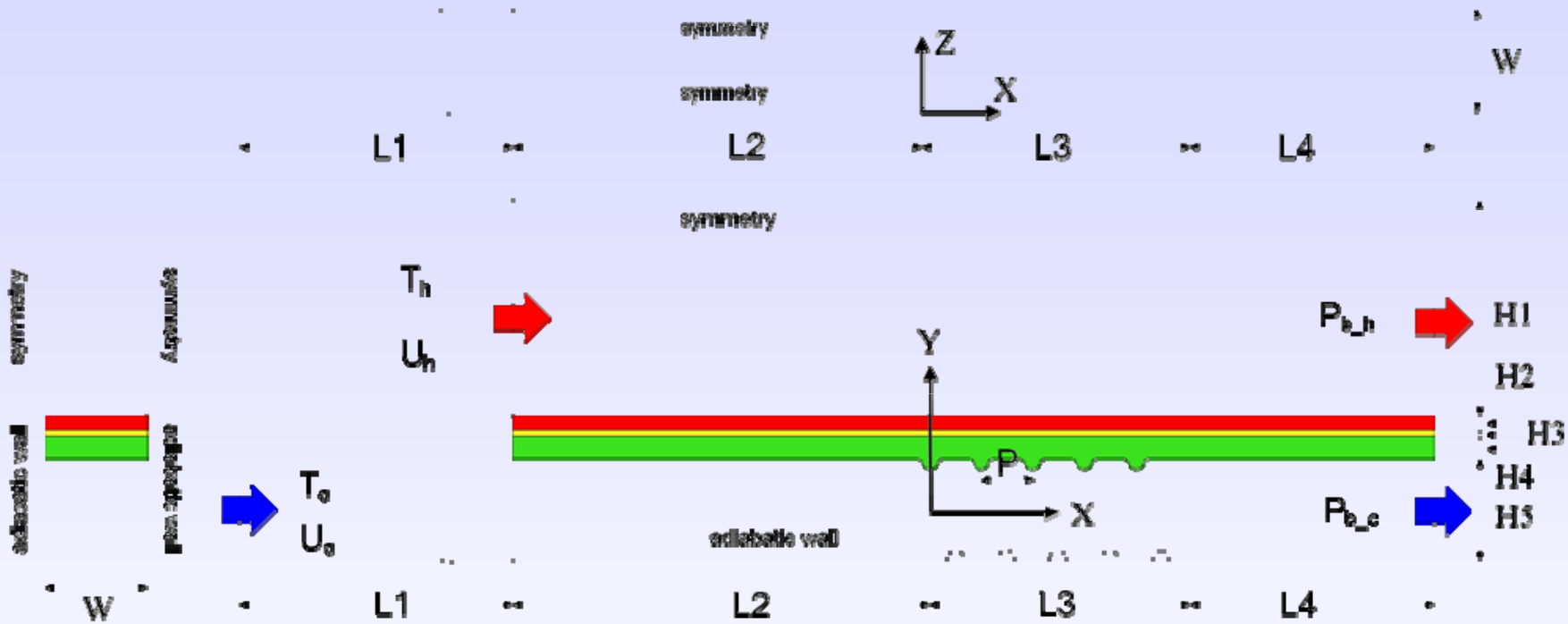


Understand meaning of the Biot number for design of experiments and interpretation of results?

Understand the effects of Biot number on local temperature and heat-flux distribution for a problem with large variations in local heat-transfer coefficients.

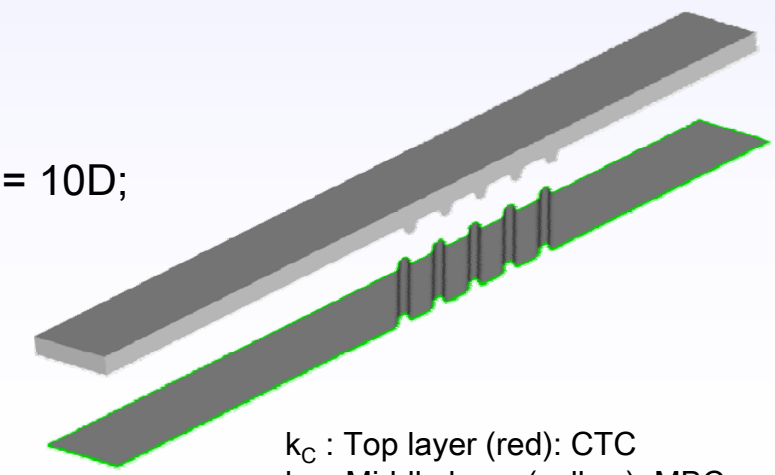
Approach: CFD

Description of Problem



D = 1.25, 12.5 mm
L1 = 200D ; L2 = 20D ; L3 = 25D ; L4 =48D;
H1 = 30D ; H2 = 0.4D; H3 = 0.1D; H4 = 2D; H5 = 10D;
P = 5D, W = 10D

T_h = 1755, 450 K ; T_c = 450, 300 K
U_h = 100, 44 m/s; U_c = 50, 31m/s
P_{b_h}= P_{b_c}= 1, 10 atm
$k_S/k_M/k_C$= 8/7.2/0.8, k_S = 1, 2, 4, 8



k_C : Top layer (red): CTC
k_M : Middle layer (yellow): MBC
k_S : Bottom layer (green): Super Alloy

Summary of Cases Studied

Case No.	T_h (K)	T_c (K)	U_h (m/s)	U_c (m/s)	P_{b_h} & P_{b_c} (atm)	H4 (mm)
1	**1755**	**550**	**100**	**50**	**10**	**2.5**
2	**1755**	**550**	**100**	**50**	**1**	**25**
3	**450**	**300**	**44**	**31**	**1**	**25**

Outline of Talk

Introduction

Objectives and Approach

Description of Problem

Basis of CFD Study

Results

Summary and Conclusions

Basis of CFD

CODE

FLUENT: version 12.1.4, single precision

FORMULATION

Gas Phase: ensemble-averaged continuity, full compressible Navier-Stokes, and total energy

- Ideal Gas
- Sutherland's model for the coefficient of molecular viscosity
- Temperature dependent Cp and k
  - **$Cp(T) = (1.0575e3) - (0.4489)*T + (1.1407e-3)*T^2 - (7.999e-7)*T^3 + (1.9327e-10)*T^4$**
  - **$k(T) = -(3.9333e-4) + (1.0184e-4)*T - (4.8574e-8)*T^2 + (1.5207e-11)*T^3$**

closed by realizeable k-ε model + 2-layer model in the near-wall region

Solid Phase: Fourier Law

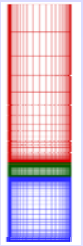
ALGORITHM

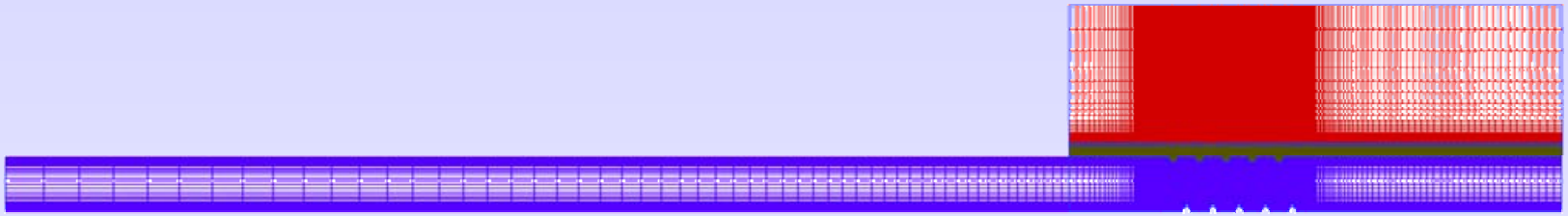
Coupled pressure-velocity scheme (2nd order for pressure and 2nd order upwind for density, momentum, and energy)

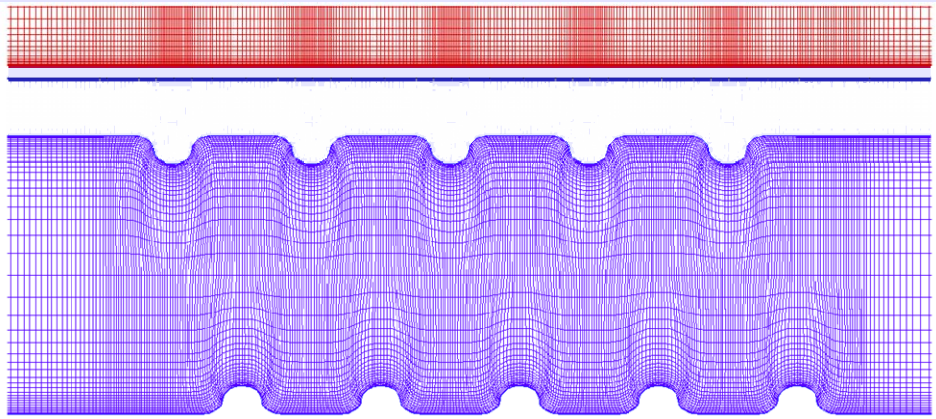
CONVERGENCE CRITERIA

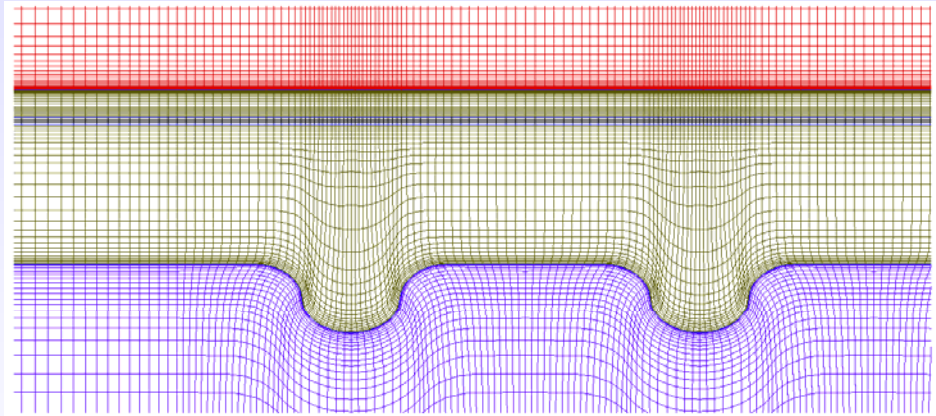
All residual plateaus ($<10^{-6}$ for density &$< 10^{-6}$ for momentum, $< 10^{-8}$ for energy, &$< 10^{-5}$ for k & ε)

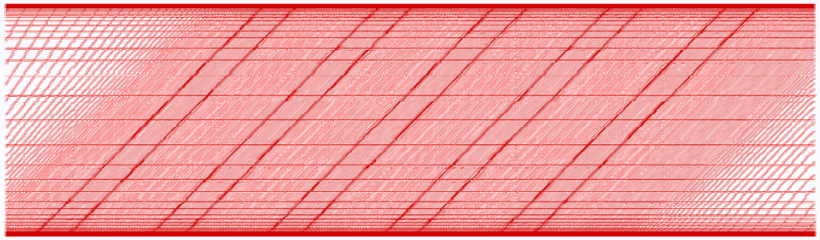
Grid











Cell numbers: 3,376,240
Face numbers: 10,310,362
Cells cluster near walls (y+ < 1 for 1st layer)

Outline of Talk

Introduction

Objectives and Approach

Description of Problem

Basis of CFD Study

Results

Summary and Conclusions

Results

Case No.	T_h (K)	T_c (K)	U_h (m/s)	U_c (m/s)	P_{b_h} & P_{b_c} (atm)	H4 (mm)
1	1755	550	100	50	10	2.5
2	1755	550	100	50	1	25
3	450	300	44	31	1	25

Understand meaning of the Biot number for design of experiments and interpretation of results?

Understand the effects of Biot number on local temperature and heat-flux distribution for a problem with large variations in local heat-transfer coefficients.

Meaning of Biot Number

$$q'' = \frac{T_h - T_c}{\frac{1}{h_h} + \frac{L_C}{k_C} + \frac{L_M}{k_M} + \frac{L_S}{k_S} + \frac{1}{h_c}}$$

$$Bi = \left(\frac{L_C}{k_C} + \frac{L_M}{k_M} + \frac{L_S}{k_S} \right) \Big/ \left(\frac{1}{h_h} + \frac{1}{h_c} \right)$$

$$Bi = h_h \left(\frac{L_C}{k_C} + \frac{L_M}{k_M} + \frac{L_S}{k_S} \right)$$

Hot-Gas Side

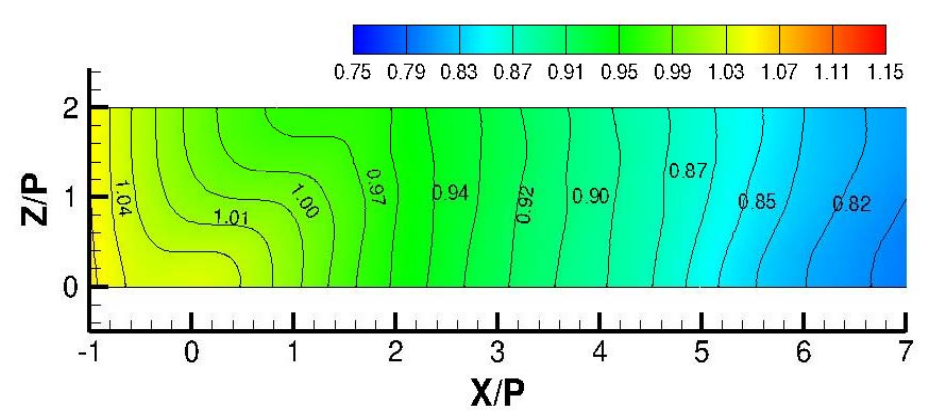
$$Bi = h_c \left(\frac{L_C}{k_C} + \frac{L_M}{k_M} + \frac{L_S}{k_S} \right)$$

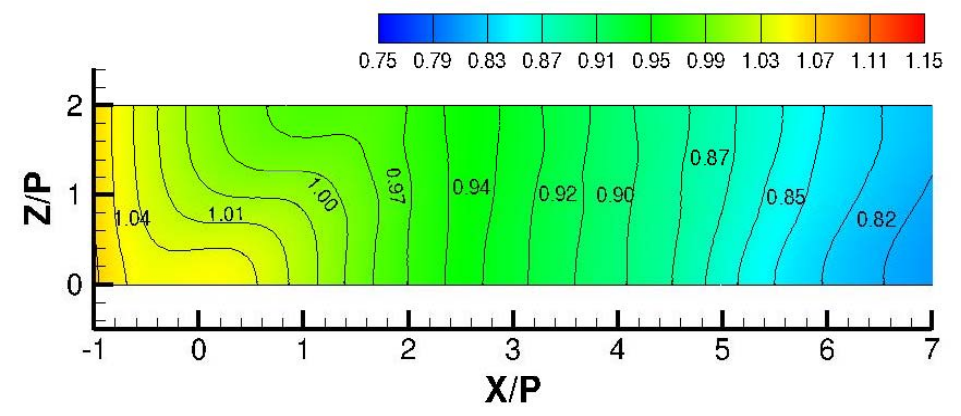
Coolant Side

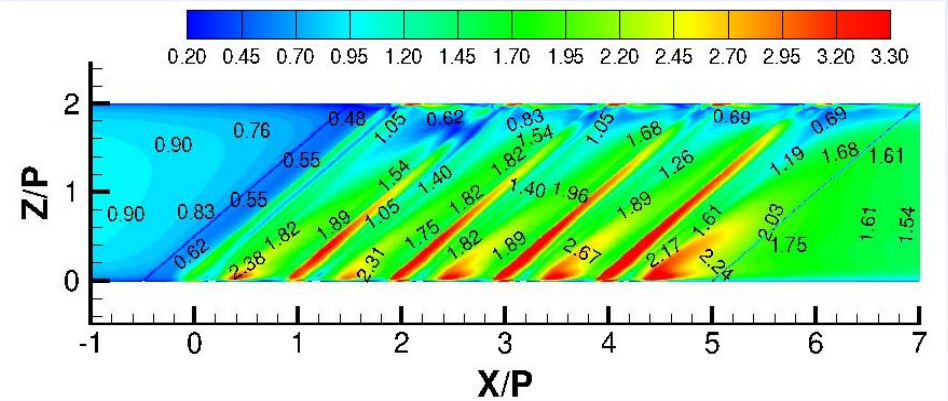
same Bi, but different size & P

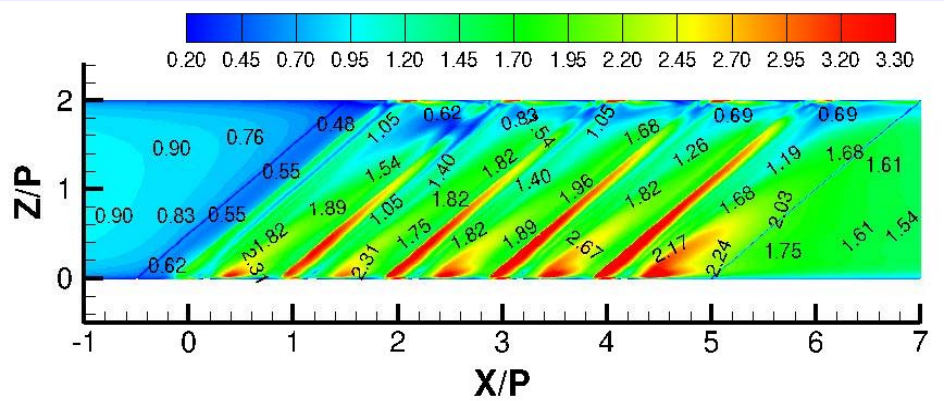
Case No.	T_h (K)	T_c (K)	U_h (m/s)	U_c (m/s)	P_{b_h} &P_{b_c} (atm)	H4 (mm)	$Re_{hot\text{-}side}$/ $Re_{cold\text{-}side}$
1	**1755**	**550**	**100**	**50**	**10**	**2.5**	**510,521/ 143,021**
2	**1755**	**550**	**100**	**50**	**1**	**25**	**510,521/ 143,021**
3	450	300	44	31	1	25	1,635,866/ 251,200

Biot Number Similarity (size & P)







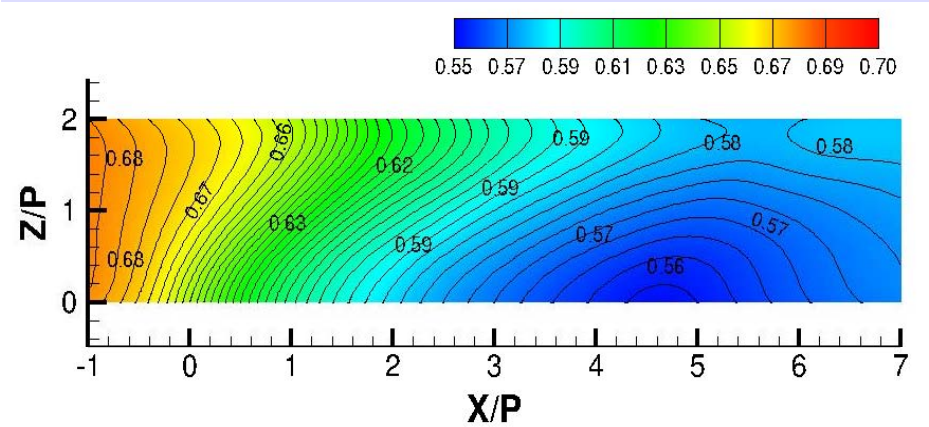


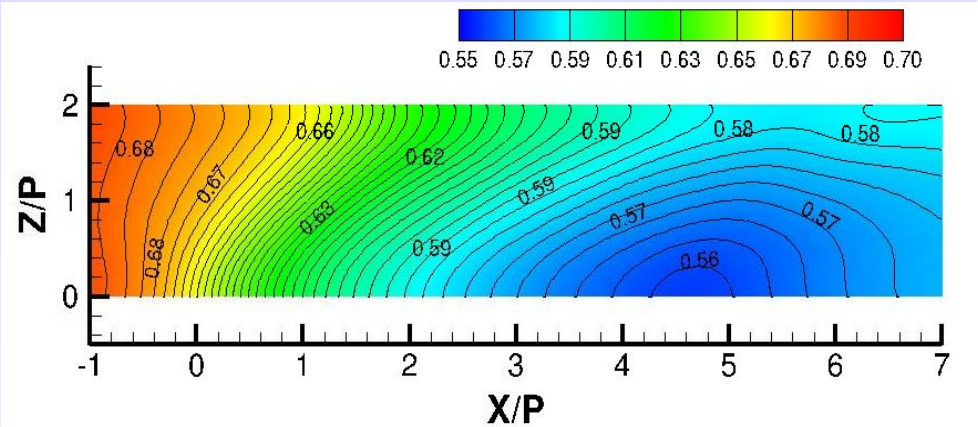
1755 K / 550 K, 10 atm, small **1755 K / 550 K, 1 atm, large**

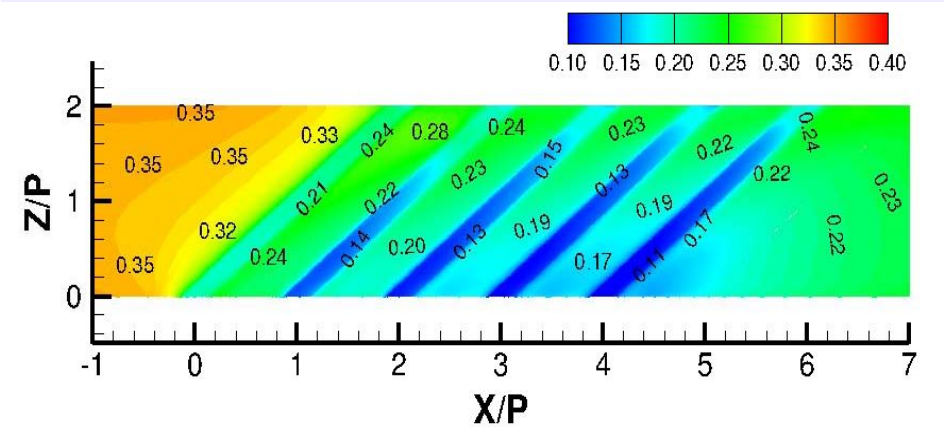
BIOT NUMBER at the hot-gas and coolant sides.

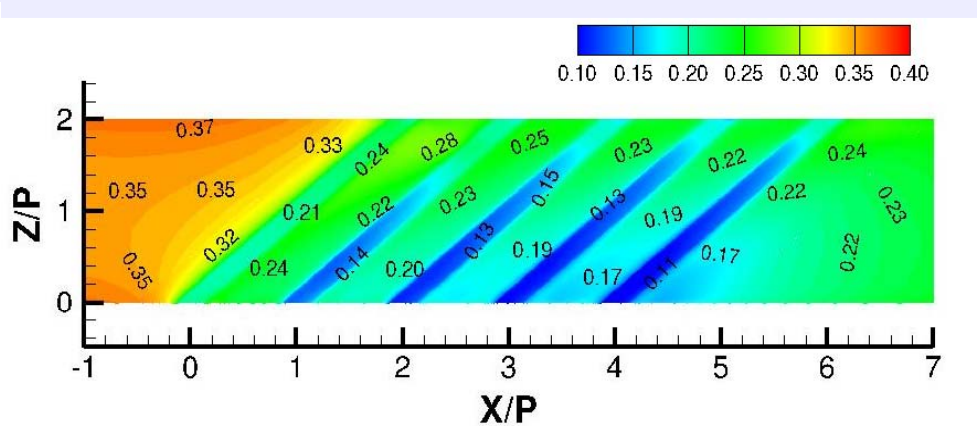
Bi Similarity & Temperature (size & P)

$$\eta = \left(T - T_{cold}\right) / \left(T_{hot} - T_{cold}\right)$$







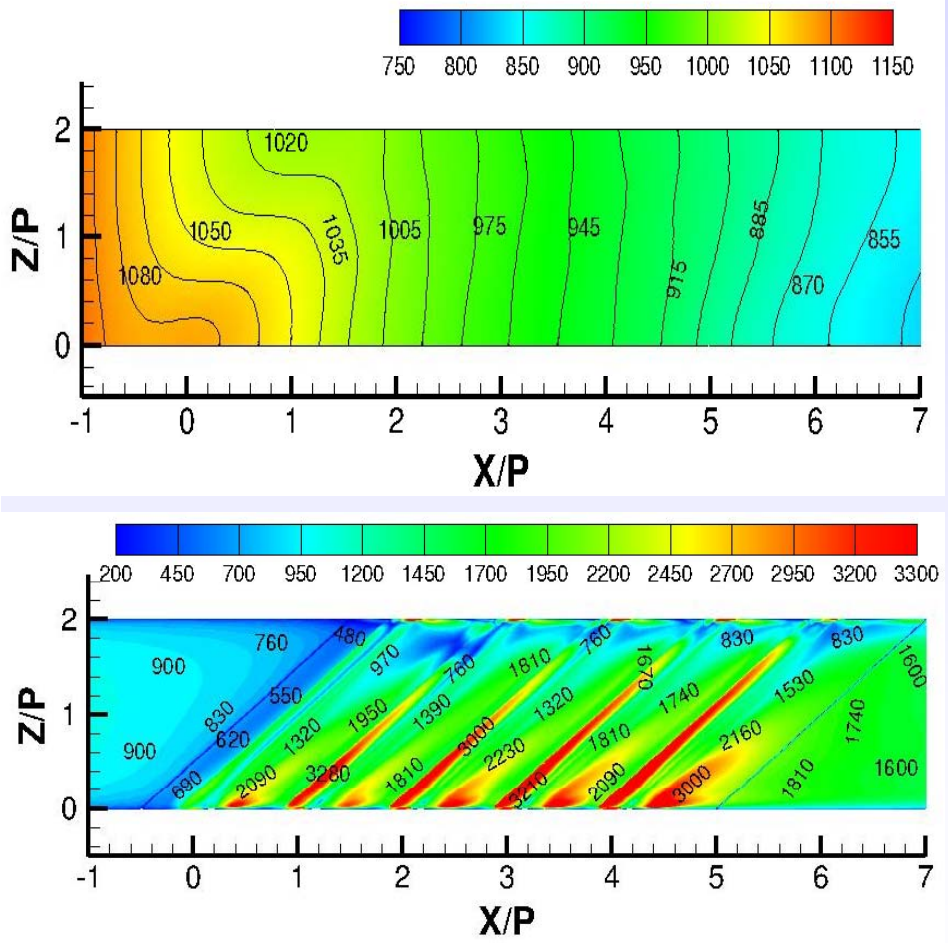


1755 K / 550 K, 10 atm, small

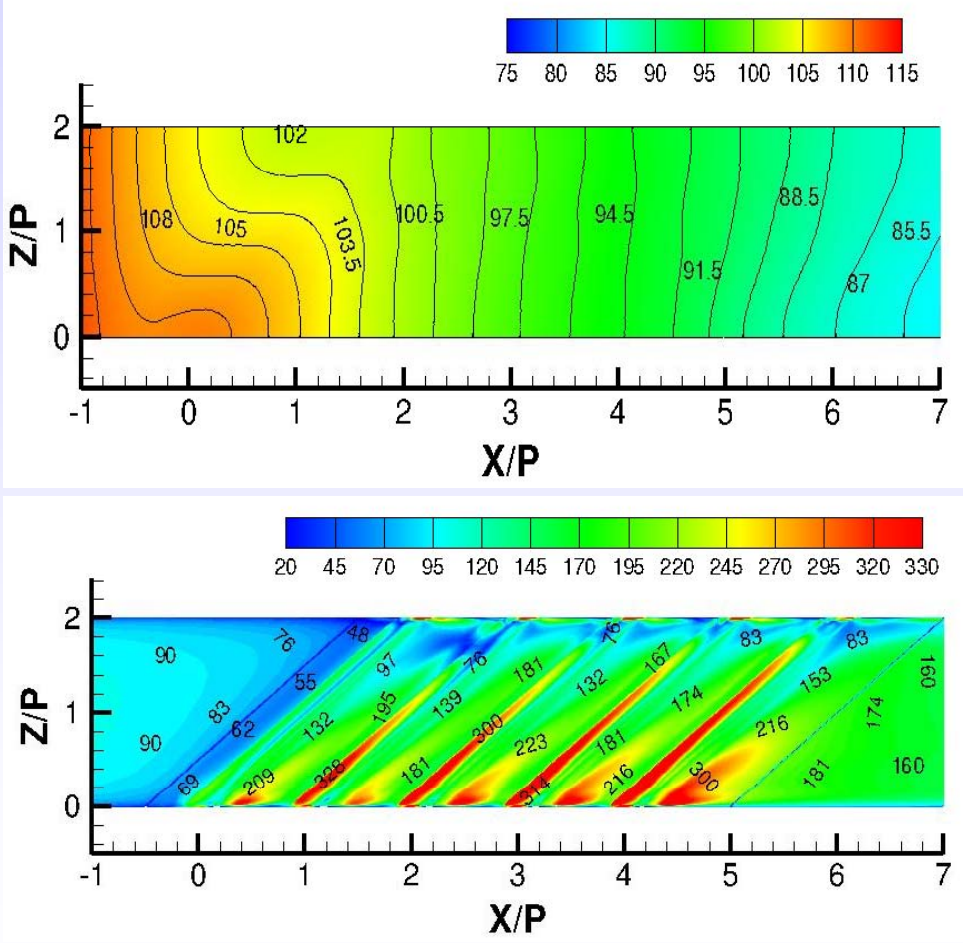
1755 K / 550 K, 1 atm, large

TEMPERATURE at the hot-gas and coolant sides.

Bi Similarity & Heat Transfer (size & P)



1755 K /550 K, 10 atm, small

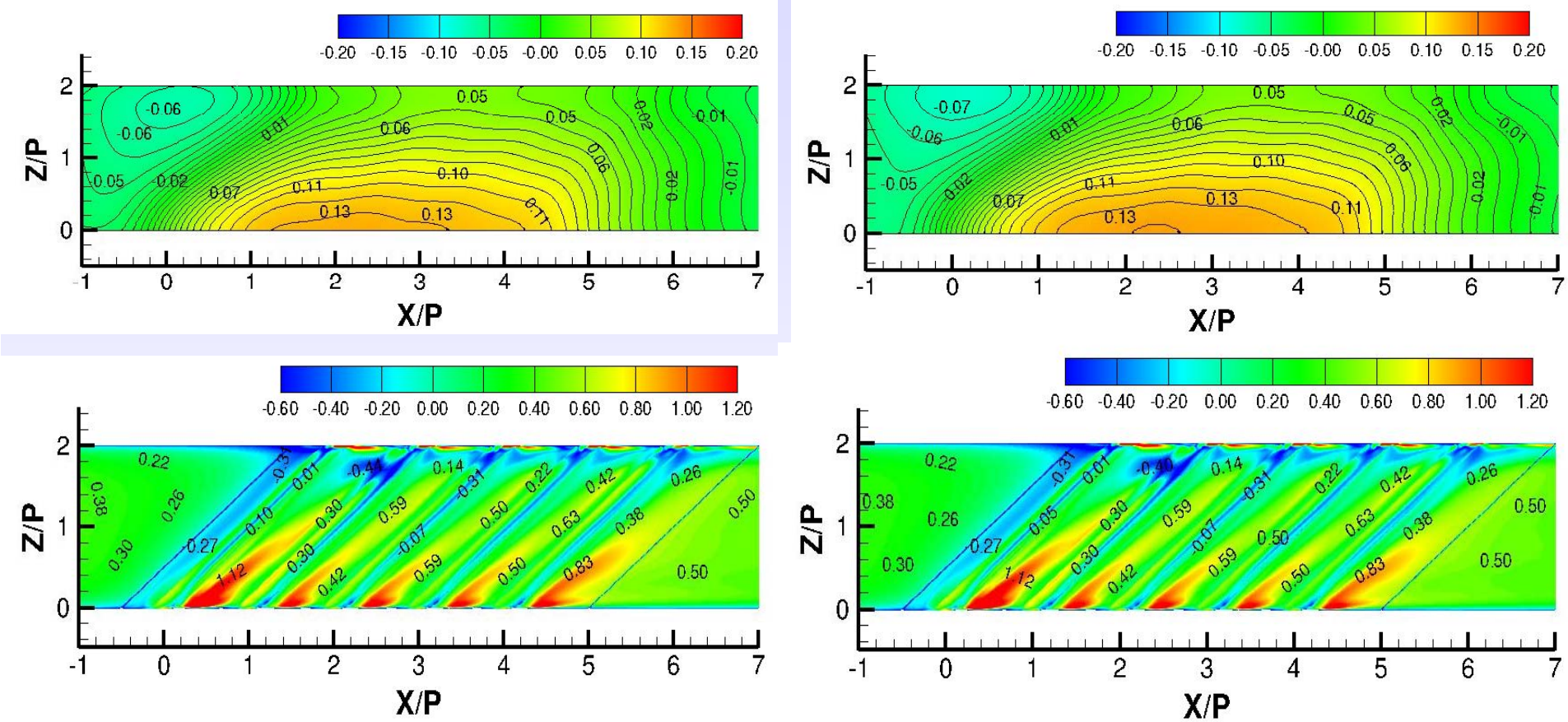


1755 K /550 K, 1 atm, large

HEAT TRANSFER COEFFICIENT at the hot-gas and coolant sides.

Bi Similarity & Heat Transfer (size & P)

$$\theta = \left(q'' - q''_{ave}\right) / q''_{ave}$$



1755 K /550 K, 10 atm, small

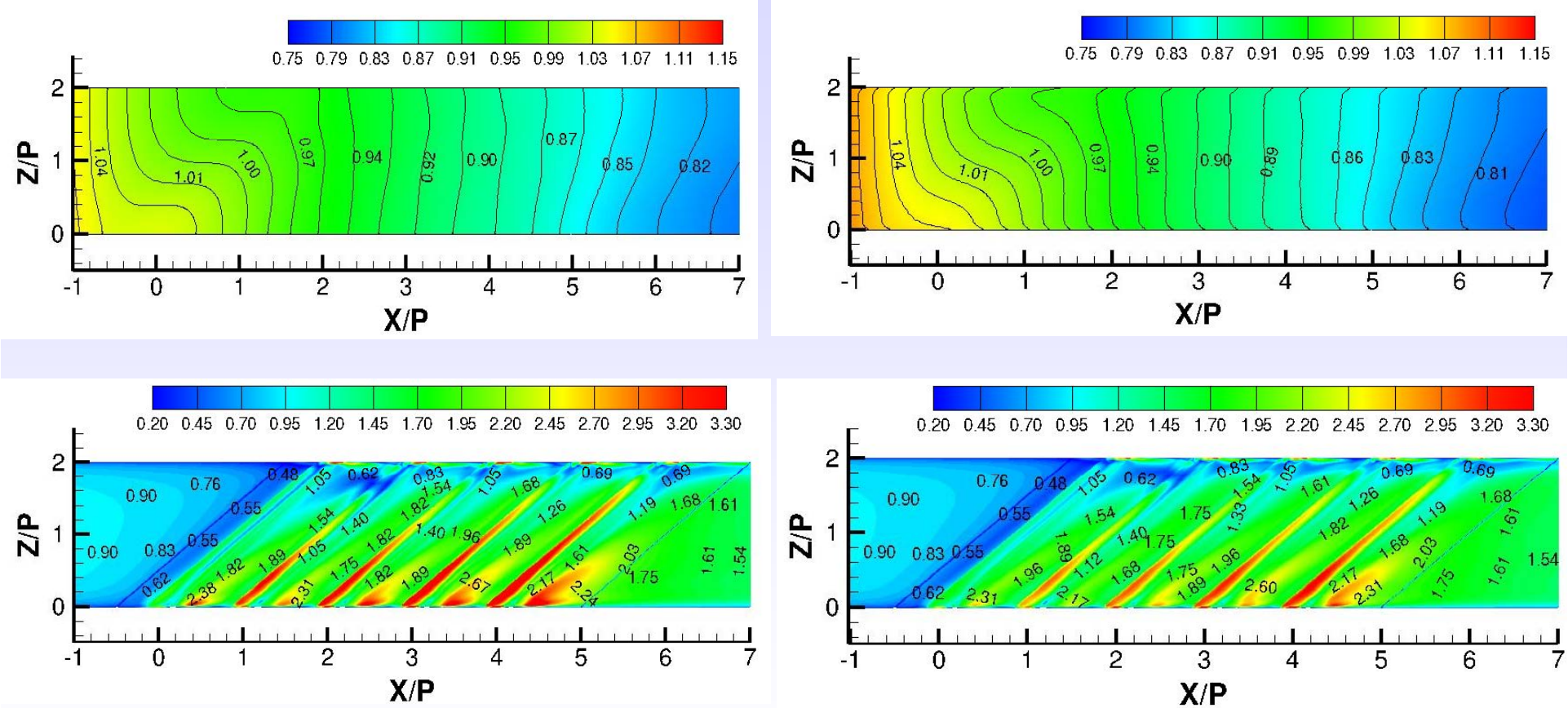
1755 K / 550 K, 1 atm, large

HEAT FLUX at the hot-gas and coolant sides.

same Bi, but different size, P, and T

Case No.	T_h (K)	T_c (K)	U_h (m/s)	U_c (m/s)	P_{b_h}&P_{b_c} (atm)	H4 (mm)	$Re_{hot\text{-}side}$/ $Re_{cold\text{-}side}$
1 (K8)	**1755**	**550**	**100**	**50**	**10**	**2.5**	**510,521/ 143,021**
2 (K8)	1755	550	100	50	1	25	510,521/ 143,021
3 (K8)	**450**	**300**	**44**	**31**	**1**	**25**	**1,635,866/ 251,200**

Biot Number Similarity (size, P, and T)



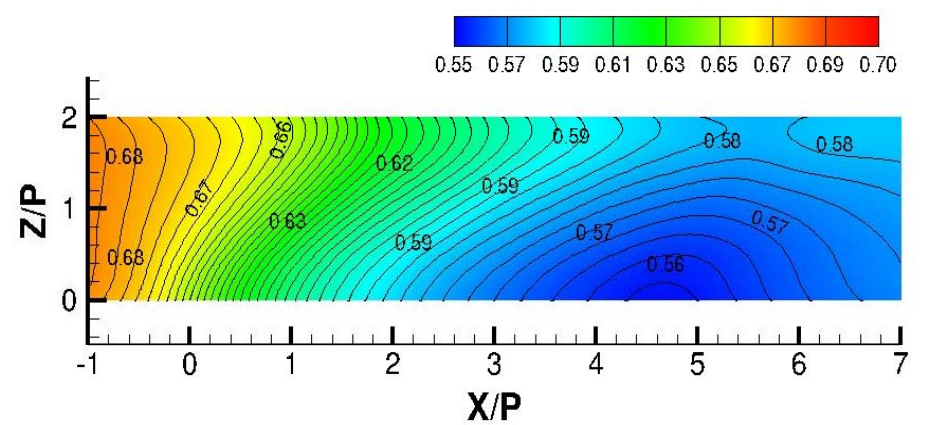
1755 K / 550 K, 10 atm, small

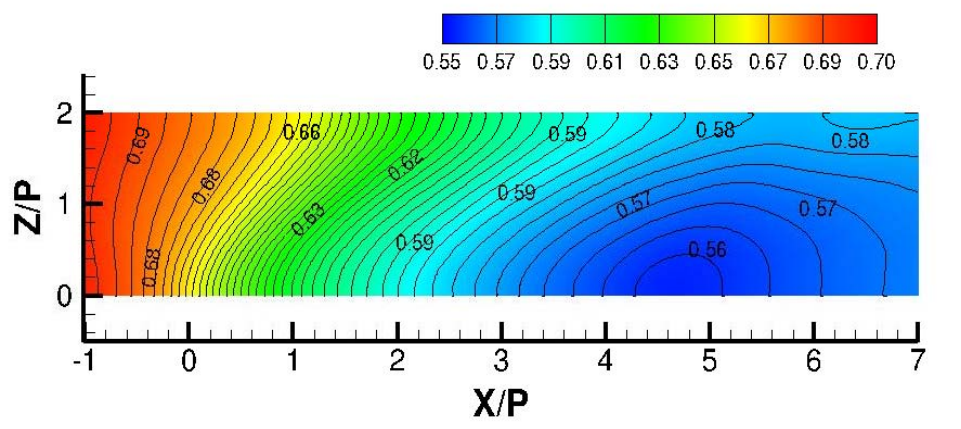
450 K / 300 K, 1 atm, large

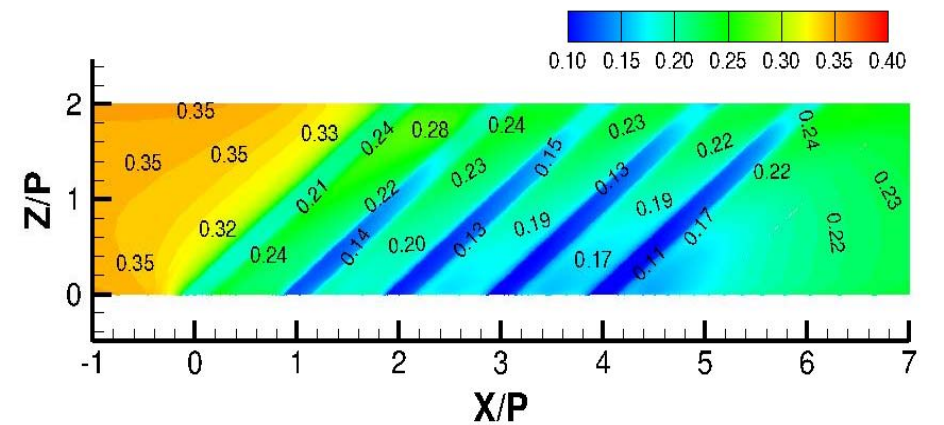
BIOT NUMBER at the hot-gas and coolant sides.

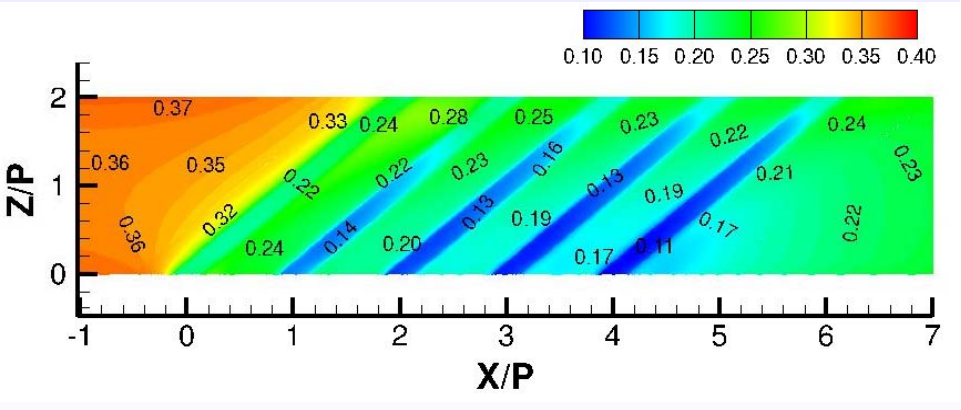
Bi Similarity & Temperature (size, P, and T)

$$\eta = \left(T - T_{cold}\right) / \left(T_{hot} - T_{cold}\right)$$







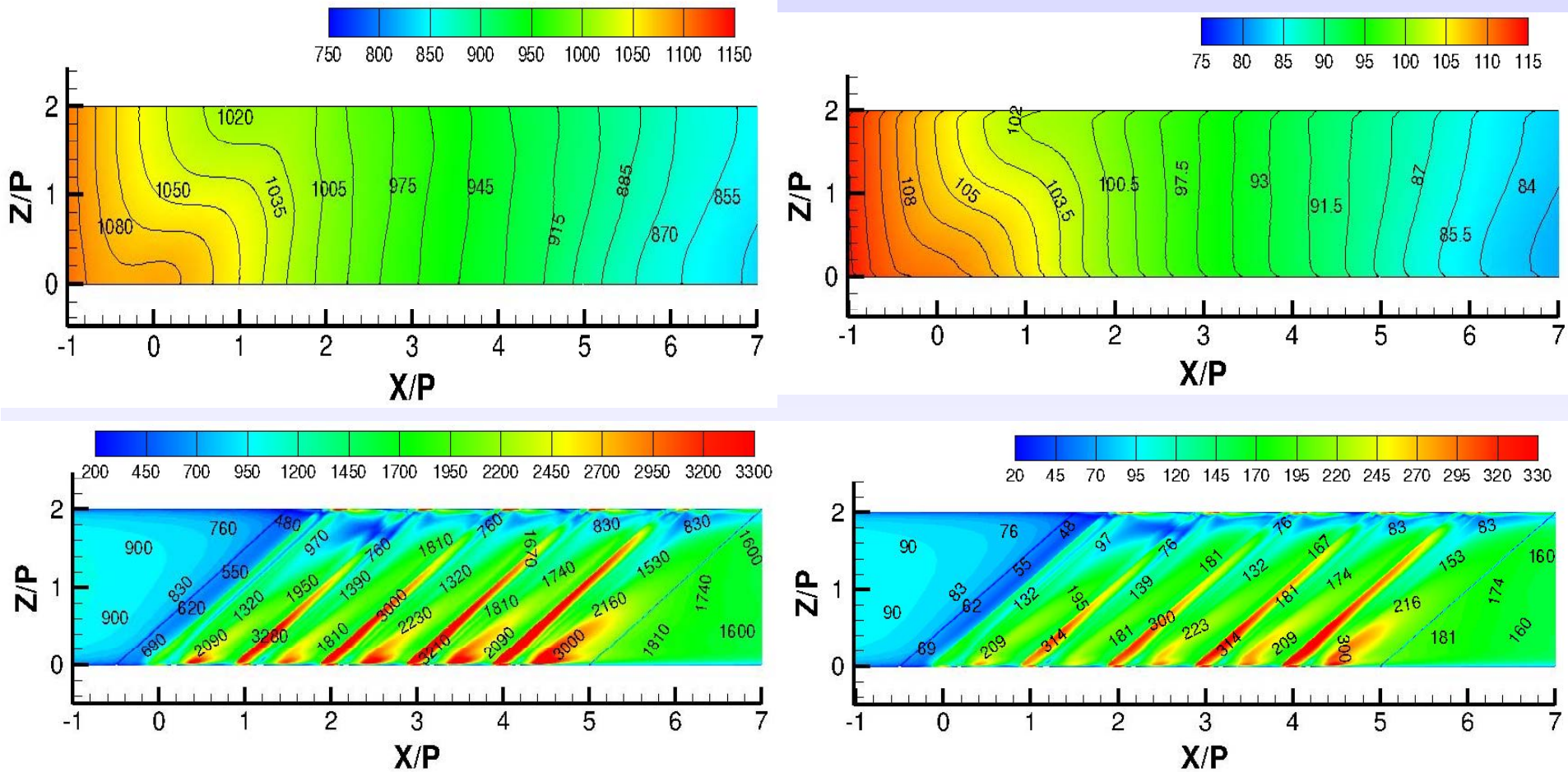


1755 K /550 K, 10 atm, small

450 K /300 K, 1 atm, large

TEMPERATURE at the hot-gas and coolant sides.

Bi Similarity & Heat Transfer (size, P, and T)



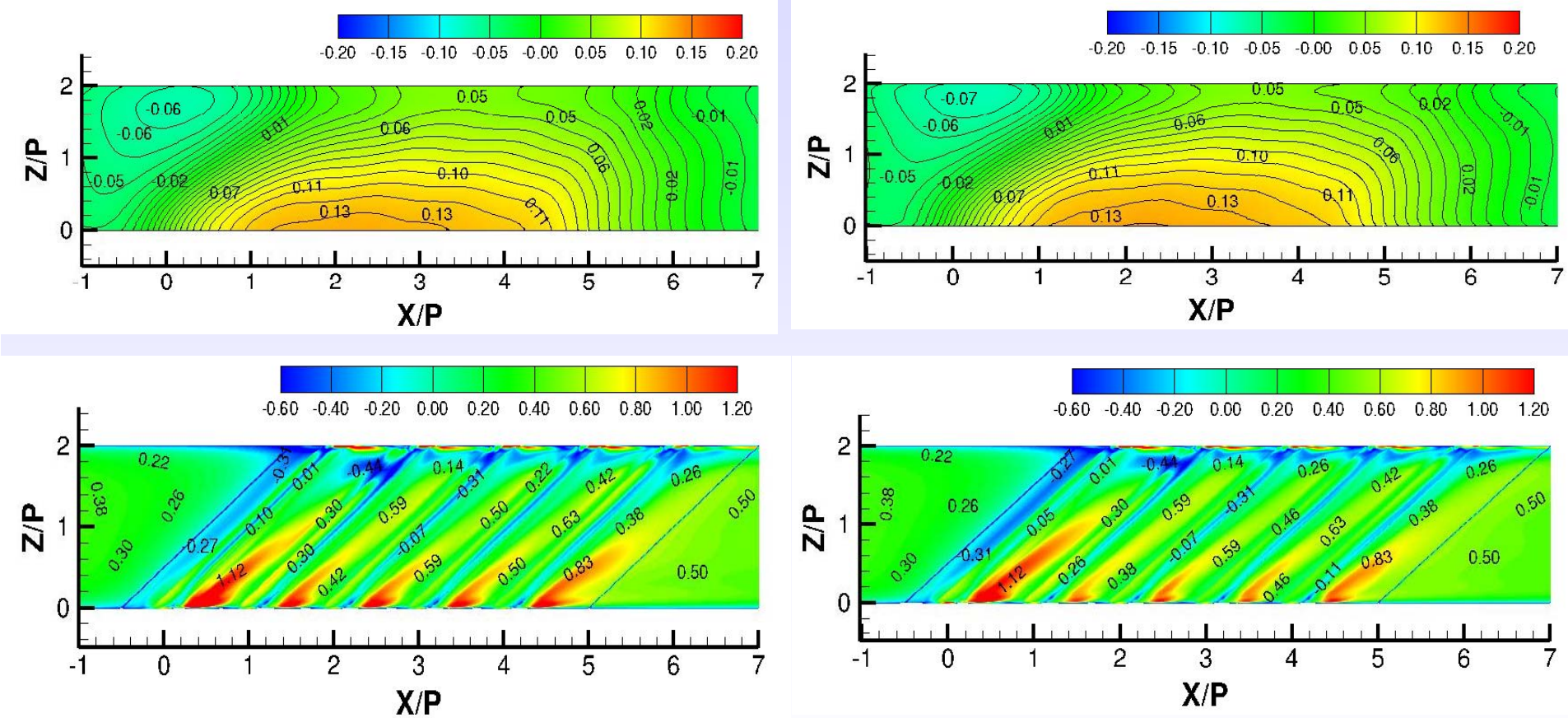
1755 K / 550 K, 10 atm, small

450 K / 300 K, 1 atm, large

HEAT TRANSFER COEFFICIENT at the hot-gas and coolant sides.

Bi Similarity & Heat Transfer (size, P, and T)

$$\theta = \left(q'' - q''_{ave}\right) / q''_{ave}$$



1755 K /550 K, 10 atm, small **450 K /300 K, 1 atm, large**

HEAT FLUX at the hot-gas and coolant sides.

Summary: Meaning of Biot No. Similarity

- If the Biot numbers of 2 geometrically similar configurations are nearly the same (magnitude and distribution on both the hot and the cold sides), then the magnitude and contours of the normalized temperatures are nearly the same.

- Though magnitude of the heat flux can differ considerably, the normalized heat-flux are nearly the same.

Thus, Biot number similarity could be used in DoE for CFD and EFD to understand temperature distribution in the material induced by the coupling of internal and external heat transfer.

This enables an experimental study of high T, high P, and small sizes in normal temperature and pressure and scaled up geometry.

Thank You