Adaptive Homogenization for Upscaling Heterogeneous Porous Media

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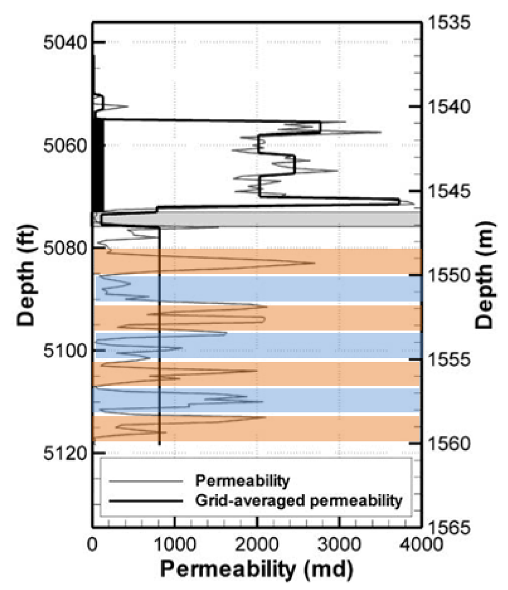
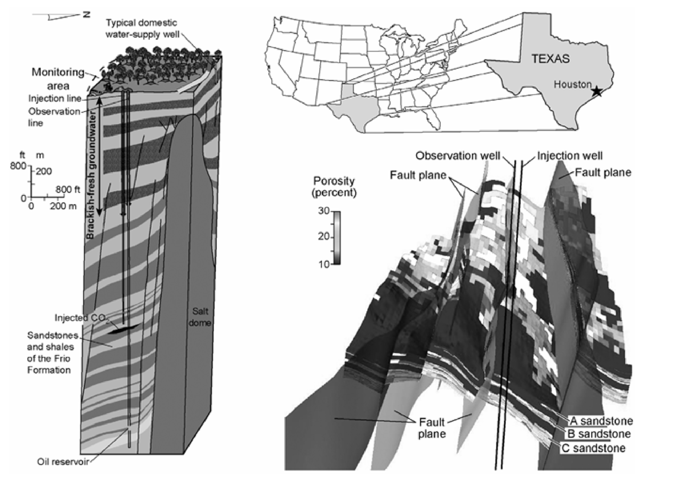
1. Introduction

The multiscale nature of the flow and transport problems in porous medium is rising attention in pratical applications

- petroleum reservoir recovery evaluations
- nuclear waste disposal systems
- CO2 sequestration
- groundwater remediation.



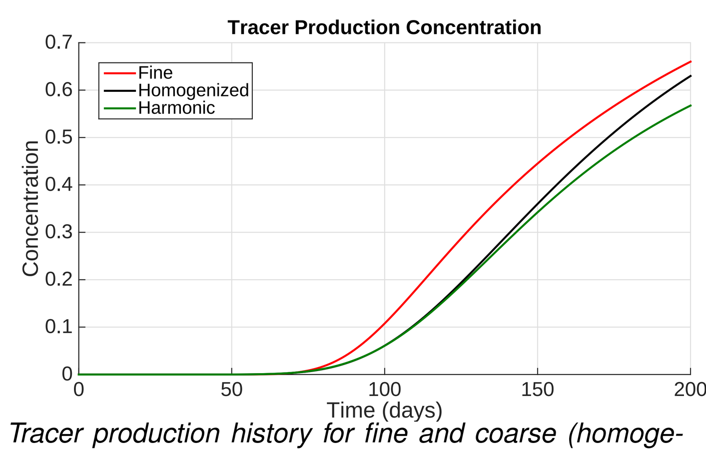


the Frio Brine Pilot experiment

We propose an adaptive multiscale approach to improve the efficiency and the accuracy of numerical computations by combining upscaling and domain decomposition methods. We use the Enhanced Velocity Mixed Finite Element Method (EVMFEM) as a domain decomposition approach to couple the coarse and fine subdomains [3].

2. Motivation

- Direct numerical computation is computationally prohibitive, since
  - Heterogeneity of porous media
  - Complexity of dynamic systems
- Capture fine scale features
  - Higher resolution only near well-bore: polymer injection, gas injection
  - Non-linear reactions with large variation in reaction rates
- Effective parameter computation precision



Tracer production history for fine and coarse (homogenized and harmonic average) scale problems.

3. Model formulation

$$\Gamma_{i,j} = \partial\Omega_i \cap \partial\Omega_j \qquad \Gamma_i = \partial\Omega_i \setminus \Omega \qquad J = (0, T)$$

Phase Mass Conservation

$$\nabla \cdot \mathbf{u} = q \qquad \text{in } \Omega_i \times \text{J}$$
$$\mathbf{u} = -\frac{K}{\mu}\nabla p \qquad \text{in } \Omega_i \times \text{J}$$

Component Mass Conservation

$$\frac{\partial(\phi c)}{\partial t} + \nabla \cdot (\mathbf{u}c - D\nabla c) = q_\alpha \hat{c} + r(c) \qquad \text{in } \Omega_i \times \text{J}$$

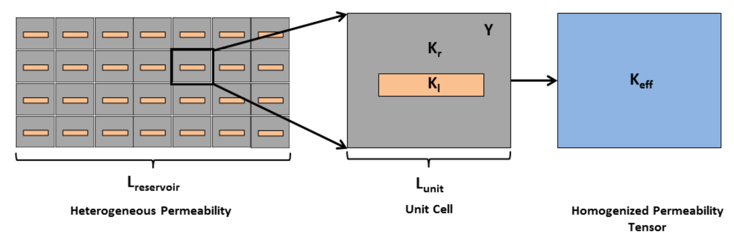
Boundary and Initial conditions

$$\mathbf{u}\cdot\mathbf{n} = 0, \qquad \mathbf{u}c\cdot\mathbf{n} = 0 \qquad \text{on} \qquad \partial\Omega \times \text{J}$$
$$c = c^0, \qquad p = p^0 \qquad \text{at} \qquad \Omega \times \{\text{t} = 0\}$$

Methods

The key objective of this work is to develop upscaling approaches to minimize the use of fine scale information for time varying initial and boundary conditions. A fine scale flow and transport problem is solved only in specific subdomains while solving a coarse scale problem in the rest of the domain. This involves special treatment of sharp interfaces associated with the transport equations. We define transient regions as subdomains where spatial changes in concentration are significant. Away from the transient regions, upscaling is performed locally by using a numerical homogenization to obtain effective equations at the macroscopic scale. A fine grid is then used in the transient regions and a coarse grid everywhere else resulting in a non-matching multi-block problem.

Numerical Homogenization

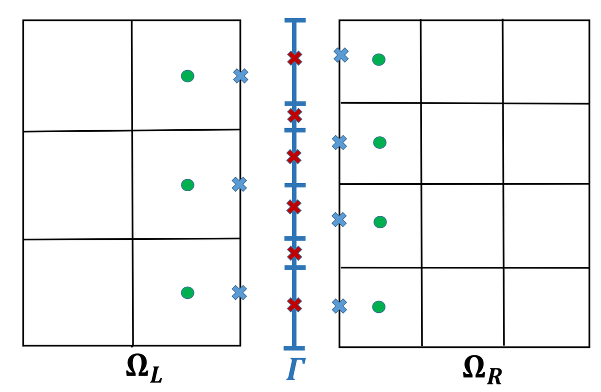


Ansatz for the solution

$$f_\varepsilon(x) = f_0(x,y) + \varepsilon f_1(x,y) + \varepsilon^2 f_2(x,y) + ..$$

where $f_i(x,y)$ is a function of both x and y, periodic in y and $y = \frac{x}{\varepsilon}$.
The gradient operator scales as $\nabla = \nabla_x + \varepsilon^{-1}\nabla_y$.
We expect $u = u_\varepsilon \to u_0 \quad as \quad \varepsilon \to 0$.

Enhanced Velocity Mixed FEM



On interface Γ

- extra velocity
- directly construct a flux-continuous velocity approximation

4. Adaptivity criteria

- **Criteria 1** (gradient in time):

$$\Omega_f = \left\{ \mathbf{x} : |c^n(x) - c^{n-1}(x)| > \epsilon_{tadap} \right\}$$

- **Criteria 2** (gradient in space): We define

$$\Omega_{neighbor}(x) = \{y : y \in E_j, |\partial E_i \cap \partial E_j| \neq \emptyset, \text{if } x \in E_i.\}$$

The criteria can then be defined as,

$$\Omega_f = \left\{ \mathbf{x} : \max|c^n(x) - c^n(y)| > \epsilon_{sadap} \quad \forall y \in \Omega_{neighbor}(x) \right\}$$

5. Numerical Experiments

5.1 Homogeneous permeability

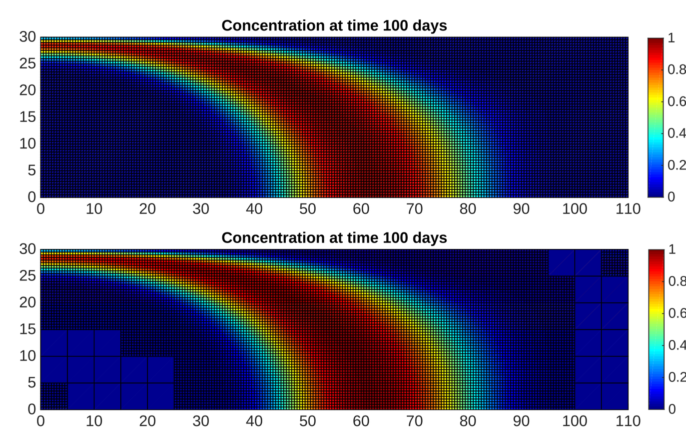


Figure 1: *Concentration profiles at 100 days for fine (top) and adaptive (bottom) approaches*

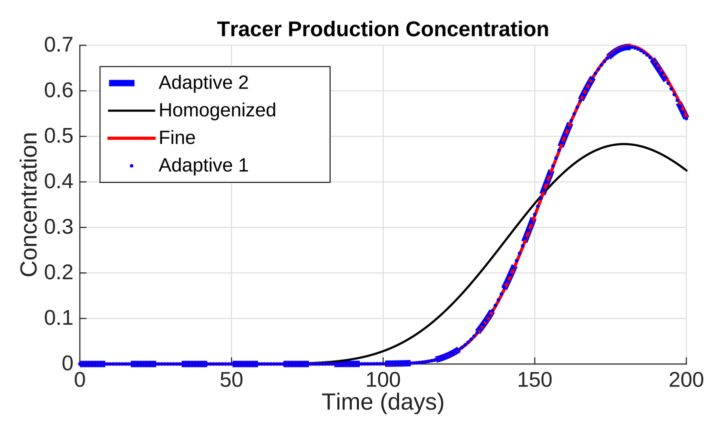


Figure 2: *Tracer concentration history at production well for a homogeneous permeability distribution of 50 mD*

5.2 Benchmark datasets: SPE 10

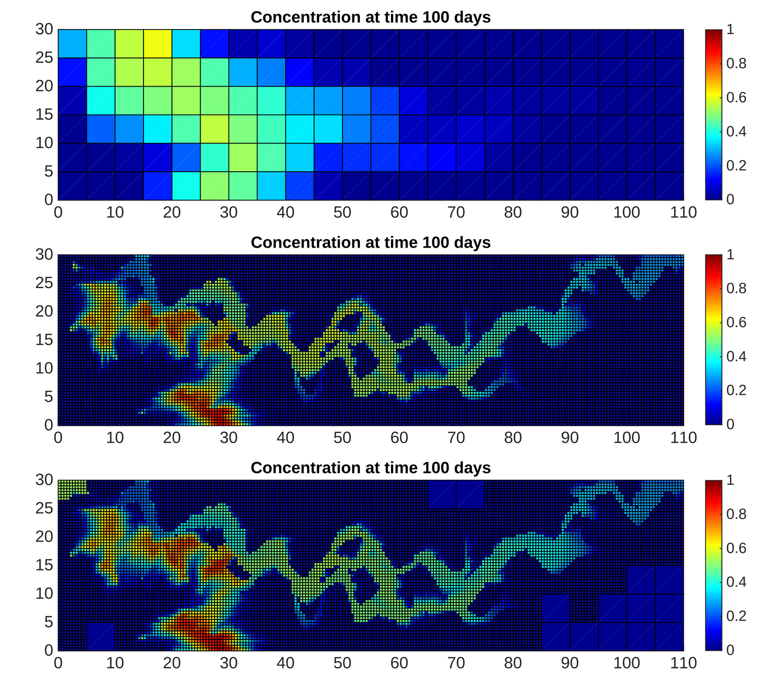


Figure 3: *Concentration profiles at 100 days for coarse (top) , fine (middle) and adaptive (bottom) approaches*

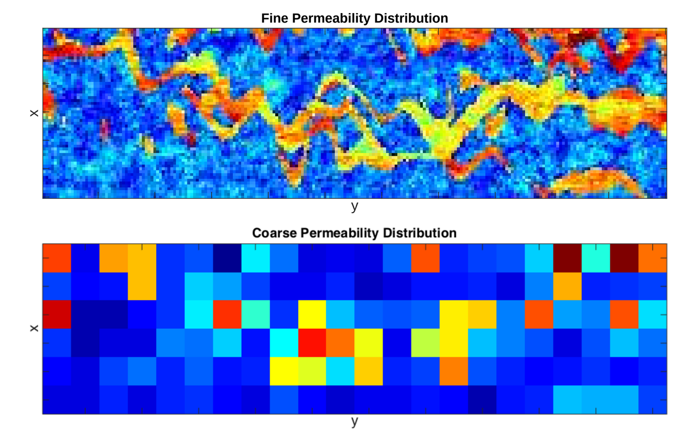


Figure 4: *Fine and Coarse scale permeability distributions for SPE 10 layer 37*

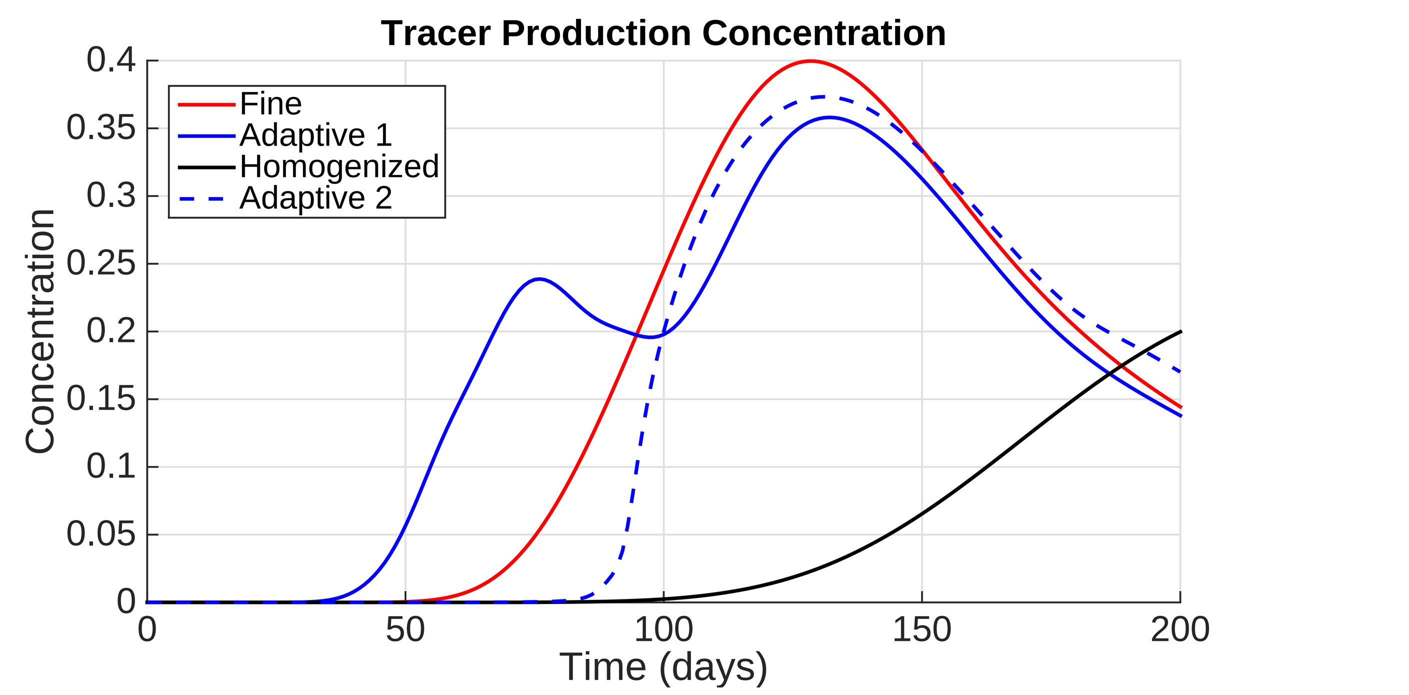


Figure 5: *Tracer concentration history at the production well for SPE layer 37.*

6. Conclusions

We developed an adaptive multiscale scheme using local numerical homogenization and EV MFEM for upscaling single phase flow and transport in a heterogeneous porous media. The numerical results on different layers of SPE10 also indicate that an upscaling based solely upon numerical homogenization is in good agreement with the fine scale solution for a Gaussian or periodic permeability distribution. However, for highly channelized for layered permeability distributions the results deviate substantially. The adaptive multiscale scheme, on the other hand, is in good agreement for Gaussian, periodic, and layered permeability distributions and is approximately faster than the fine scale simulation for a tracer slug injection of 50 days in all the above numerical tests. It is important to note that a tracer slug injection is specifically chosen to test adaptivity since there are two transient regions at the front and back of the injected slug. The accuracy of the adaptive multiscale approach presented here, compared to the fine scale solution, is dependent on the nature of physical process and validity of the adaptivity criteria in capturing fine scale physics. An optimal tolerance for the adaptivity criteria is chosen to reduce computational cost without substantial loss in solution accuracy.

References

[1] Y. Amanbek, G. Singh, M.F. Wheeler, and H. van Duijn. Adaptive numerical homogenization for upscaling single phase flow and transport. Technical report. ICES Report 17-12, June, 2017.

[2] G. Singh, Y. Amanbek, and M. F. Wheeler. Adaptive numerical homogenization for non-linear multiphase flow and transport. Technical report. ICES Report 17-13, June, 2017.

[3] John A Wheeler, Mary F Wheeler, and Ivan Yotov. Enhanced velocity mixed finite element methods for flow in multiblock domains. *Computational Geosciences*, 6(3-4):315–332, 2002.