An Information Theoretic Framework for Health and Condition Monitoring of Power Plant Equipment

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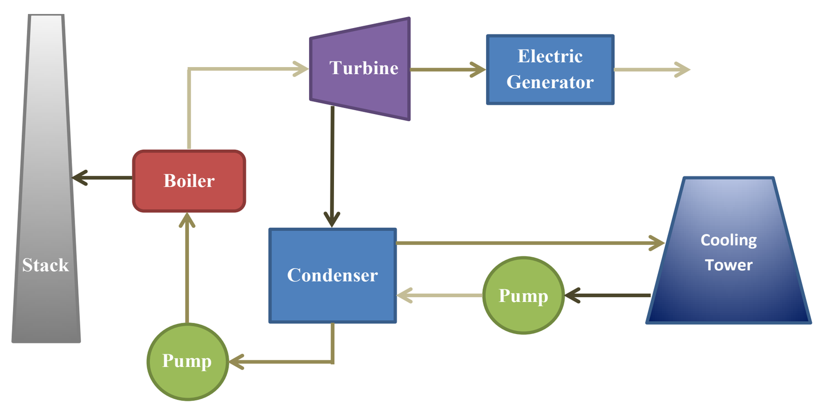
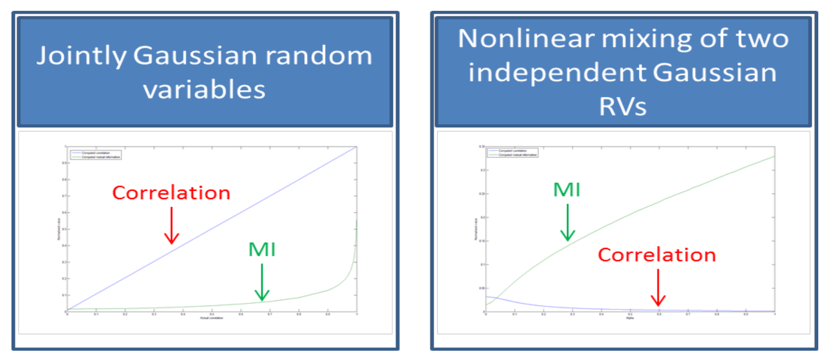
I. Introduction

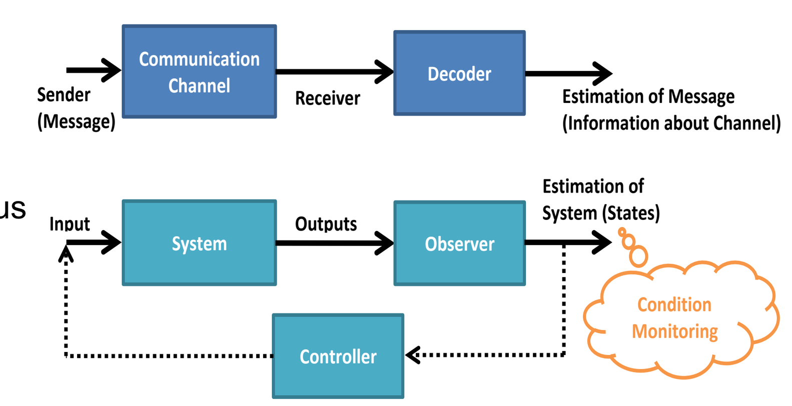
Power plants are composed of many subsystems and components. These subsystems possess their own individual dynamics, and contributing to overall system dynamics through their interactions. Sensors are used to partially monitor plant to detect and track anomalous behaviors. While sensor data from directly connected elements are related, it is also possible that other elements may be related via the intrinsic dynamics of the overall system.





- Correlation measures can perform poorly for nonlinear correlation.
- Correlation measures are insufficient to capture nonlinear underlying dynamics of power plants
- We propose mutual information to capture this nonlinear interconnection.

- From information theory perspective, inputs and outputs for any subsystem are analogous to a transmitted message and the corresponding message received, respectively,
- The system and the observer play roles analogous to that of the communication channel and a decoder,
- Based upon this correspondence, we investigate an information-theoretic framework for capturing system structure and its evolution.



II. Relationship between Information Measures and Power Plant Equipment Monitoring: A Control Theory Perspective

Mathematical Model of Power Plant:

$$x_{k+1} = f(x_k, u_k) + v_k$$
$$y_k = h(x_k) + w_k$$

Connection between Inputs and System

$$I(y_{1:k}, u_{1:k}) = I(x_{1:k}, u_{1:k}) - I(x_{1:k}, u_{1:k} \mid y_{1:k})$$

Connection between Inputs and Outputs

Connection between System and Outputs (Inverse relationship)

- Mutual information between inputs and outputs are directly related to system structure,
- With appropriate sensing, plant condition will be reflected in the sensor data.

Linear System:

$$x_{k+1} = Ax_k + B\begin{bmatrix} u_k^{(1)} \\ u_k^{(2)} \end{bmatrix} + v_k$$
$$\begin{bmatrix} y_k^{(1)} \\ y_k^{(2)} \end{bmatrix} = Cx_k + w_k$$

$$I\left(y_{1:k}^{(i)}, u_{1:k}^{(j)}\right) = I\left(\tilde{x}_{1:k}^{(c,o)}, u_{1:k}^{(j)}\right) - I\left(\tilde{x}_{1:k}^{(c,o)}, u_{1:k}^{(j)} \mid y_{1:k}^{(i)}\right)$$

- Mutual information between input and output is an explicit function of input and controllable/ observable structure (A,B,C),
- ***Changes in structure of linear systems can be detected by examining mutual information,***
- ***We hypothesize that this result holds for nonlinear systems as well [1,2].***

III. Computation of Mutual Information

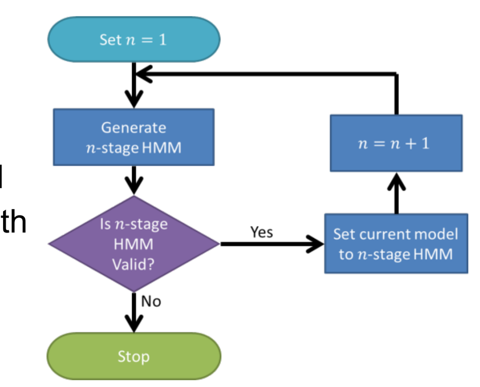
A key challenge is the computation of mutual information as existing algorithms require strict assumptions [3,4,5,6] and violation of these assumptions can lead to ambiguous interpretation.

A) Continuous-Valued Representation

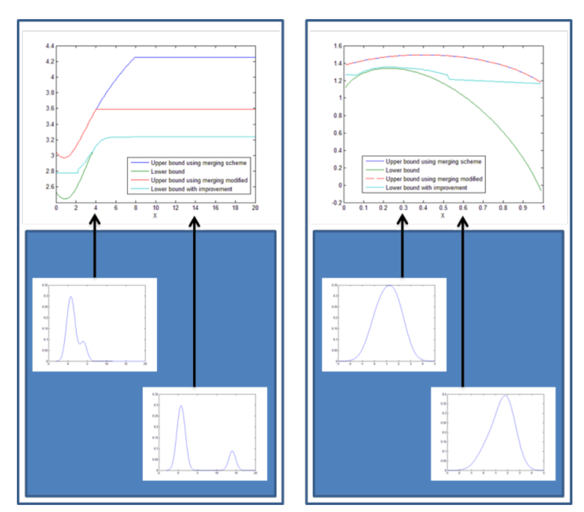
- Underlying dynamic of complex plant are continuous,
- Sensor data generally treated as continuous-valued,
- Use Hidden Markov Models (HMMs) to capture coarse approximation of underlying dynamics,
- Data under HMM with Gaussian emission is Gaussian Mixture (GM) => approximate large class of probability distributions.
- Model construction performed in two steps;
  - Model order selection and parameterization for given data,
  - Computation of mutual information given HMM.
- Interpretation of information measure for continuous RVs can differ from original definition in discrete-valued space.

A.1) Hidden Markov Model Construction

1. Parameters of HMM can be computed by Baum-Welch method given model order.
2. Robust implementation (Mann [8]) applied to counter underflow issues associated with long training data sequences.
3. Iterative method used to determine model order.



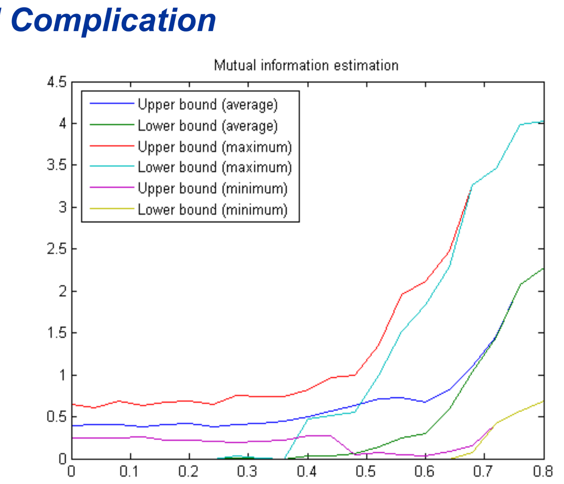
A.2) Mutual Information Computation for Data Generated by HMM



- $h(O_{1:T} \mid Model) = \sum h(O_t \mid O_{1:t-1}, Model)$, where each term in the sum on the right-hand side is a GM,
- We compute the entropy of GM using improvement for upper and lower bounds estimation based on [7].

A.3) Results from Simulations and Complication

- Computationally expensive,
- Degenerate solution lead to use low order model,
- Gaps between the upper and lower bounds of mutual information are large,
- ***These three problems need to be solved for feasibility of this approach.***

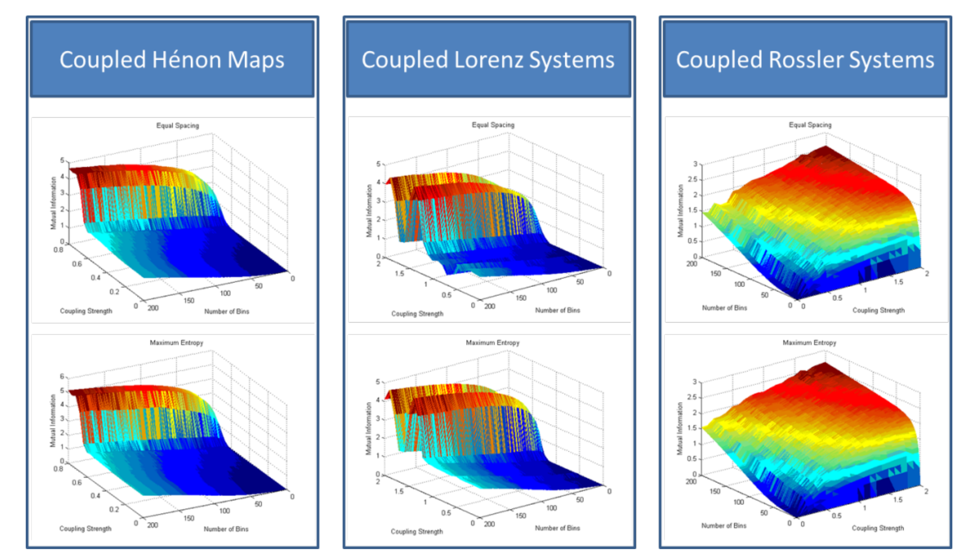


B) Discrete-Valued Representation

- Results from computational algorithms upon violation of strict assumptions, can have completely different interpretation,
- Mathematically proven that Shannon mutual information give meaningful result even if i.i.d. assumption is not satisfied,
- We focus on this simplest method for mutual information computation.

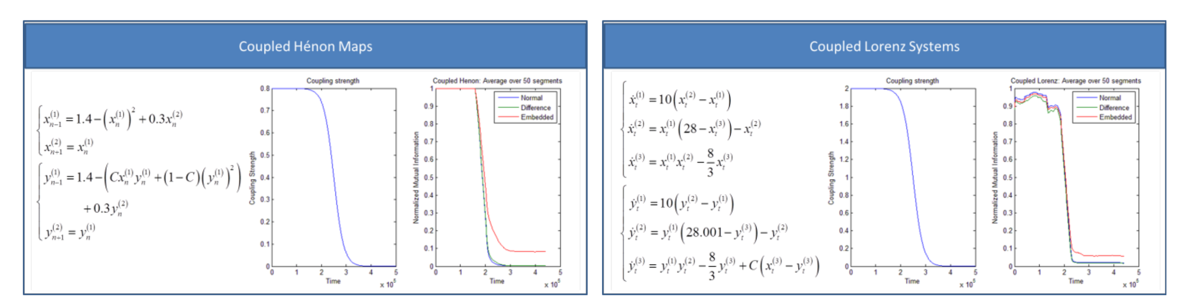
B.1) Shannon Mutual Information and Quantization

- Discrete-valued representation requires quantization,
- Selecting methods for quantization can be challenging,
- Simulations show that computation of mutual information is not overly sensitive to quantization method,
- Simulations also suggest that normalized mutual information can be used to reduce the effect of different number of bins..



B.2) Simulation Results on Varying Parameter Dynamical Systems

- Normalized mutual information is computed for blocks of size 8,000 data on two non-stationary coupled chaos systems .
- The average from 50 consecutive windows are used as an estimation.
- We computed $I(\tilde{X}_{1:T};\tilde{Y}_{1:T})$ $I(\check{X}_{1:T};\check{Y}_{1:T})$ and $I(X_{1:T};Y_{1:T})$ where $\tilde{Z}_i=[Z_i,Z_{i+1}]$ and $\check{Z}_i = Z_{i+1} - Z_i$.

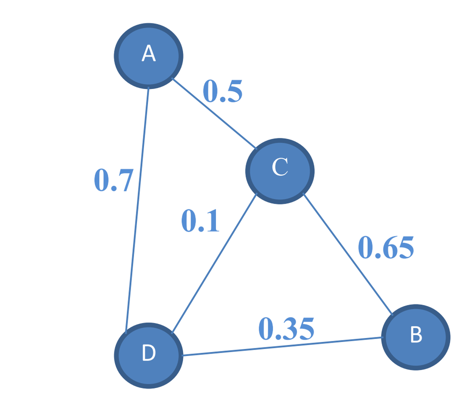


- Shannon mutual information is applicable for non-stationary and complex dynamics to certain degree.
- This method works well for coupled Hénon maps while some complication exists for more complicated systems (coupled Lorenz systems).
- ***We strongly believe that this computation will be able to detect malfunction in the power plant .***

IV. System Structure Representation

Simplest representation of system structure given information measure is graph and/or adjacency matrix where the weight between each node (sensor) is mutual information.

Undirected Weighted Graph



Weighted Adjacency Matrix

	A	B	C	D
A	0	0	0.5	0.7
B	0	0	0.65	0.35
C	0.5	0.65	0	0.1
D	0.7	0.35	0.1	0

Extension on this topic is our project [9] which focus the method for discovering and constructing sensor network using mutual information.

V. Conclusions and Future Work

- Theoretically, information measure can be used to detect the change in the structural of the systems,
- We propose, study and compare the computation approach for mutual information,
- Simulation results show that Shannon mutual information can detect the change in the parameter of coupling chaos systems,
- We propose the use of adjacency matrix for health condition monitoring

- ❑ Due the symmetric property, mutual information cannot be used to detect the direction of influence.
- ❑ We are currently investigate the new alternative quantity which emerges from mutual information in order to detect the directionality.

VI. References

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