

# Phase-Field Model Development for Plasticity/Creep

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## INTRODUCTION

Components in fossil energy (FE) power generation devices are often subject to high temperatures for hundreds of thousands of hours. Consequently, creep is a major concern for design of alloy toward more efficient FE applications. Common creep models are, unfortunately, largely empirical in nature. Physics-based phase-field modeling has attracted increasing interests for its capability of modeling the kinetics of materials at microscale. In the literature, there has been models that attempt to couple phase-field modeling with (classical) plasticity or crystal viscoplasticity. These models normally directly invoke the plasticity theories. However, they lack a unified thermodynamic potential that governs both plastic flow and microstructure evolution. Here we develop a thermodynamically consistent crystal plasticity phase-field framework in which the plastic strain is taken as a phase-field variable subject to the time-dependent Ginzburg-Landau equation. This way, the plasticity is fully coupled with microstructure evolution through a common free energy functional. In addition, in this modeling framework,  $J_2$  plasticity can coexist with crystal plasticity. Such a feature is utilized to model grain boundary sliding (GBS), which is also an important mechanism for creep. In the GBS model, the grain boundary region behaves more of  $J_2$  plasticity and the bulk crystal constrained to the crystallographic slip systems (crystal plasticity). The modeling results are carefully validated against analytical solutions, finite element solutions (M. Ashby *et al.*) and the FFT-EVP algorithm (R. Lebensohn *et al.*).

## APPROACH

### Formulating $J_2$ plasticity and crystal plasticity in a unified Ginzburg-Landau framework

- Phase-field microelasticity theory (PFME) (Khachaturyan 1983)

$$E^d = \frac{1}{2} \int_{\Omega} C_{ijkl}(\mathbf{r}) \epsilon_{ij}(\mathbf{r}) \epsilon_{kl}(\mathbf{r}) dV + \frac{1}{2} \int_{\Omega} C_{ijkl}(\mathbf{r}) \bar{\epsilon}_{ij}(\mathbf{r}) \bar{\epsilon}_{kl}(\mathbf{r}) dV$$

$$\frac{1}{2} \int_{\Omega} \frac{d^2 k}{(2\pi)^2} n_i \bar{\sigma}_{ij}(\mathbf{k}) \Omega_{ij}(\mathbf{n}) \bar{\sigma}_{kl}(\mathbf{k}) n_l, \quad (\epsilon_{ij}^0 = \epsilon_{ij}^p + \epsilon_{ij}^e)$$

- Imposing constraints: Lagrange multiplier

$$\mathcal{L} = F - \int_{\Omega} \lambda^p \epsilon_{ij}^p(\mathbf{r}) dV - \sum_{\alpha} \int_{\Omega} \lambda_{ij}^{(\alpha)} (\epsilon_{ij}^{(\alpha)} - \epsilon_{ij}^{p(\alpha)} - m_{ij}^{(\alpha)} m_i^{(\alpha)} m_j^{(\alpha)}) dV$$

- Condition of thermodynamic equilibrium under constraint:

$$\sigma_{ij}^p(\mathbf{r}) = 0 \quad \text{Zero deviatoric stress: Isotropic } (J_2) \text{ plasticity}$$

$$\sigma'_{ij} m_i^{(\alpha)} m_j^{(\alpha)} = 0 \quad \text{Zero resolved shear stress: Crystal plasticity}$$

- Lagrange multipliers solved to be hydrostatic pressure & negative stress

$$\lambda^p(\mathbf{r}) = -\frac{1}{3} \sigma_{kk}(\mathbf{r}) = p(\mathbf{r}) \quad \lambda_{ij}^{(\alpha)}(\mathbf{r}) = -\sigma_{ij}(\mathbf{r})$$

- Time-dependent Ginzburg-Landau (TDGL) equation

$$\frac{\partial \phi_{ij}^{(\alpha)}}{\partial t} = \frac{\partial}{\partial t} \left( \epsilon_{ij}^{p(\alpha)} + \sum_{\beta=1}^N \phi_{ij}^{(\beta)} \right) = -L_{ij}^{(\alpha)} \left( \frac{\delta \mathcal{L}}{\delta \phi_{ij}^{(\alpha)}} \right) - \sum_{\beta} L_{ij}^{(\beta)} \left( \frac{\delta \mathcal{L}}{\delta \phi_{ij}^{(\beta)}} \right)$$

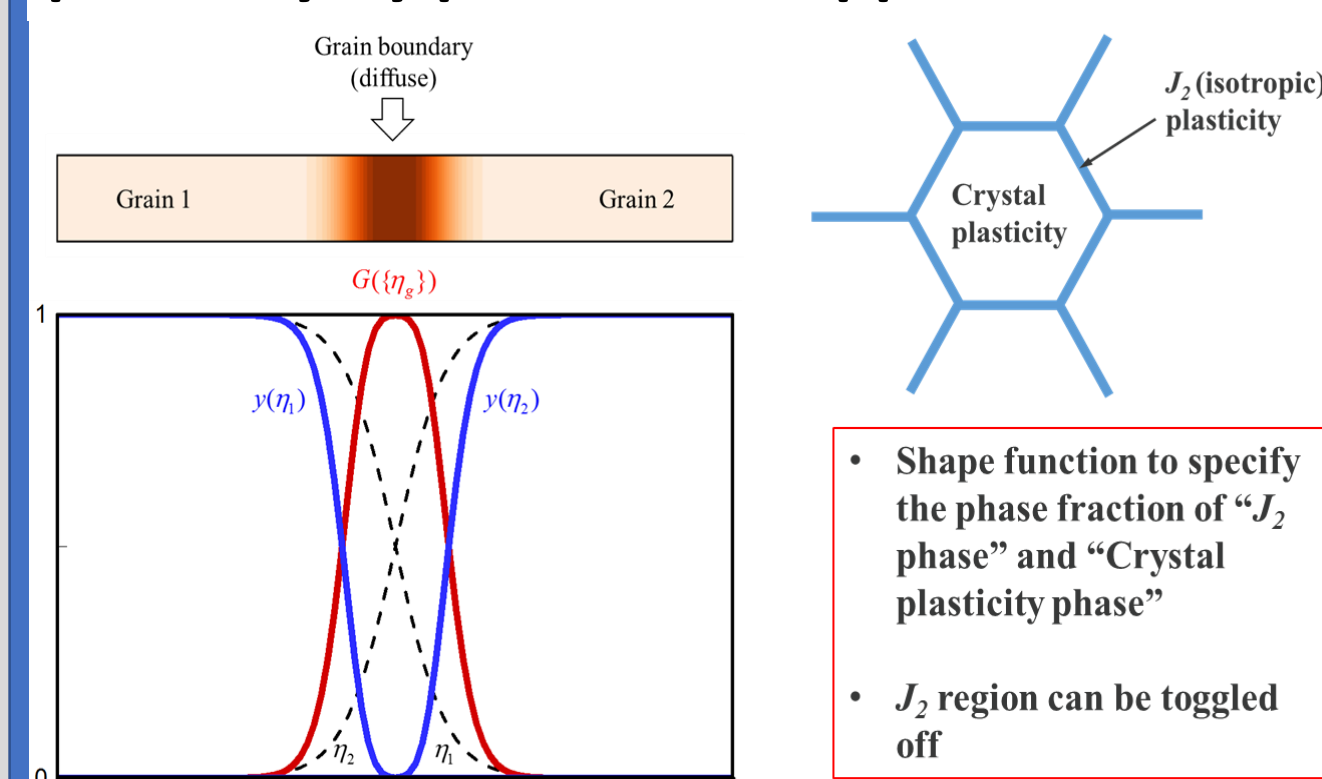
$$L_{ij}^{(\alpha)} = \frac{3\gamma_{ij}^2}{2J_2} \left( \frac{J_2}{\Lambda} \right)^{n-1} \delta_{ij} \delta_{ij} [1 - \sum_{\beta} \gamma_{ij}^{(\beta)}]$$

$$L_{ij}^{(\alpha)} = \gamma_{ij}^2 \frac{1}{\Lambda} \left| \frac{\partial \phi_{ij}^{(\alpha)}}{\partial x} \right|^{n-1} \text{sign}(\epsilon^{(\alpha)}) \delta_{ij} \delta_{ij} \gamma_{ij}^{(\alpha)}$$

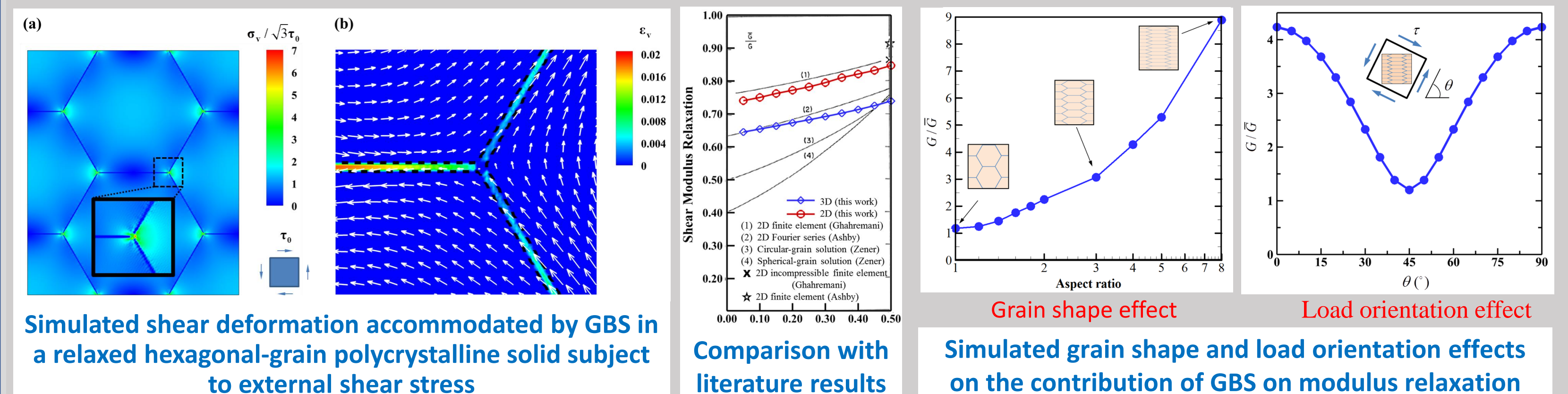
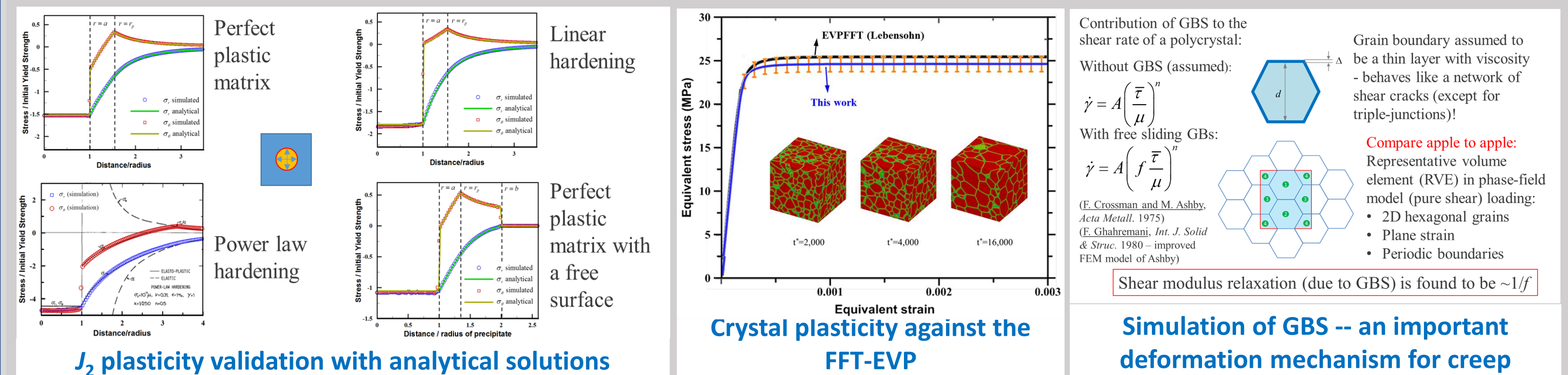
$$\frac{\partial \epsilon_{ij}^p}{\partial t} = \frac{3}{2} \left( \frac{J_2}{\Lambda} \right)^{n-1} \frac{\sigma'_{ij}}{J_2(\mathbf{r})} \quad \text{Odqvist's law } (J_2 \text{ plasticity})$$

$$\frac{\partial \epsilon_{ij}^{(\alpha)}}{\partial t} = \gamma_{ij}^{(\alpha)} \left| \frac{\partial \phi_{ij}^{(\alpha)}}{\partial x} \right|^{n-1} \text{sign}(\epsilon^{(\alpha)}) m_i^{(\alpha)} m_j^{(\alpha)} \quad \text{Asaro \& Needleman (crystal plasticity)}$$

### Defining the diffuse interface as $J_2$ plasticity and bulk region as crystal plasticity by phase-field approach



## RESULTS



## SUMMARY

- Developed a thermodynamically consistent crystal-plasticity phase-field framework that can couple plasticity (viscoplasticity) and crystal viscoplasticity with phase transformation and microstructure evolution [1,2].
- Applied this modeling framework to model grain boundary sliding coupled with crystal plasticity, and the results agree with the finite element simulations of Ashby (1975) and Ghahremani (1980) in the limit of free GBS, and conform to the FFT-EVP result of Lebensohn (2012) in the limit of zero GBS [2].
- Paved the way for developing a physics-based creep model that provides quantitative understanding for the correlation between macroscopic creep performance and underlying mechanisms at microstructure level.

## ACKNOWLEDGEMENTS

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