2016 UTSR Workshop

PURDUE UNIVERSITY

Flow and Heat Transfer in a Smooth U-Duct with Trapezoidal Cross Section

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Introduction

The thermal efficiency of gas turbines increases with the gas temperature at the turbines' inlet. The temperatures sought far exceed the melting temperature of the best superalloys. Thus, cooling is needed to ensure that all material that come in contact with the hot gases never exceeds the maximum allowable temperature for strength and desired service life. Since cooling requires work, the amount of cooling flow used should be kept to a minimum. Also, since tremendous advances have already been made in turbine cooing, to make the next advance, a leap is needed in our understanding on how geometry affects the flow and surface heat transfer Thus, high fidelity design tools are of interest.

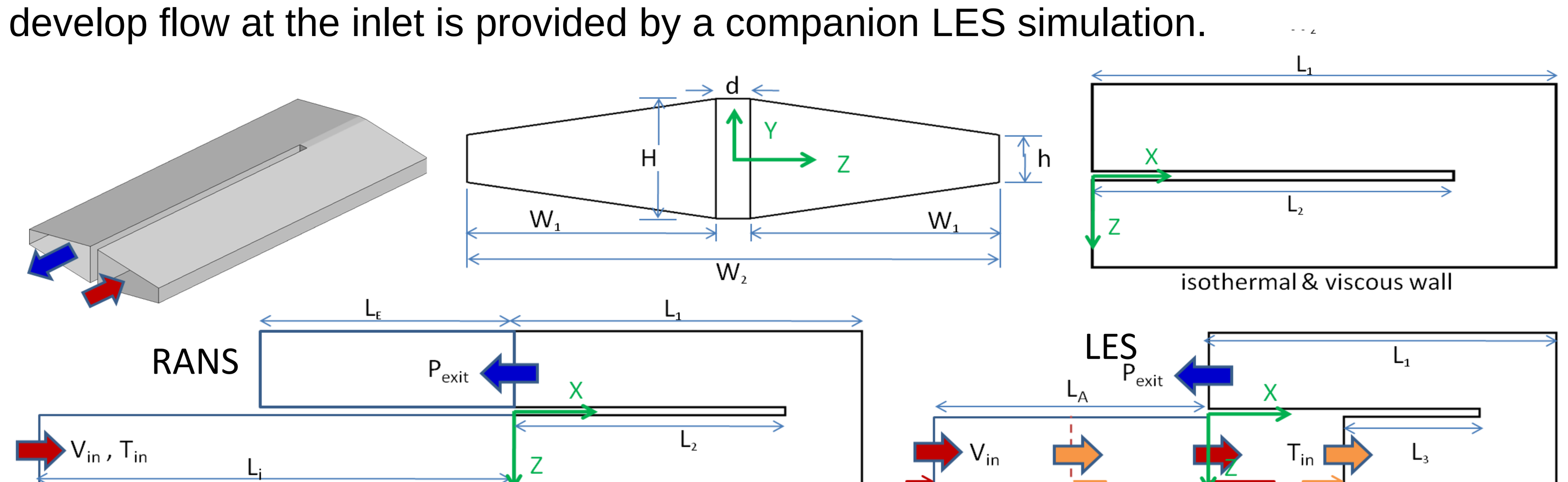
CFD based on RANS are now widely used to design internal and film-cooling strategies. However, the accuracy of the turbulence models is a major challenge. Though LES and DNS have the potential to offer the accuracy needed, performing LES or DNS of a cooled turbine blade with internal and film cooling in reasonable time is not yet practical. Thus, LES and DNS are used to assess and guide the development of turbulence models for RANS and URANS. Though a number of investigators have performed research in this area, few have performed LES of U-duct and none on high aspect ratio and ducts with cross sections that are not rectangular.

Objective

Use grid-independent LES to assess the ability of three RANS models – k-ε, SST, and stress-omega Reynolds stress models – in predicting the turbulence characteristics and how turbulence affects heat-transfer in a U-duct with trapezoidal cross section.

Problem Description

The U-duct with a trapezoidal cross section along with the operating conditions are given in the figure below. For simulations that use RANS, an upstream and a downstream duct are appended. The upstream duct is appended to ensure fully-developed flow at the U-duct's inlet. The downstream duct is appended to ensure no reverse flow at the outflow boundary. For LES, these are not used. For LES, the fully-develop flow at the inlet is provided by a companion LES simulation.



Unit: mm

d = 6.35	L_i = 40 D_h	L_2 = 192	**Re = 20,000**
D_h = 29.04	L_E = 384	L_3 = 96	V_{in} = 12.65 m/s
H = 28.48	L_A = 6.6 D_h	W_1 = 54.24	T_{in} = 343.15 K
h = 11.72	L_1 = 246	W_2 = 114.83	T_{wall} = 313.15 K
			P_{exit} = 101325 Pa

Formulation and Numerical Method

Solver: Fluent 16.2.0

Assumptions: incompressible flow of air with constant properties at

$T = (T_{inlet} + T_{wall})/2 = 328.15$ K

(ρ = 1.0753 kg/m^3, C_p = 1007 J/kg-K, k = 0.028332 W/m-K, μ = 1.9765X10^{-5} kg/m-s)

neglect viscous dissipation.

RANS Algorithm

- Double precision, pressure-based, absolute velocity formulation, SIMPLE
- Second-order upwind for convective terms.

Convergence Criteria:

- compute until all "scaled" residuals plateau
- plateaued residual: continuity < 10^{-3}, momentum < 10^{-5}, energy < 10^{-7}, turbulence quantities < 10^{-5}

LES Algorithm

- Double precision, pressure-based, absolute velocity formulation, SIMPLE
- PRESTO pressure, 2nd order central, bounded 2nd order in time

Governing Equations

RANS: **Reynolds-Averaged Continuity & Navier-Stokes equations**

Continuity: $\frac{\partial u_i}{\partial x_i} = 0$

Momentum: $\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{u}_i}{\partial x_j} + \boxed{\tau_{ij}}\right)$

Energy: $\frac{\partial \bar{T}}{\partial t} + \bar{u}_i \frac{\partial \bar{T}}{\partial x_i} = \Gamma \frac{\partial}{\partial x_i}\left(\frac{\partial \bar{T}}{\partial x_i}\right) - \frac{\partial \overline{u_i' T'}}{\partial x_i}$

Turbulence Models: **Realizable k-ε**, **SST-kω**, and **RSM-τω**

Eddy Diffusivity Models

Two equation models:

Boussinesq Hypothesis: $-\rho\overline{u_i' u_j'} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\left(\rho k + \mu_t \frac{\partial u_k}{\partial x_k}\right)\delta_{ij}$

Realizable k-ε: $\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \Leftrightarrow \mu_t^{LES} = \rho C_\mu^{LES}\frac{(k^{LES})^2}{\varepsilon^{LES}}$

Shear-Stress Transport (SST)-kω:

$$\mu_t = \frac{\rho k}{\omega}\frac{1}{\max\left[1, \frac{SF_2}{a_1\omega}\right]} \Leftrightarrow \mu_t^{LES} = \frac{\rho k^{LES}}{\omega}\frac{1}{\max\left[1, \frac{SF_2^{LES}}{a_1\omega}\right]}$$

Reynolds Stress Models

Seven equation model:

Transport equation for $\overline{u_i' u_j'}$

$$\underbrace{\frac{\partial}{\partial t}(\rho\,\overline{u_i' u_j'})}_{\text{Local Time Derivative}} + \underbrace{\frac{\partial}{\partial x_k}(\rho u_k \overline{u_i' u_j'})}_{C_{ij} \equiv \text{Convection}} = \underbrace{-\rho\left(\overline{u_i' u_k'}\frac{\partial u_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial u_i}{\partial x_k}\right)}_{P_{ij} \equiv \text{Stress Production}} - \underbrace{2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}}_{\epsilon_{ij} \equiv \text{Dissipation}} + \underbrace{\overline{p\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}}_{\phi_{ij} \equiv \text{Pressure Strain}}$$

$$+ \underbrace{\frac{\partial}{\partial x_k}\left[\mu\frac{\partial}{\partial x_k}(\overline{u_i' u_j'})\right]}_{D_{L,ij} \equiv \text{Molecular Diffusion}} - \underbrace{\frac{\partial}{\partial x_k}\left[\rho\,\overline{u_i' u_j' u_k'} + \overline{p\left(\delta_{kj}u_i' + \delta_{ik}u_j'\right)}\right]}_{D_{T,ij} \equiv \text{Turbulent Diffusion}}$$

exact terms

modeled terms

Reynolds stress model (RSM-τω):

$$\frac{\partial \tau_{ij}}{\partial t} + \underbrace{\bar{u}_k\frac{\partial \tau_{ij}}{\partial x_k}}_{Convection} = \underbrace{-\mathbb{P}_{ij}}_{Production} + \underbrace{\frac{2}{3}\beta_o^* f_{\beta^*} k\omega\delta_{ij}}_{Dissipation} - \underbrace{\Pi_{ij}}_{Pressure-Strain} + \underbrace{\frac{\partial}{\partial x_k}\left((\nu + \sigma^*\nu_t)\frac{\partial \tau_{ij}}{\partial x_k}\right)}_{Transport\ by\ Diffusion}$$

where

$$\Pi_{ij} = \underbrace{\beta_o^* f_{\beta^*} C_1\omega\left(\tau_{ij} + \frac{2}{3}k\delta_{ij}\right)}_{Slow\ Term} \underbrace{- \hat{a}\left(\mathbb{P}_{ij} - \frac{1}{3}\mathbb{P}_{kk}\delta_{ij}\right) - \hat{\beta}\left(D_{ij} - \frac{1}{3}\mathbb{P}_{kk}\delta_{ij}\right) - \hat{\gamma}k\left(S_{ij} - \frac{1}{3}S_{kk}\delta_{ij}\right)}_{Rapid\ Term}$$

LES: **Filtered Navier-Stokes Equations**

$$\frac{\partial \hat{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\hat{u}_i\hat{u}_j) = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x_i} - \frac{\partial \tilde{\tau}_{ij}}{\partial x_j} + \nu\frac{\partial^2 \hat{u}_i}{\partial x_j \partial x_j}$$

$$\tilde{\tau}_{ij} = -2\nu_{sgs}\hat{S}_{ij} + \frac{1}{3}\tilde{\tau}_{kk}\delta_{ij}$$

WALE SGS Model:

$$\nu_{sgs} = (C_w\Delta)^2\frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(\hat{S}_{ij}\hat{S}_{ij})^{5/2} - (S_{ij}^d S_{ij}^d)^{5/4}}$$

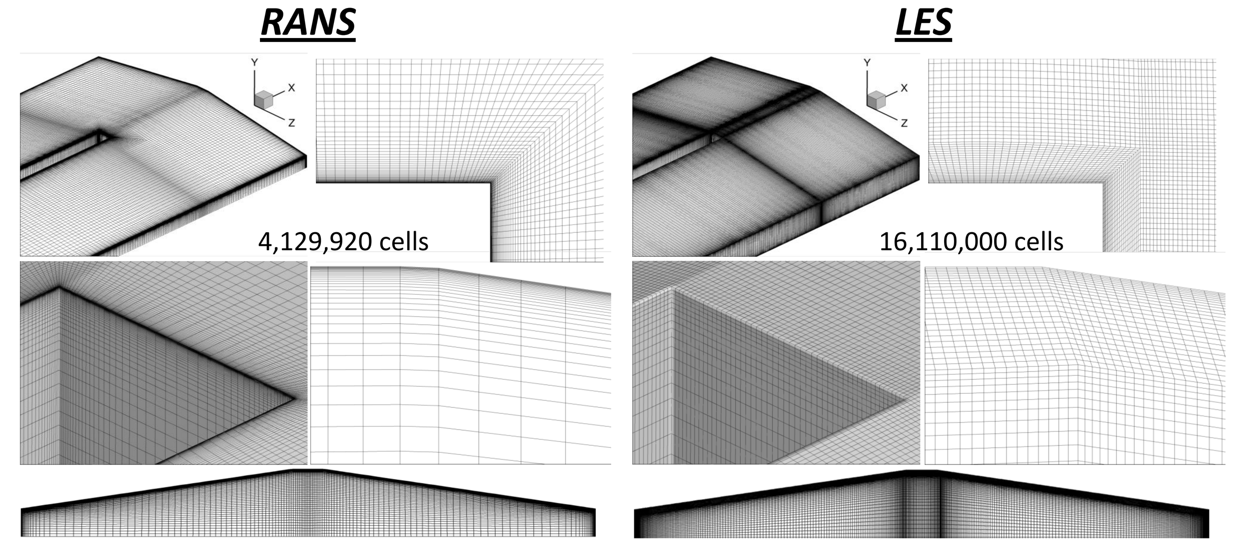
$$S_{ij}^d = \hat{S}_{ik}\hat{S}_{kj} + \hat{\Omega}_{ik}\hat{\Omega}_{kj} - \frac{1}{3}(\hat{S}_{mn}\hat{S}_{mn} - \hat{\Omega}_{mn}\hat{\Omega}_{mn})\delta_{ij}$$

Eddy Diffusivity Hypothesis (EDH):

$$\overline{u_i' T'} = -\Gamma_t\frac{\partial T}{\partial x_i} \qquad \Gamma_t = \frac{\mu_t}{\sigma_t}$$

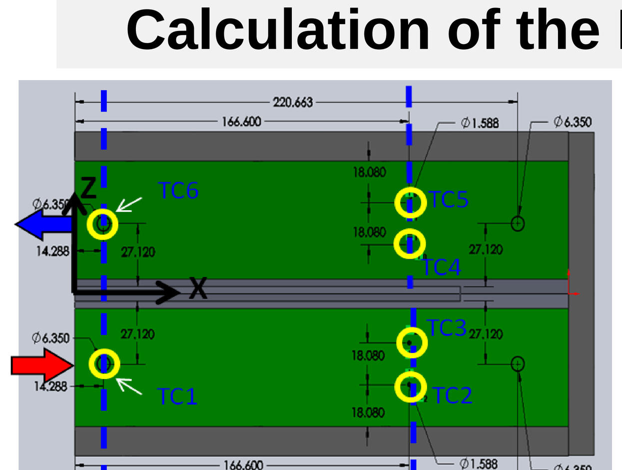
Grid System (y^+<1 for all cells next to all walls)

RANS ***LES***



Grid arrived via a grid-sensitivity study involving 3 grids.

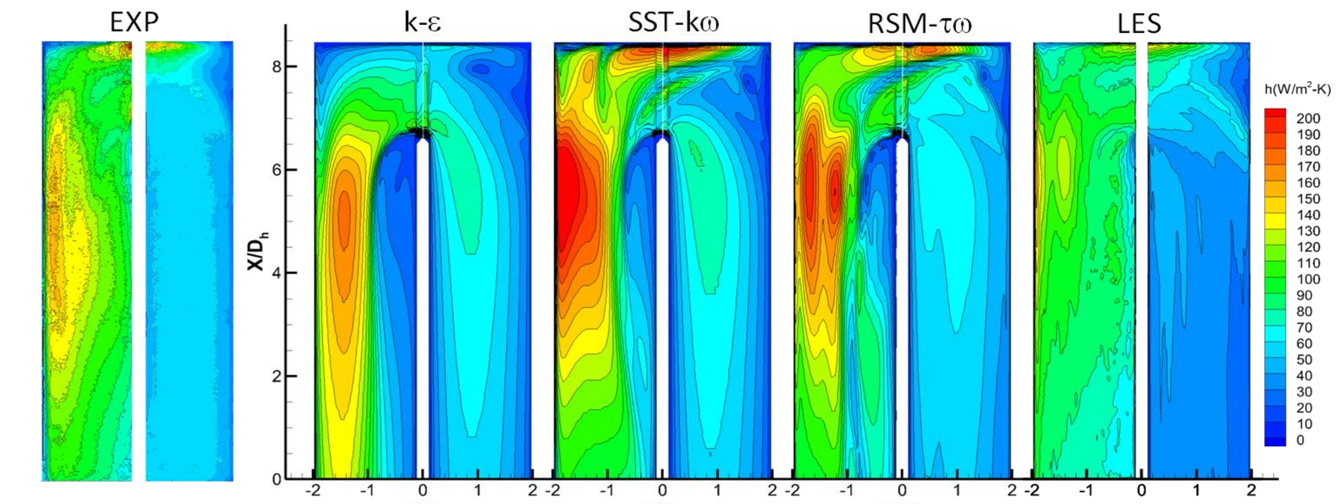
Calculation of the Bulk Temperature (T_b)

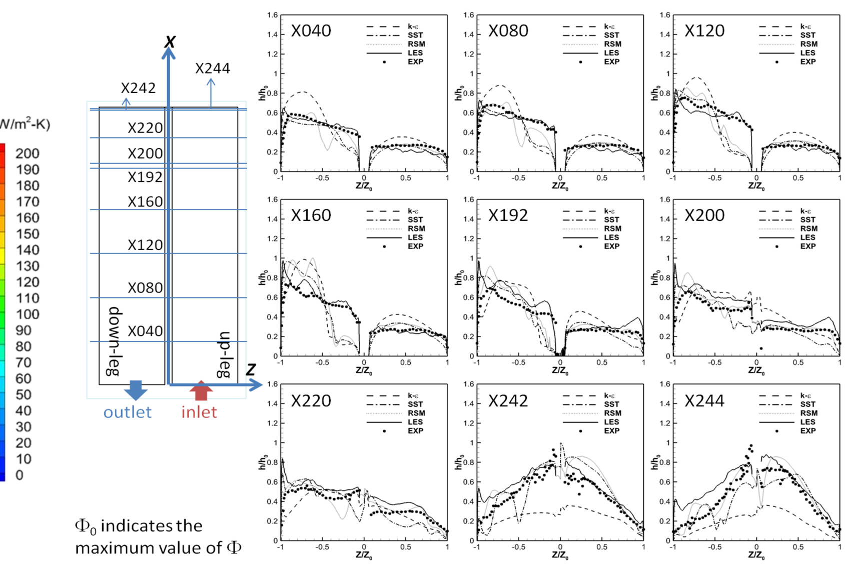


Same procedure as experiment

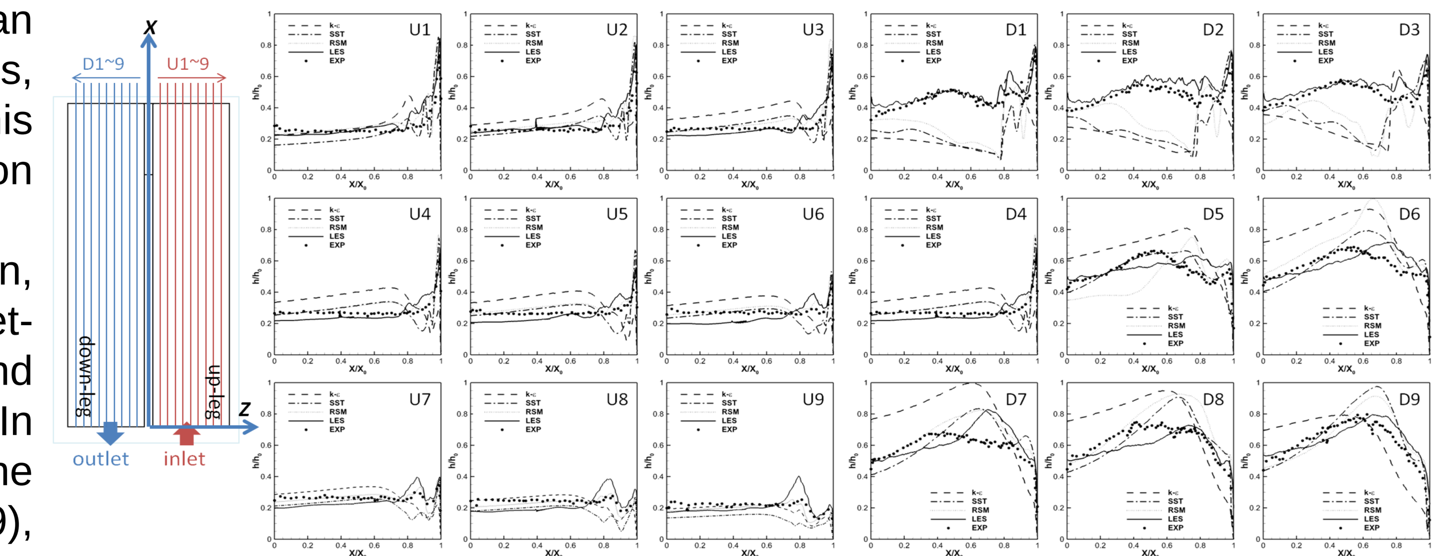
- T_{turn}= (T_{TC2}+T_{TC3}+T_{TC4}+T_{TC5})/4
- $T_{b,upleg}$ is calculated by linear interpolation between T_{TC1} and T_{turn} along X
- $T_{b,downleg}$ is calculated by linear interpolation between T_{turn} and T_{TC6} along X

Computed & Measured Heat Transfer Coef.

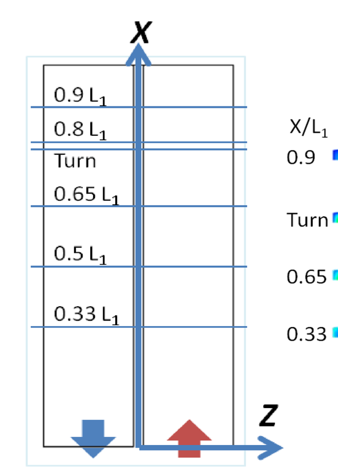




- Along the upleg(U), only LES can match EXP. Among RANS models, RSM is the most close one to EXP. This could be due to the ability of prediction for secondary flow.
- When the flow goes into the turn region, dean type secondary flow, jet-impingement-like flow, separation, and recirculation flow dominate the flow. In the separation region(D1 to D3) and the jet-impingement-like region(D8 to D9), only LES can predict EXP result. But the locations of highest heat transfer of D8 and D9 are shifted. Among RANS models, k-ε is way off away from the EXP result. SST and RSM give similar results but still not comparable to LES and EXP.

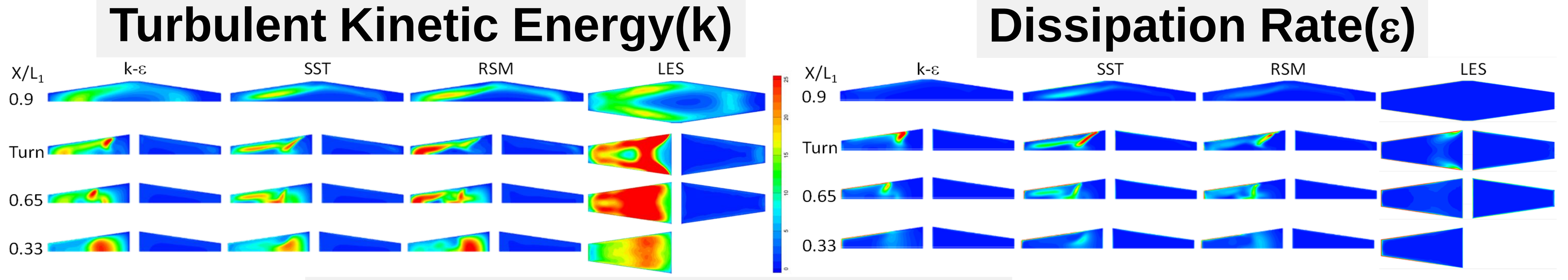


- RANS models are able to predict the turbulent kinetic energy(TKE) and dissipation(ε) in the straight duct and in the turn region except in the separation region. They significantly under-predict TKE and over-predict ε in all the regions with separation. According to the data on all the planes, it can be told that the heat transfer coefficient is under-predicted in consequence of the prediction of TKE and ε.

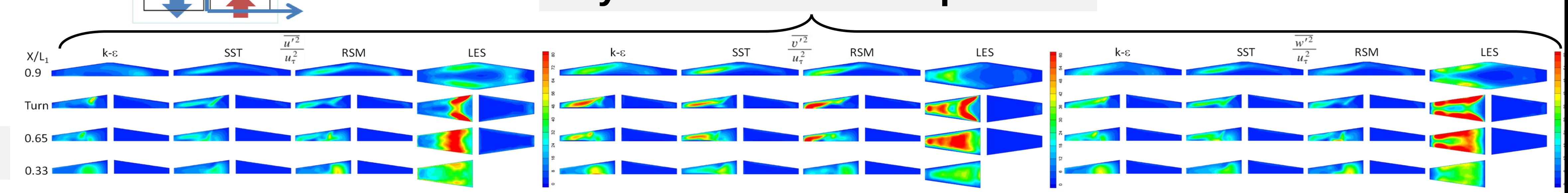


Turbulent Kinetic Energy(k)

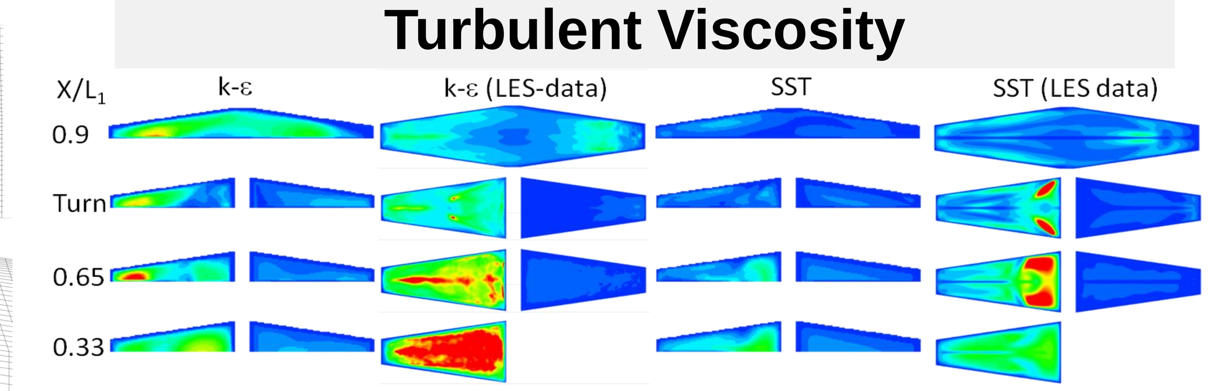
Dissipation Rate(ε)



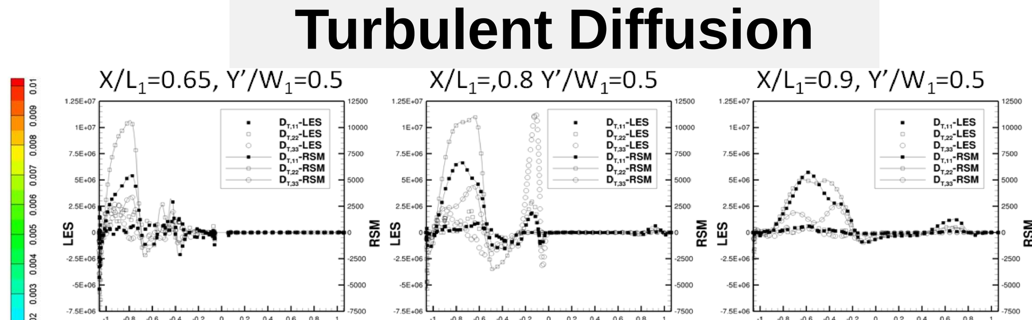
Reynolds Stress Components



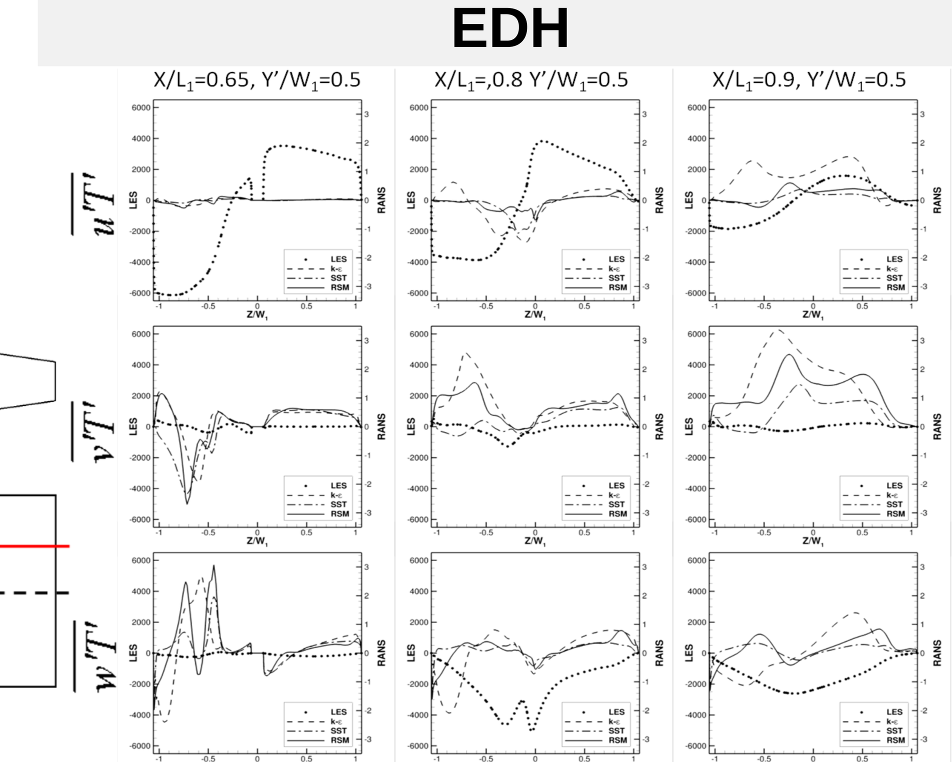
Turbulent Viscosity



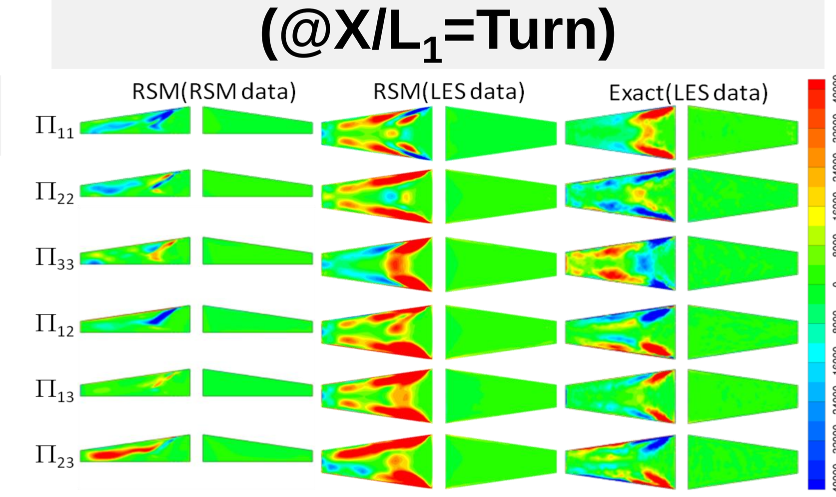
Turbulent Diffusion



EDH



Pressure Strain Rate (@X/L1=Turn)



Mapping trapezoidal duct to square duct with the length of a side= W_1