

## Development of Pore-Scale Models for Diffusion-Reaction Systems with Application to CO<sub>2</sub> Adsorption

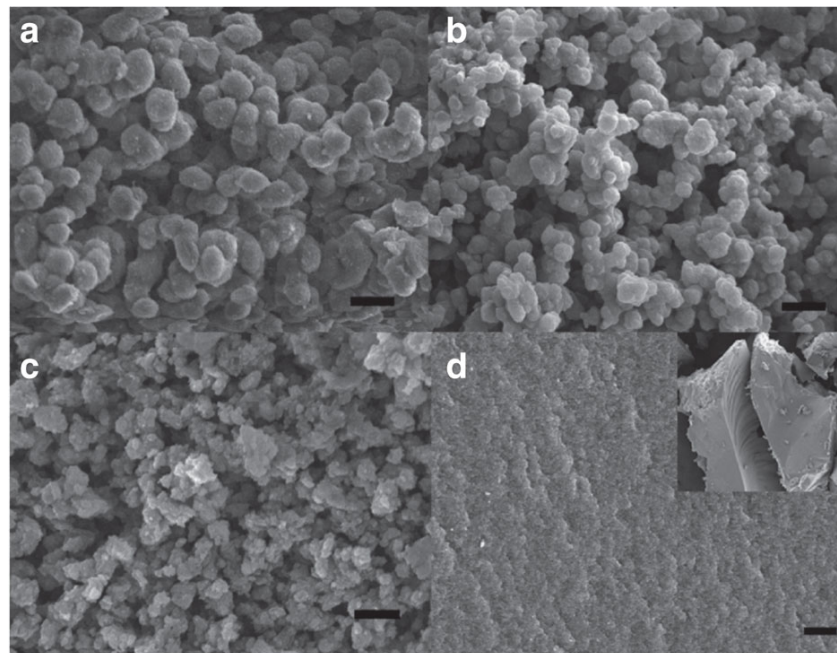
*Support:  
NETL-ORD's Innovative  
Process Technologies  
Field Work Proposal  
through the RES site  
support contract at NETL*

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# Motivation

- Porous particles
  - Widely used in chemical processes
  - High surface area and selectivity
  - Categorized based on pore size
    - Microporous (< 2 nm)
    - Mesoporous (2 nm – 50 nm)
    - Macroporous (> 50 nm)



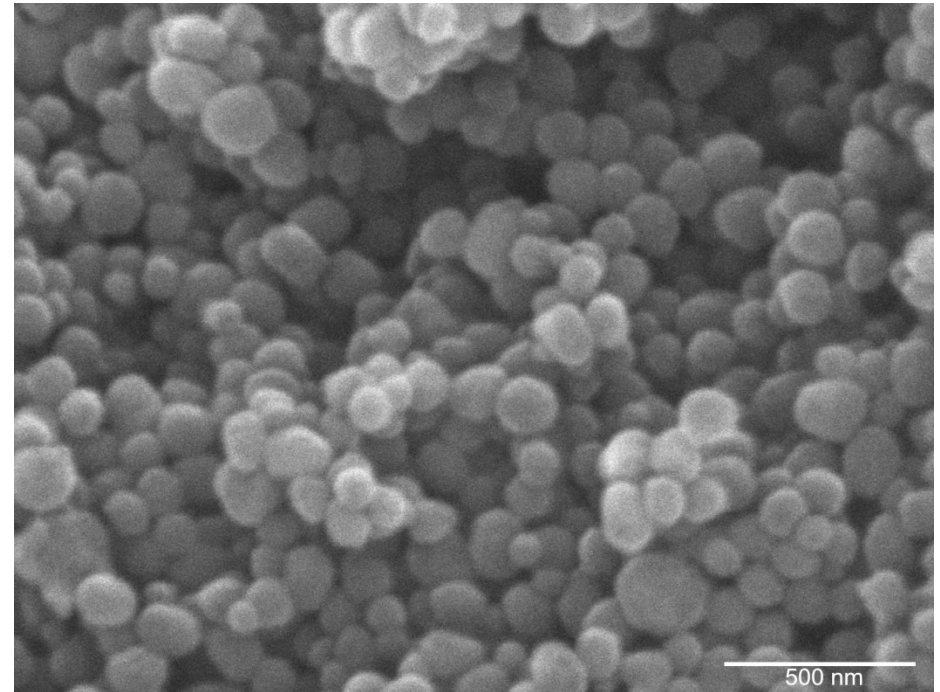
*SEM images of macroporous structures of weakly intergrown spherical particles (source: [1])*

[1] Mohanty et al (2011)

# Motivation

- Mesoporous silica particles, impregnated with polyethylimine (PEI) used for carbon capture process
- One of the technologies being pursued for carbon capture and storage (CCS) process

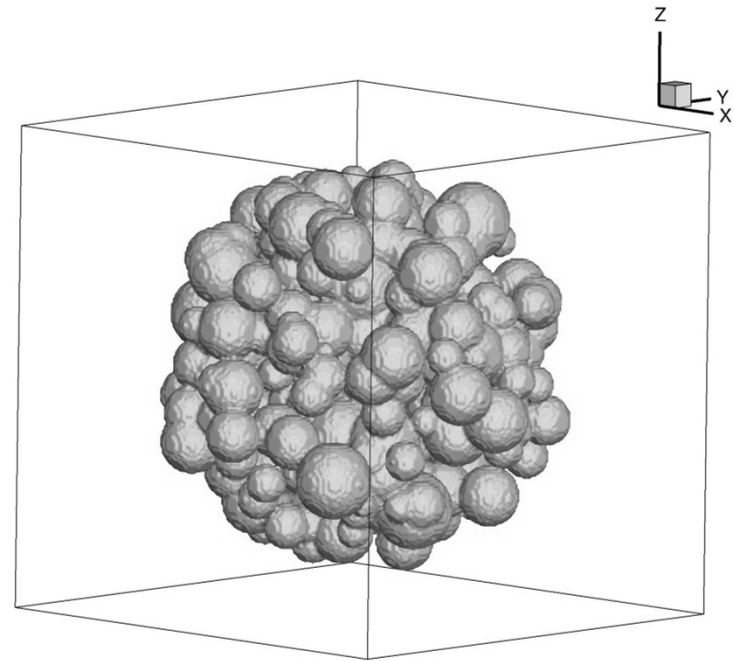
Develop numerical tools to quantify the heat and mass transfer phenomena occurring in porous particles



*SEM image of mesoporous silica (source: Wikipedia)*

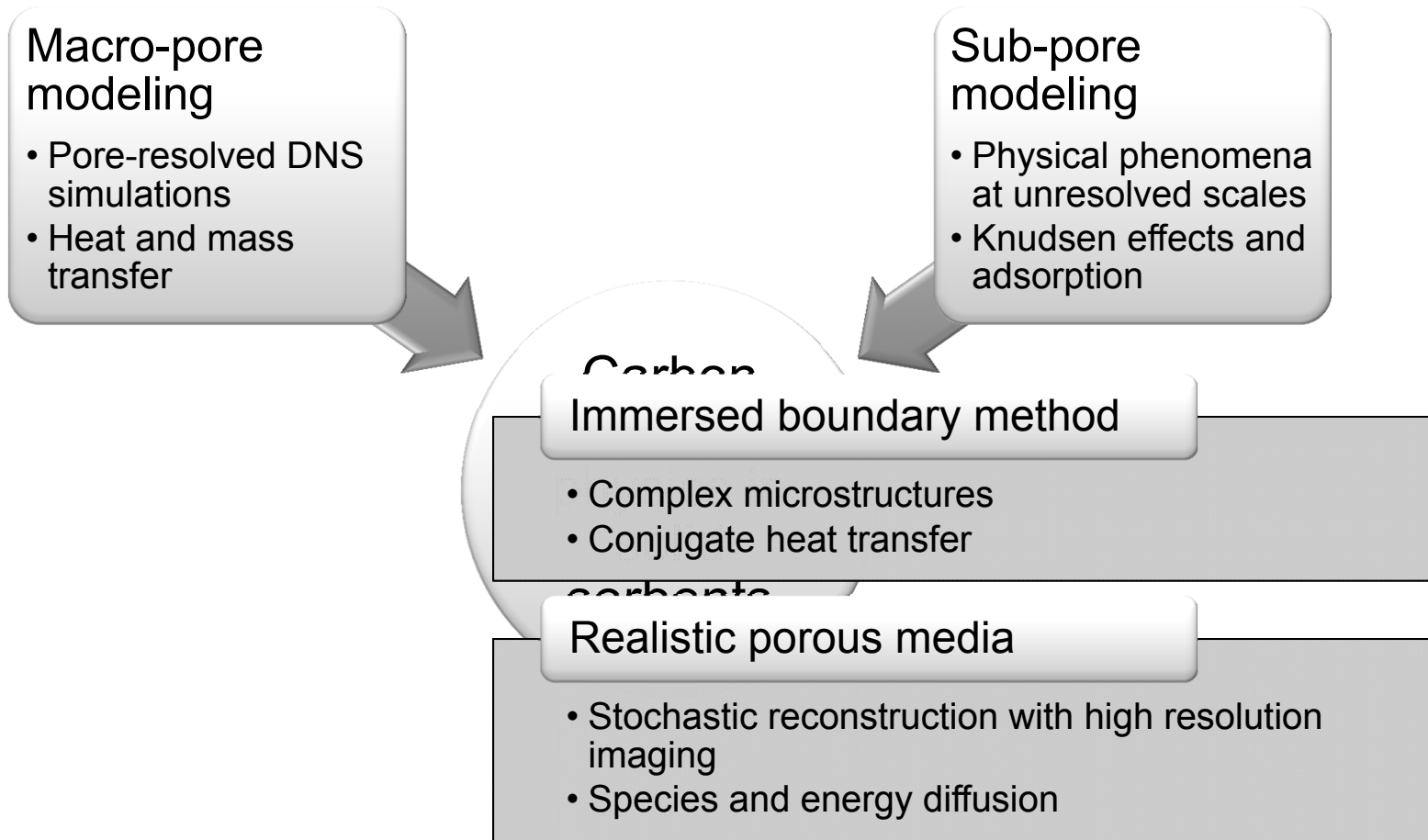
# Challenges

- Complex network of porous microstructures
- A multi-physics problem
  - CO<sub>2</sub> diffusion
  - Heat Transfer
  - Adsorption-desorption kinetics
- Multi-scale nature
  - Particles are sized  $\sim 100\ \mu\text{m}$  while smallest pore channels are  $\sim 30\ \text{nm}$ !



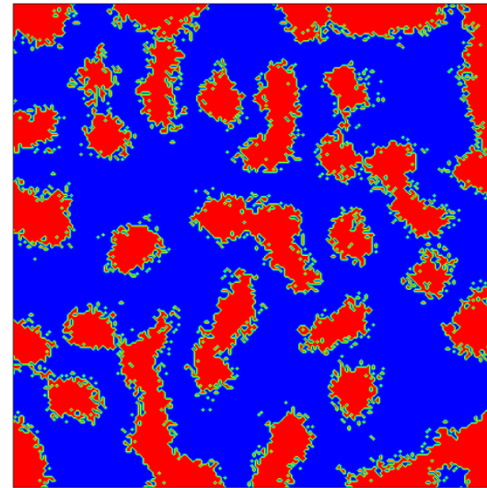
*A porous spherical particle created using stochastic reconstruction with a porosity of 0.40*

# Approach



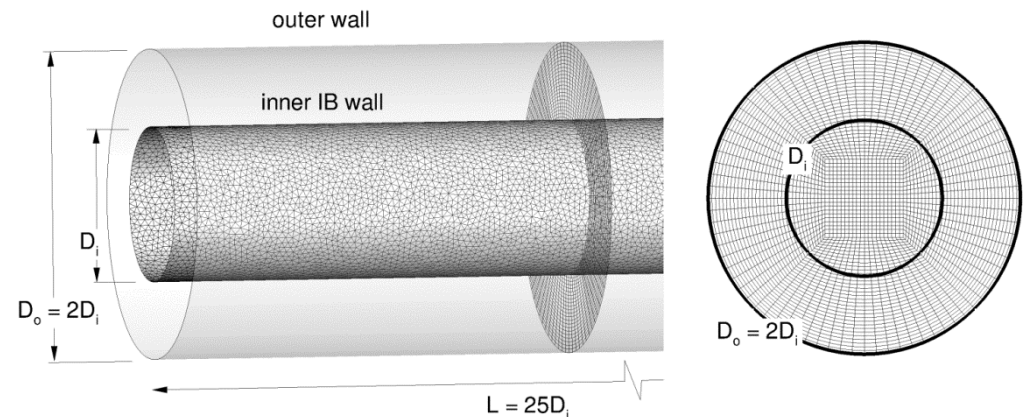
# Immersed Boundary Method (IBM)

- Implemented in our in-house code GenIDLEST
- A finite-volume code with **non-staggered** grid formulation
- Use of **curvilinear coordinates**
- Capabilities
  - Conjugate heat transfer
  - Species diffusion
  - Wall modeling

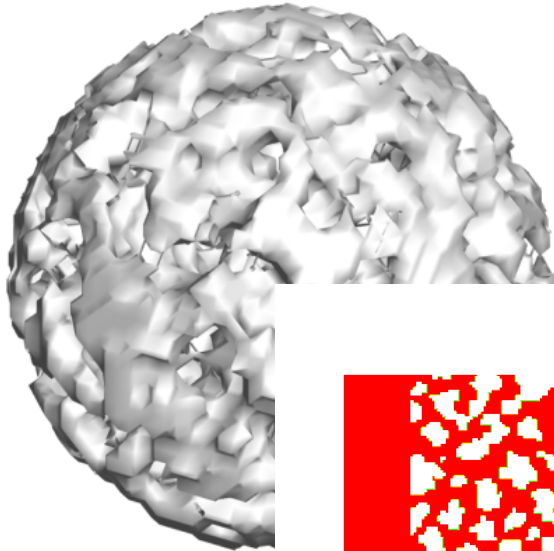


*A 2D porous medium created using stochastic reconstruction with a porosity of 0.70*

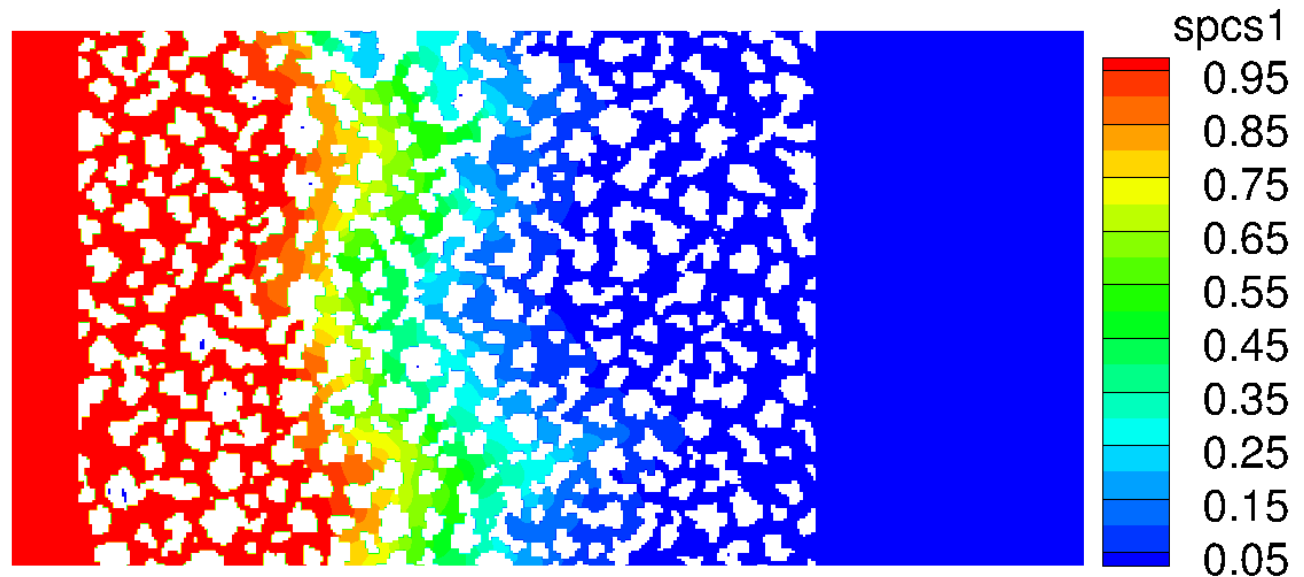
*A thick circular tube simulation with conjugate heat transfer*



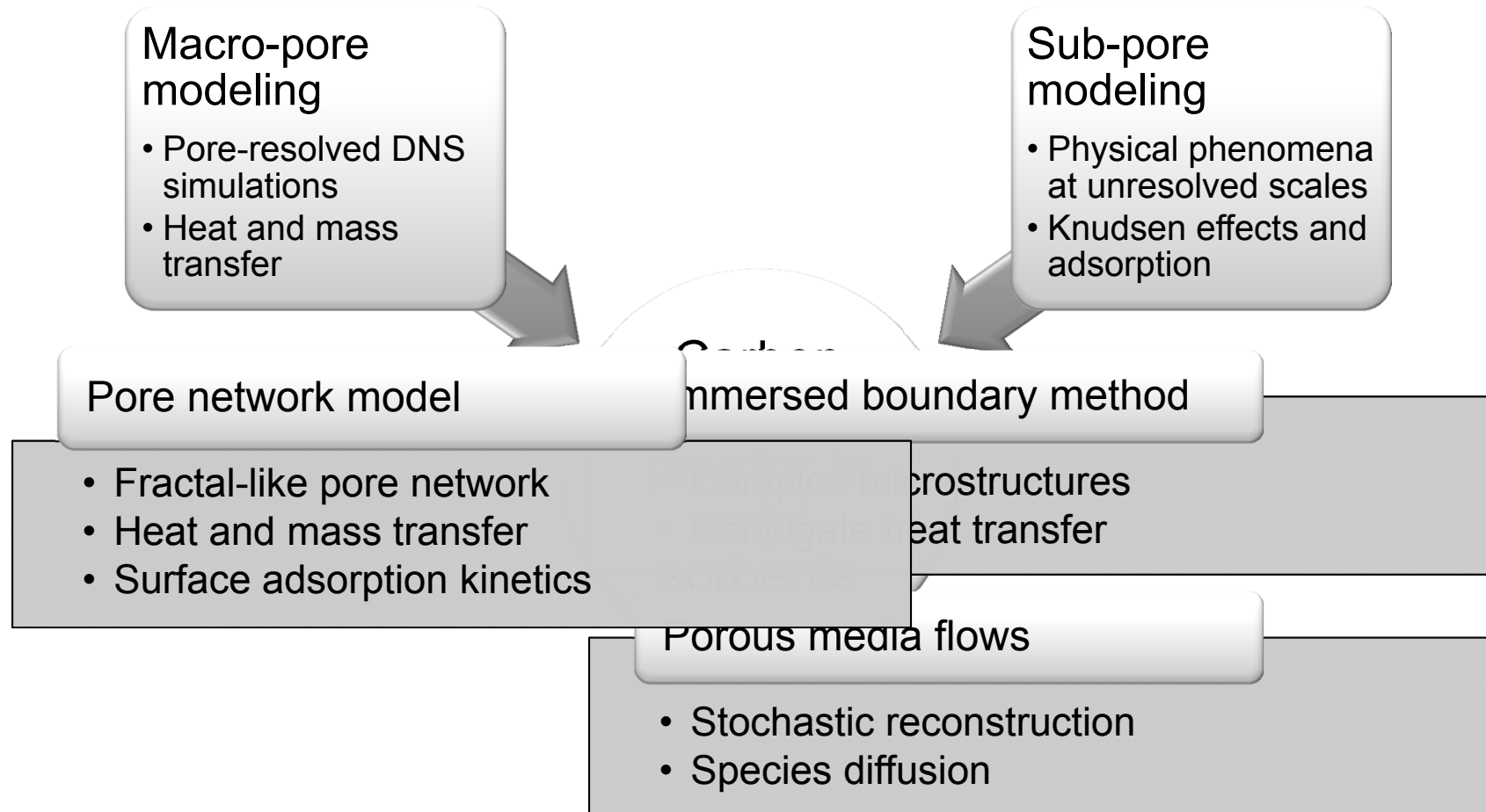
# Immersed Boundary Method (IBM)



*Flows through 2D and 3D reconstructed porous geometries*



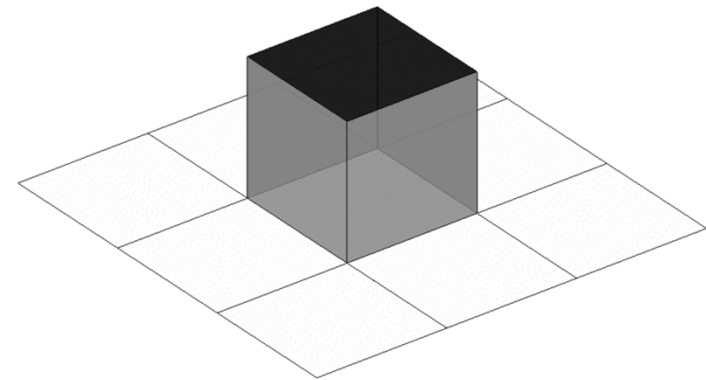
# Approach





# Pore Network Model

- A multi-level **hierarchical system** composed of cylindrical pores
- In the pore network model solve equations for
  - Species and heat diffusion
  - Surface adsorption kinetics
- **Coupling** with macro-pore system for mass and energy conservation

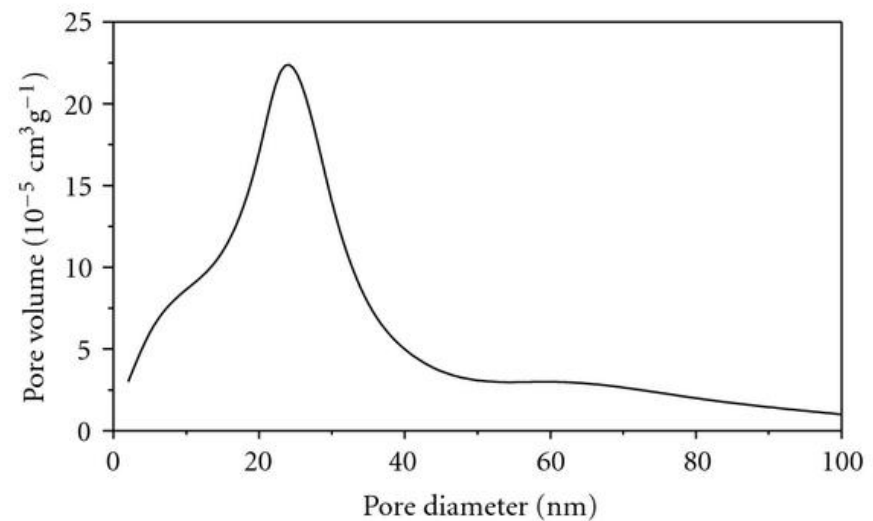
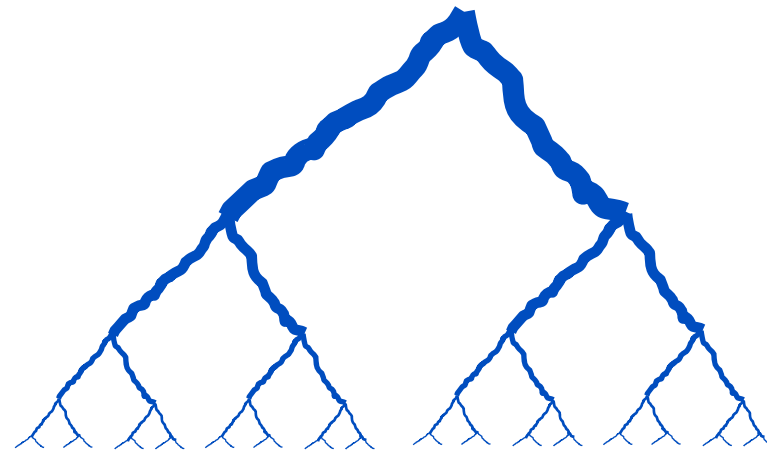


*Example Van Koch surface with cubic structures\*\**

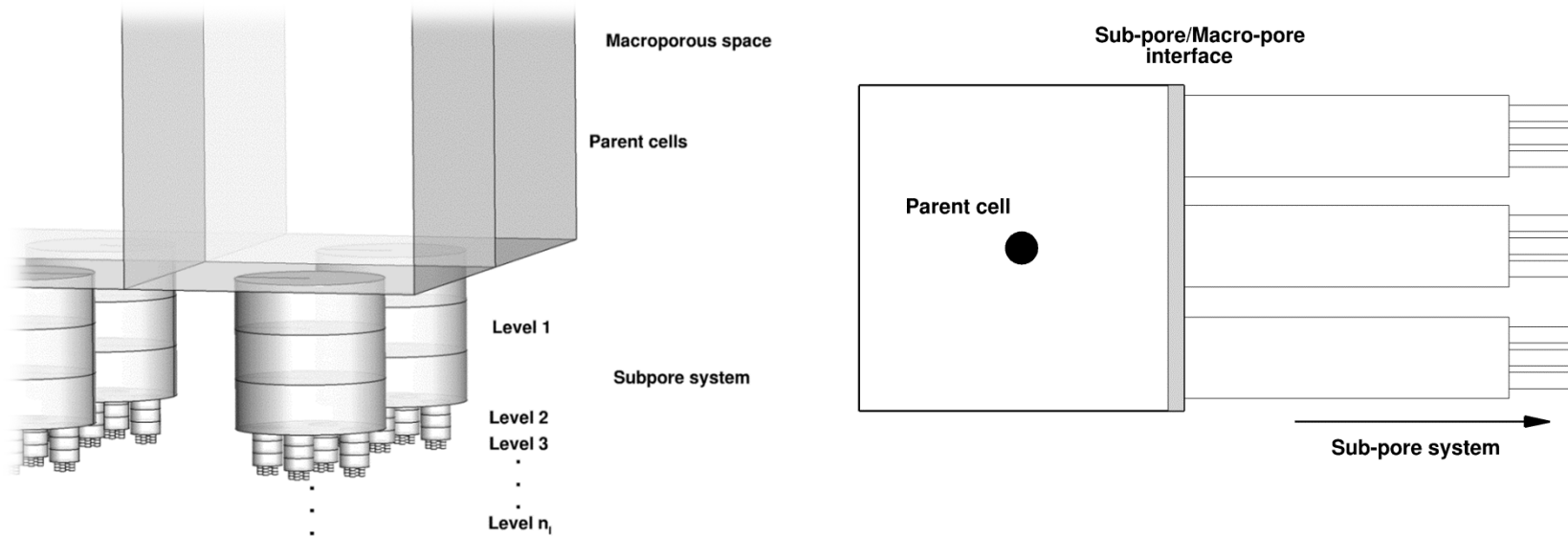
*\*\* Source: Wikipedia*

# Pore Network Model

- Geometric parameters
  - Hierarchical levels with larger channels branching into smaller ones
  - Desired pore channel size range and specific area specified based on experimental measurements
  - Number of levels and branches derived to satisfy given pore size distribution (PSD) and porosity



# Pore Network Model



- Diffusion (heat and mass) into sub-pore channels
- Adsorption on the sub-pore channel surface
- Sub-pore/macro-pore interface – coupling with macro-pore system
  - Parent cell included in solution for coupling with macro-system (ensures implicit conservation)
  - Source terms to include the adsorption/desorption effects

# Mass and Energy Diffusion Modeling

- 1D unsteady governing equation
- Composition dependent mixture property calculation
- Mass transport
  - Bosanquet formula for effective diffusion coefficient
  - Temperature dependence based on kinetic theory considerations

$$\frac{\partial(\phi)}{\partial t} = \frac{\partial}{\partial x} \left( \alpha_{eff} \frac{\partial \phi}{\partial x} \right) + \omega$$

- Bosanquet formula

$$\frac{1}{D_{eff}} = \frac{1}{D_{bulk}} + \frac{1}{D_{Kn}} + \dots$$

- Chapman and Cowling correlation:

$$D_{bulk} = 0.001858 T^{\frac{3}{2}} \sqrt{\frac{M_1 + M_2}{M_1 M_2}}$$

- Knudsen diffusion:

$$D_{Kn} = \frac{1}{3} d_p \sqrt{\frac{8R_u T}{\pi M}}$$

# Mass and Energy Diffusion Modeling

- 1D unsteady governing equation
- Composition dependent mixture property calculation
- Mass transport
  - Bosanquet formula for effective diffusion coefficient
  - Temperature dependence based on kinetic theory considerations
- Energy transport
  - Sutherlands law for temperature dependence of thermal conductivity
  - Knudsen number dependence of the bulk conductivity

$$\frac{\partial(\phi)}{\partial t} = \frac{\partial}{\partial x} \left( \alpha_{eff} \frac{\partial \phi}{\partial x} \right) + \omega$$

- Sutherland's law:

$$\frac{\kappa_{bulk}^c}{\kappa_0^c} = \left( \frac{T}{T_0} \right)^{\frac{3}{2}} \cdot \frac{T_0 + S}{T + S}$$

- Reduced thermal conductivity

$$\kappa_{gas} = \frac{\kappa_{bulk}}{1 + Kn \cdot \frac{2 - \alpha_T}{\alpha_T} \cdot \frac{9\gamma - 5}{\gamma + 1}}$$

# Adsorption/Desorption Modeling

- Rate equation derived from first principles

- Molecular wall impact rate

$$F = \frac{p}{\sqrt{2\pi m R_u T}}$$

- Sticking model

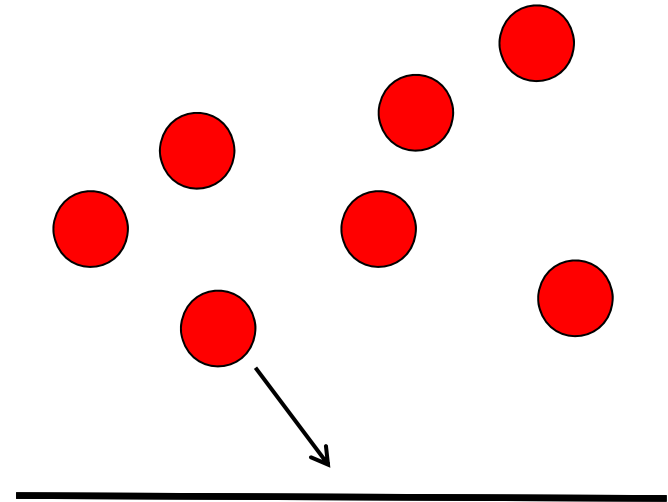
$$S = f(\theta) \cdot \exp\left(-\frac{E_{ads}}{R_u T}\right)$$

- Rate of adsorption

$$R_{ads} = S \cdot F$$

- Rate of desorption

$$R_{des} = g(\theta) \cdot \exp\left(-\frac{E_{des}}{R_u T_s}\right)$$



*Surface with adsorption sites*

- Final form for rate equation

$$\frac{d\theta}{dt} = R_{ads} - R_{des}$$

# Adsorption/Desorption Modeling

- Linear sticking model for CO<sub>2</sub> molecules is used
- Second order rate equation
- Resulting equation is **non-linear** in  $\theta$  (form is similar to **\*\* Lee et al.**)
- Parameters  $N_s, E_{ads}, \Delta S, E_{des}$  will be from **experimental calibration**
- Calibration using equilibrium surface coverage values ( $\theta_{eq}$  obtained by using  $\frac{d\theta}{dt} = 0$ )

*\*\* CO<sub>2</sub> adsorption measurements conducted at NETL by Lee et al. (2011 PCC)*

$$\frac{d\theta}{dt} = K_{ads}(1 - 2\theta)^2 - K_{des}\theta^2$$

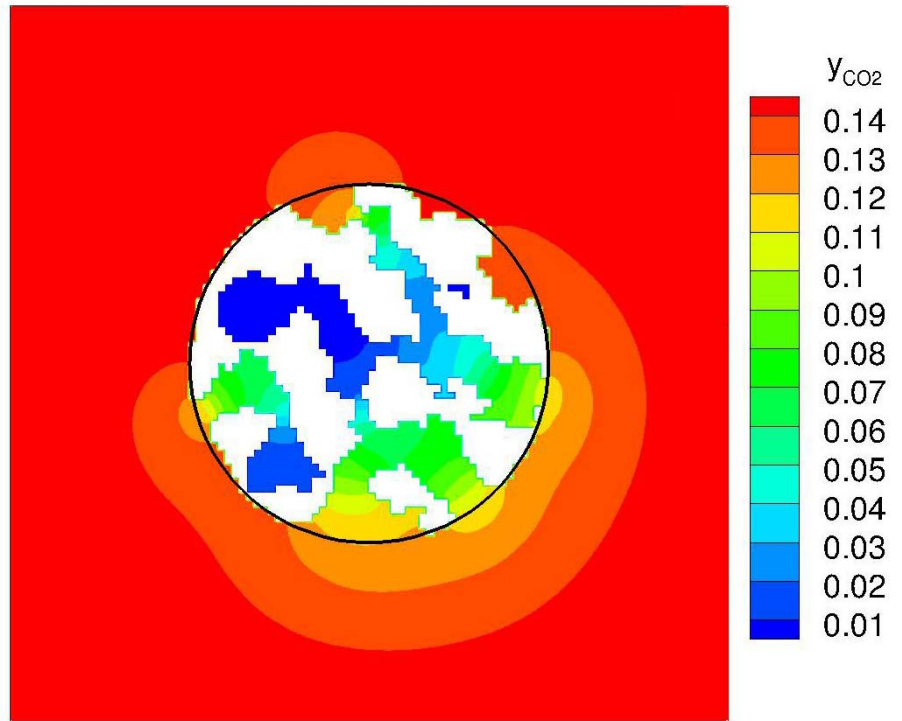
$$K_{ads} = \frac{p}{\sqrt{2\pi mk_B T}} \cdot \frac{1}{N_s} \cdot \exp\left(-\frac{E_{ads}}{R_u T}\right)$$

$$K_{des} = -\frac{e\kappa k_B T}{h} \cdot \exp\left(\frac{\Delta S}{R_u}\right) \cdot \exp\left(-\frac{E_{des}}{R_u T_s}\right)$$

$\theta$	surface coverage
$p$	partial pressure of CO <sub>2</sub>
$N_s$	no. of ads. sites per unit area
$E_{ads}$	Activation energy for adsorption (J/mol)
$E_{des}$	Activation energy for desorption (J/mol)
$\Delta S$	Entropy of adsorption reaction (J/mol/K)
$T_s$	Surface temperature
$\kappa$	pre-exponential correction term
$k_B$	Boltzmann constant ( $1.38 \times 10^{-23}$ J/K)
$h$	Planck's constant ( $6.63 \times 10^{-34}$ J s)

# Problem Description

- 2D porous particle of porosity 60%
- Particle pore space initially with 0% CO<sub>2</sub>
- Ambient conditions
  - 15% CO<sub>2</sub> (and 85% N<sub>2</sub>)
  - Temperature – 300K
- Domain boundaries at ambient condition – acting as infinite sources

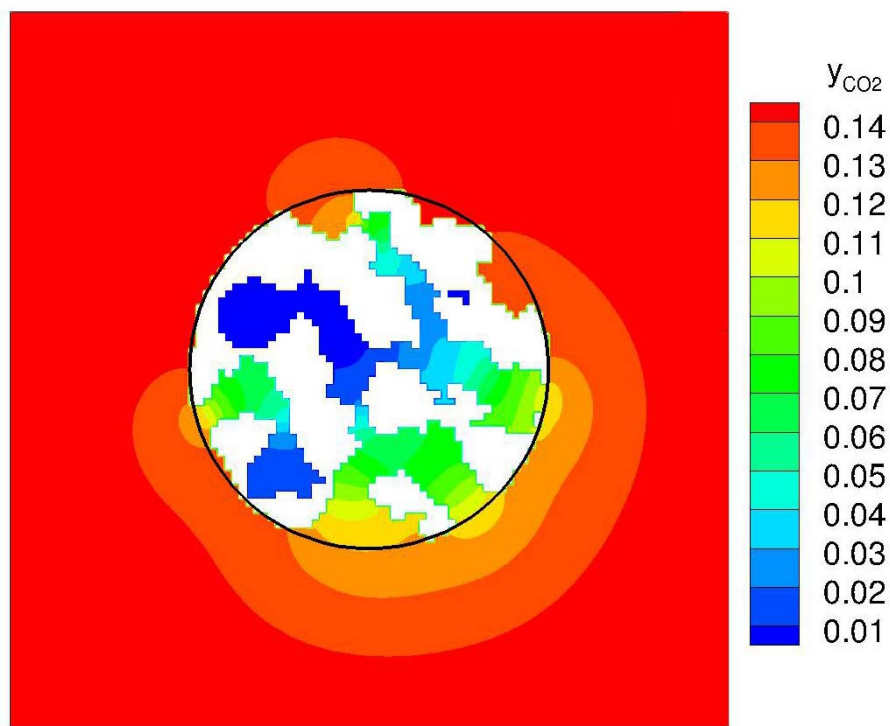


*Sectional view of diffusion of CO<sub>2</sub> into a porous particle*



# Problem Description

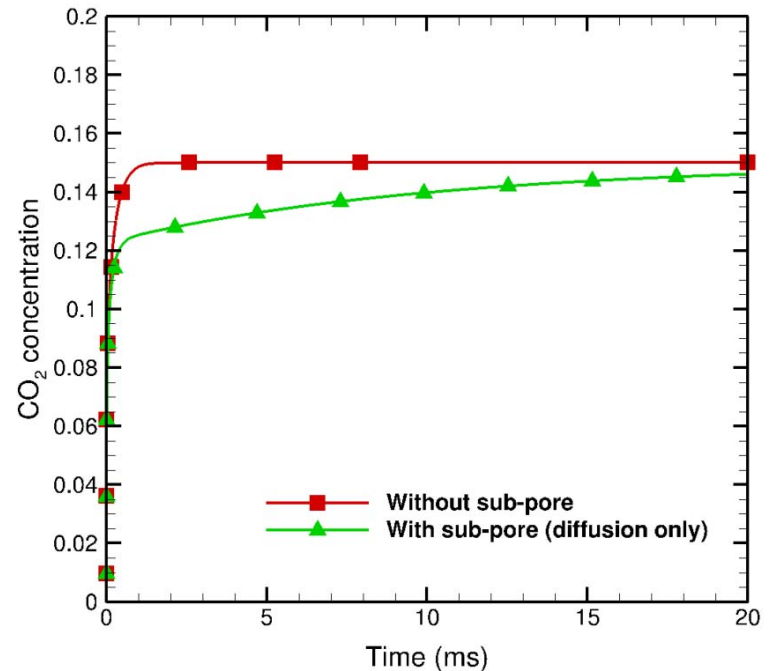
- Particle properties
  - Particle diameter - **100 microns**
  - Macro-pore porosity – 50%
  - Sub-pore – 20%
  - Sub-pore channel size range – 1 micron to 30 nm
  - Specific area – 50 m<sup>2</sup>/g (increase of 100 times than macro-particle)
  - Four levels with uniform scaling ratio



*Sectional view of diffusion of CO<sub>2</sub> into a porous particle*

# Effect of Sub-pore System

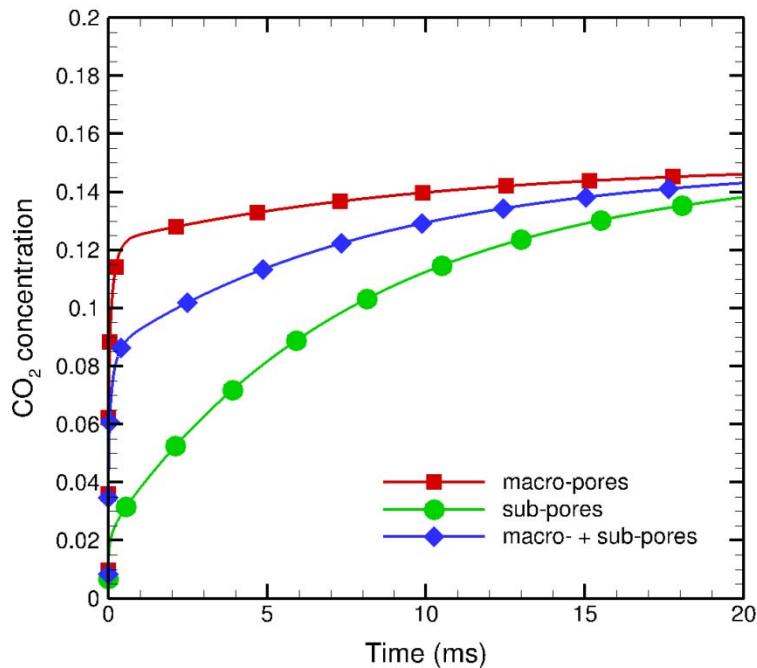
- Four types of simulations
  - No sub-pore system
  - Sub-pore system with mass diffusion only
  - Sub-pore system with adsorption only
  - Sub-pore system with mass diffusion and adsorption
- Macro-pore diffusion expected to be faster
- Sub-pores affect macro-pore saturation!



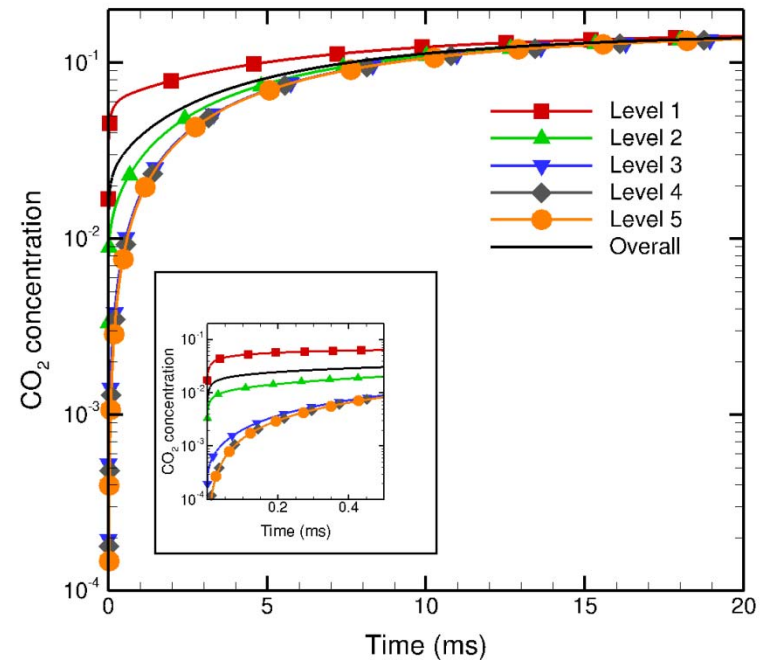
*CO<sub>2</sub> concentration level in entire particle macro-pore space*

# Diffusion-only Case

CO<sub>2</sub> concentration averaged over particle pore-space



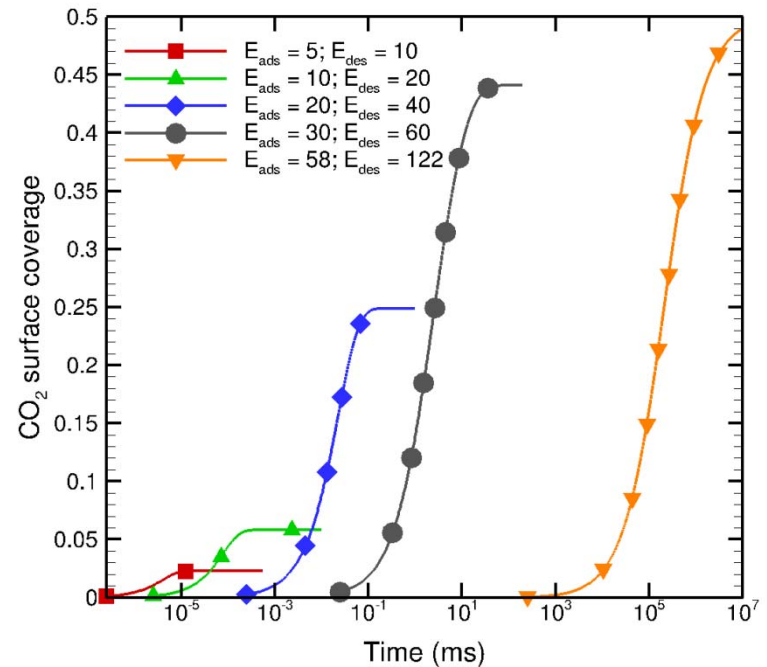
CO<sub>2</sub> concentration averaged on a per-level basis



- Sub-pore space saturation takes much longer!
- Smallest channels take longest to saturate...

# Adsorption-only Case

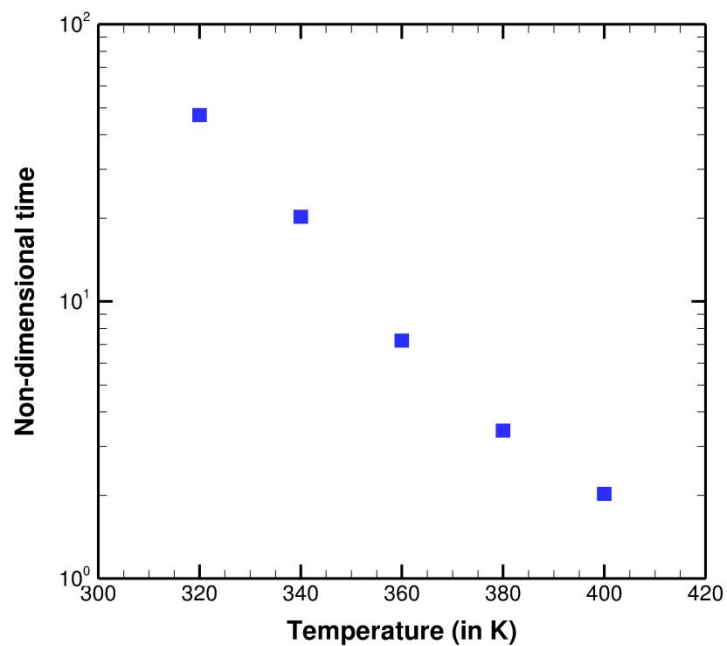
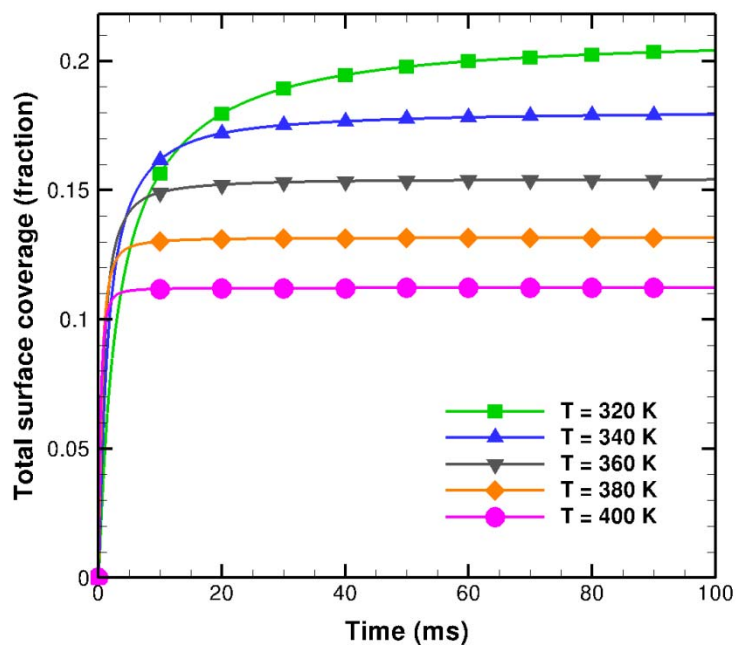
- CO<sub>2</sub> concentration within the sub-pore space is 0.15 (constant)
- No dependence on levels because of same pressure/temperature
- Asymptotic value expected to depend on ambient conditions (pressure/temperature) and kinetic parameters
- Expected saturation time depends on the parameters in the surface kinetics



*CO<sub>2</sub> surface coverage at different adsorption rates*

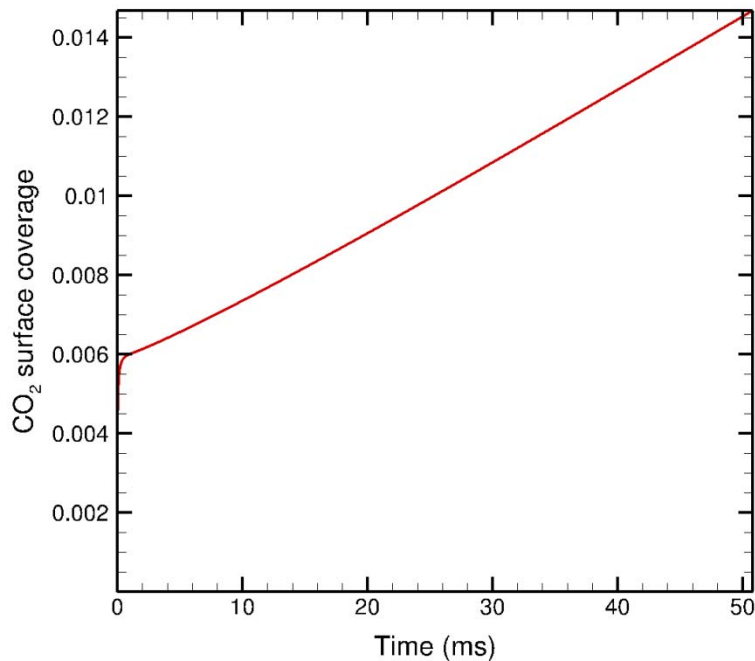
# Effect of Temperature – Adsorption only

- Adsorption on the particle surface
- Effect of temperature on saturation time

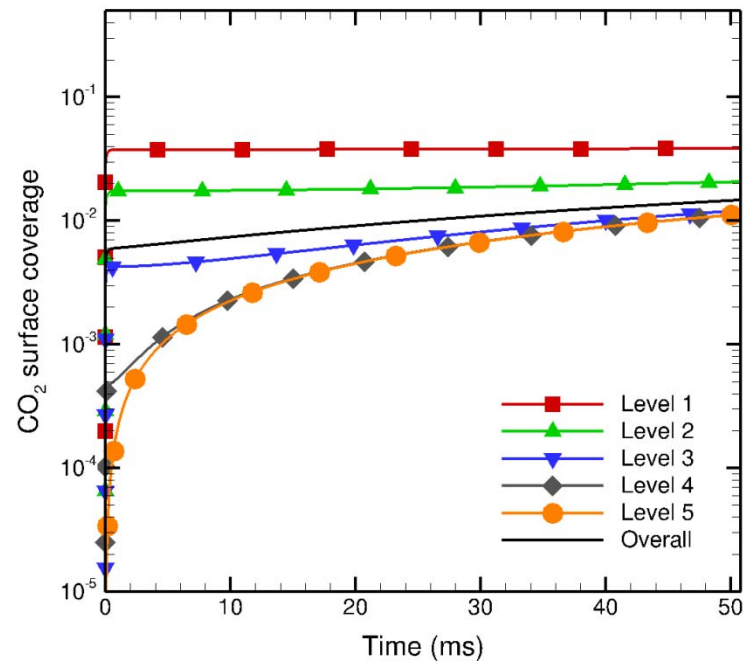


# Diffusion + Adsorption Case

*CO<sub>2</sub> surface coverage averaged over all of the sub-pore surface*



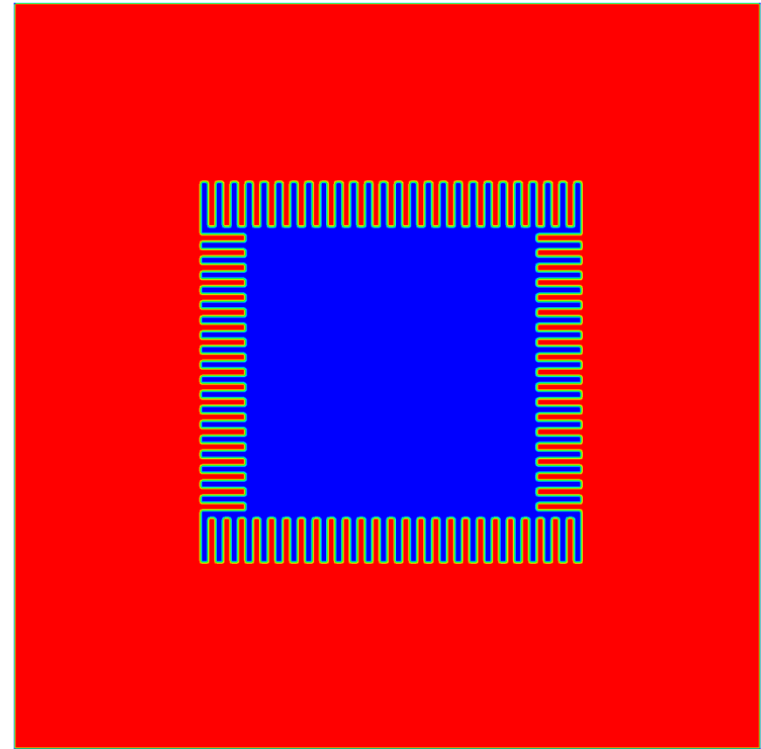
*CO<sub>2</sub> surface coverage averaged on a per-level basis within the sub-pore system*



- For  $E_{\text{ads}} = 10 \text{ kJ/mol}$ ,  $E_{\text{des}} = 20 \text{ kJ/mol}$  case
- Similar trend follows at individual levels
- Final **expected value** of saturation is around **0.05** surface coverage
- **A diffusion-limited case** – adsorption behavior is completely different than in adsorption-only case!

# Macro-/Sub-pore Interchangeability

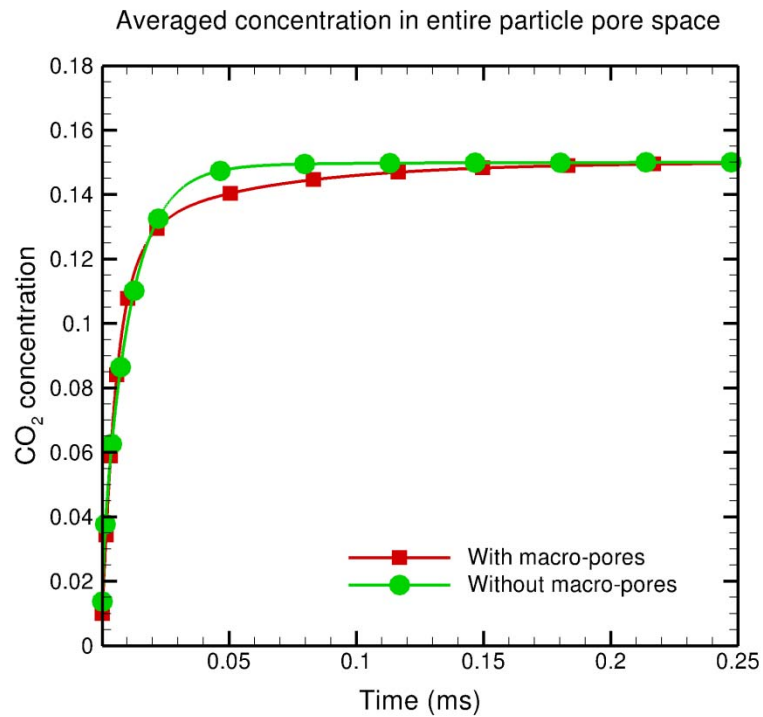
- Square particle with only two channel sizes – 1 and 2  $\mu\text{m}$
- Case 1 (with macro-pore)
  - Larger channel modeled with IBM, smaller one as a sub-pore
- Case 2 (without macro-pore)
  - Both channels modeled as sub-pores (no IBM)
- Volume contributions of the two channel sizes are maintained to be the same between two cases



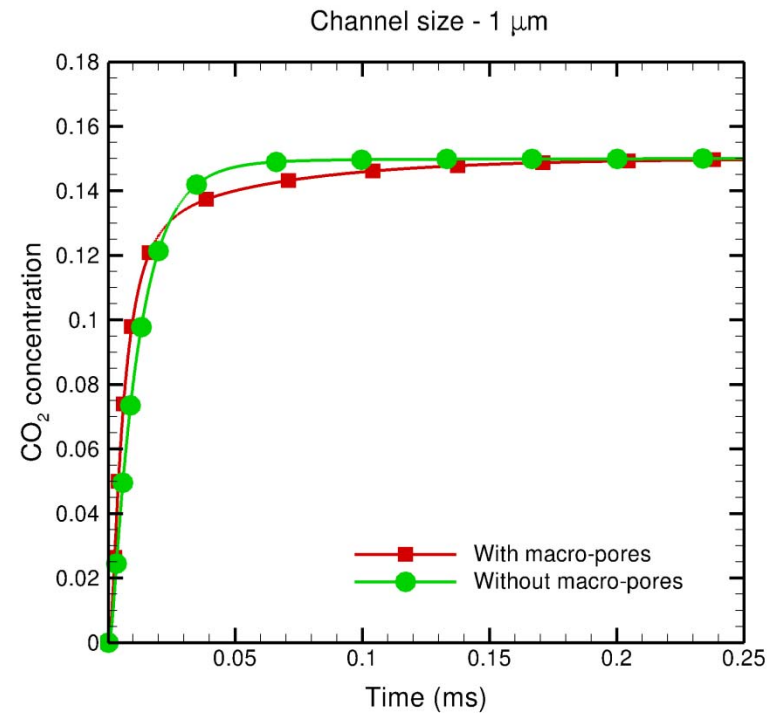
*A square porous particle within a square domain  
– the larger channels are modeled using IBM as  
macro-pores (Case 1)*

# Macro-/Sub-pore Interchangeability

*CO<sub>2</sub> concentration levels averaged over entire pore space for the two cases*



*CO<sub>2</sub> concentration levels averaged over the smaller channel only for the two cases*





# Conclusions

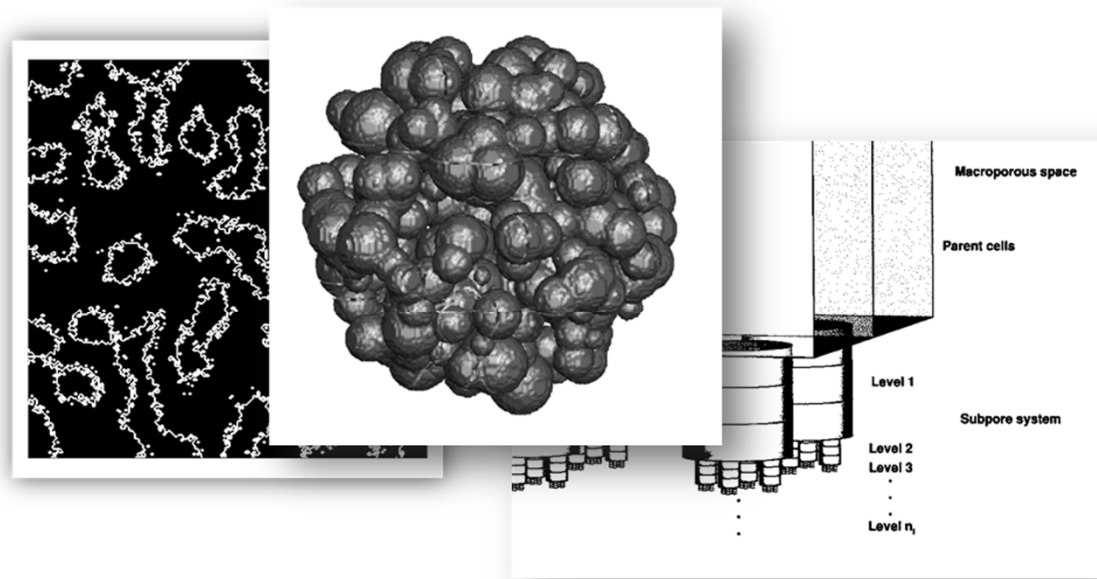
- A single framework to account for **multi-scale, multi-physics problems** in porous media
  - Immersed boundary method (IBM) for macro-pore channels
  - Sub-pore modeling methodology for unresolved channels
- A general model that is applicable to **many reaction-diffusion system** in porous media
- Sub-pore geometry based on experimental (if available) porosity measurements
- Within sub-pore system, solve for simplified governing equations – **physically accurate, yet tractable** from a practical standpoint
- Applicable to **problems without macro-porosity** – i.e., full particle porosity is modeled using pore network model

# Future Work

- Inclusion of conjugate heat transfer in the simulations
- Run single-particle simulations for different conditions of pressure and temperature to obtain CO<sub>2</sub> adsorption isotherms
- Modularize pore network model for use to model full particle in other software frameworks such as MFIX.

# Publications

- **Peer-review journals**
  - “Flows Through Reconstructed Porous Media using Immersed Boundary Methods”, K. Nagendra and D.K. Tafti, Journal of Fluids Engineering, 2014.
  - “A Novel Approach for Conjugate Heat Transfer Problems in Immersed Boundary Framework”, K. Nagendra and D.K. Tafti, Journal of Computational Physics, 2014.
- **Conference proceedings**
  - “Flows Through Reconstructed Porous Media Using Immersed Boundary Methods”, K. Nagendra and D.K. Tafti, FEDSM2012-72128, FEDSM 2012, Puerto Rico.
- **Presentations**
  - “Heat and Mass Transfer in Porous CO<sub>2</sub> Sorbent Particles”, K. Nagendra and D.K. Tafti, NETL Multi-phase Flow Workshop 2011, Pittsburgh PA.
  - “Direct Numerical Simulation of CO<sub>2</sub> Diffusion in Reconstructed Solid Sorbent Particles”, K. Nagendra and D.K. Tafti, NETL Multi-phase Flow Workshop 2012, Morgantown WV.
  - “A Novel Sub-pore Modeling Methodology for CO<sub>2</sub> Capture in Meso-porous Particle Systems”, K. Nagendra and D.K. Tafti, NETL Multi-phase Flow Workshop 2013, Morgantown WV.
- **Under preparation**
  - “A subgrid pore network model for reaction-diffusion phenomena in porous media”, K. Nagendra and D. K. Tafti
  - “Modeling CO<sub>2</sub> diffusion and adsorption in mesoporous silica particles”, K. Nagendra and D. K. Tafti

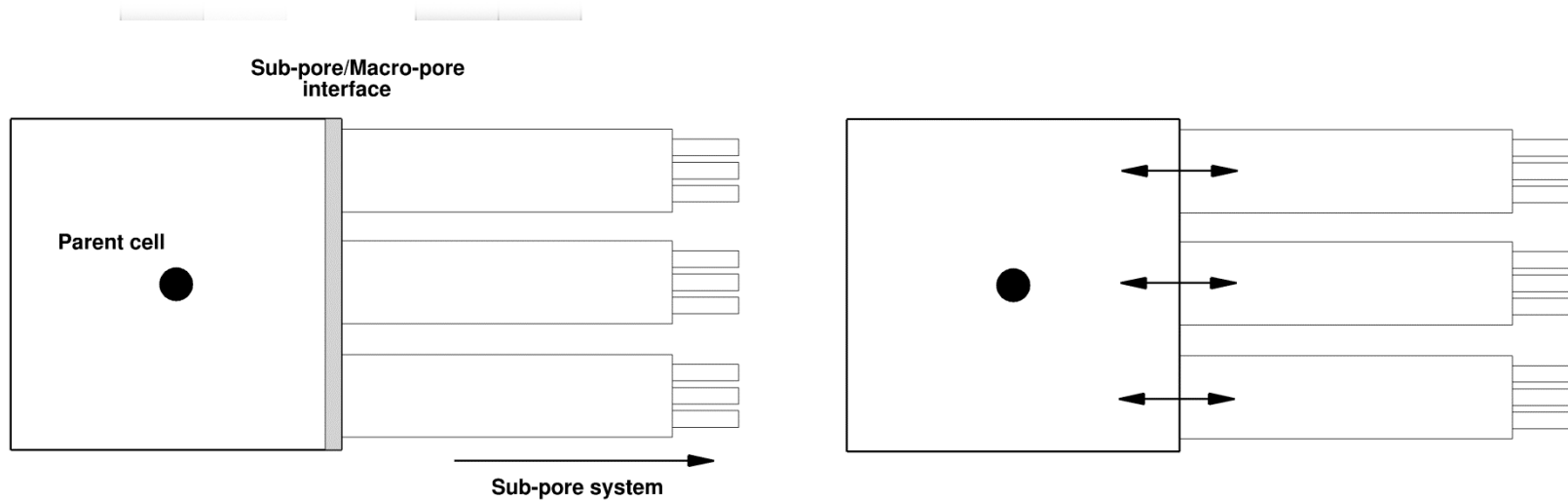


Thank you!

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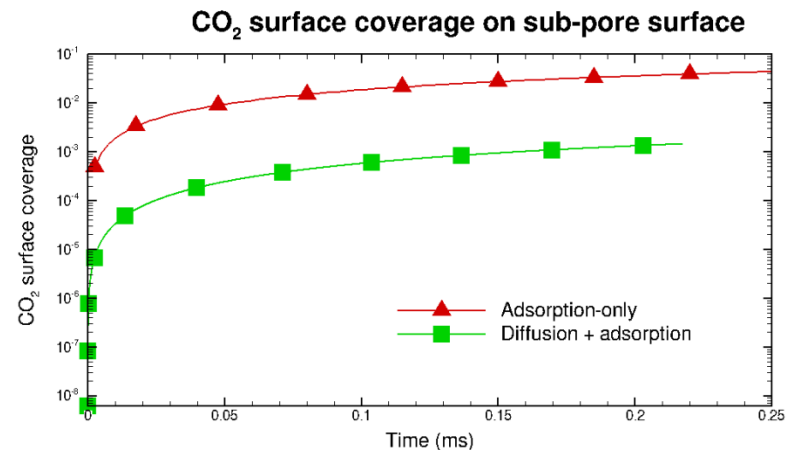
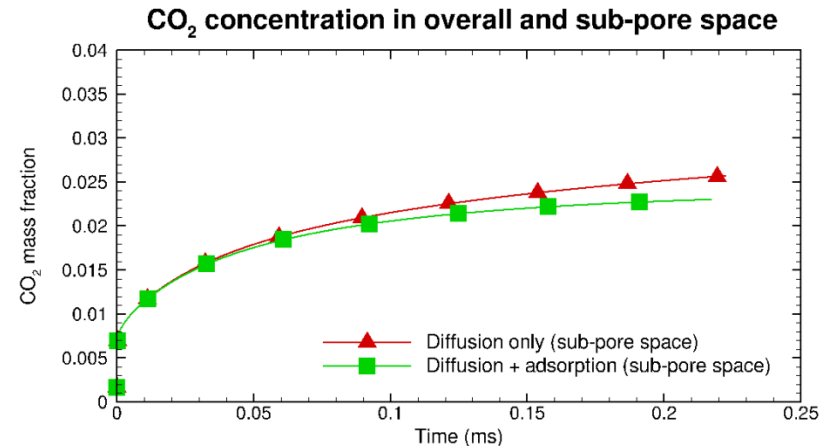
Questions?

# Sub-pore Model



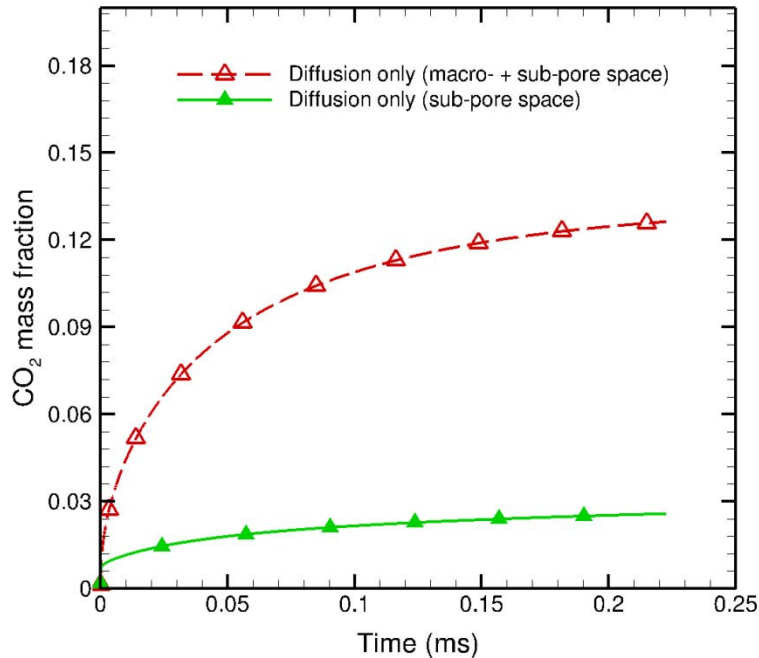
# Multi-scale Challenges

- Widely varying time-scales
  - Diffusion at macro-pore – ~1 ms
  - Diffusion at sub-pore – ~100 ms (expected)
  - Adsorption at surface - ~1-100 s (expected)
- Results in the time-step being extremely small – currently using 1 ns
- Combined behavior is very different than the constituent physical models

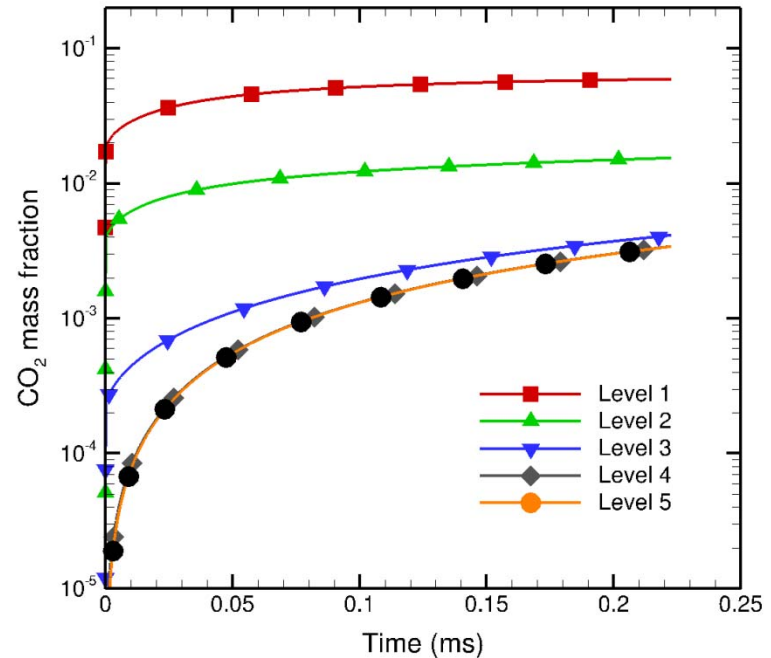


# Diffusion-only Case

CO<sub>2</sub> concentration in overall and sub-pore space



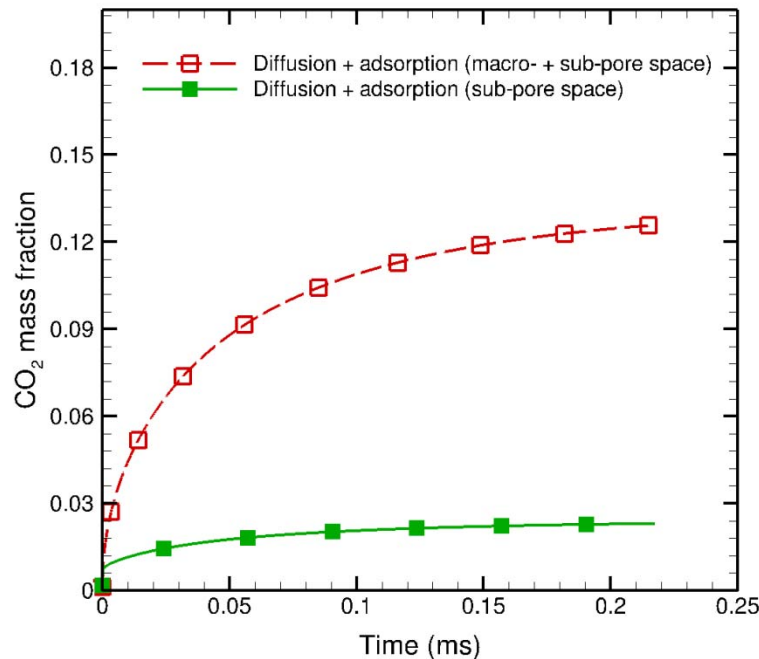
CO<sub>2</sub> concentration at different sub-pore levels



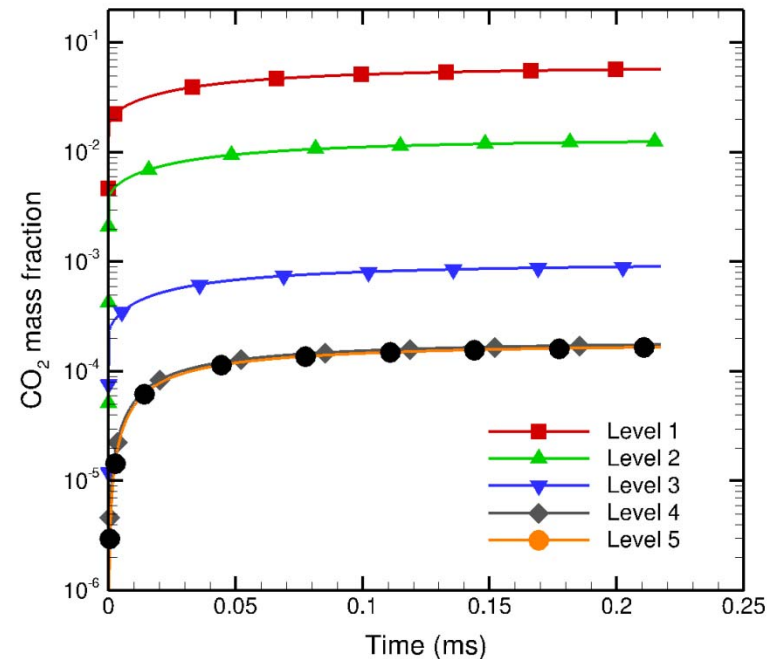
- Sub-pore space saturation takes much longer!
- Smallest channels take longest to saturate...

# Diffusion + Adsorption Case

CO<sub>2</sub> concentration in overall and sub-pore space



CO<sub>2</sub> concentration at different sub-pore levels



- Sub-pore space seems to be asymptote at a much lower value of CO<sub>2</sub> than ambient
- Similar trend follows (as expected) at different levels



# Effect of Temperature – Diffusion only

- Diffusion (only) into particle pore space
- Effect of temperature on saturation time

