

design of multiscale systems

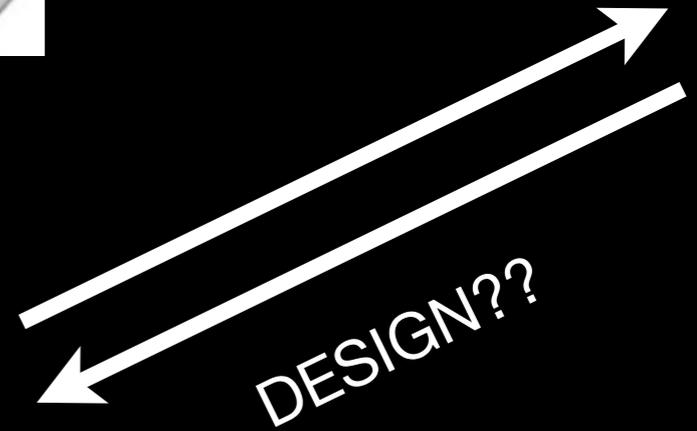
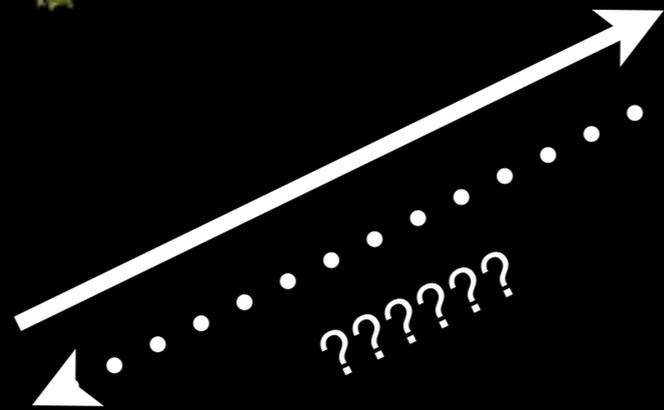
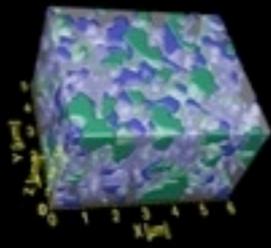
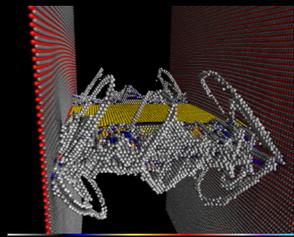
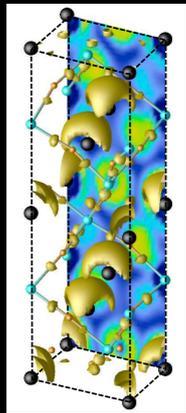
Richard LeSar & Mark Bryden

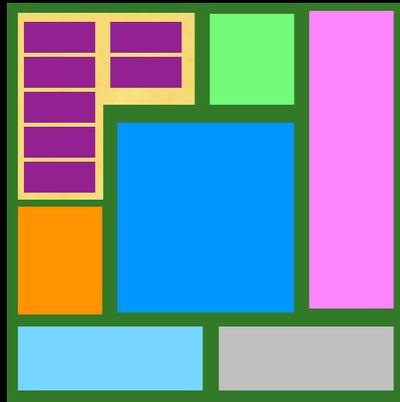
Ames Laboratory
Simulation, Modeling, and Decision Science Program



design

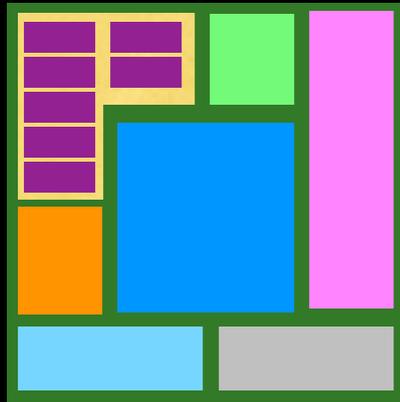
- the materials challenge





three big questions ...

- what is the nature of multiscale design?
- how we mathematically represent multiscale systems?
- how do we interact with multiscale systems?



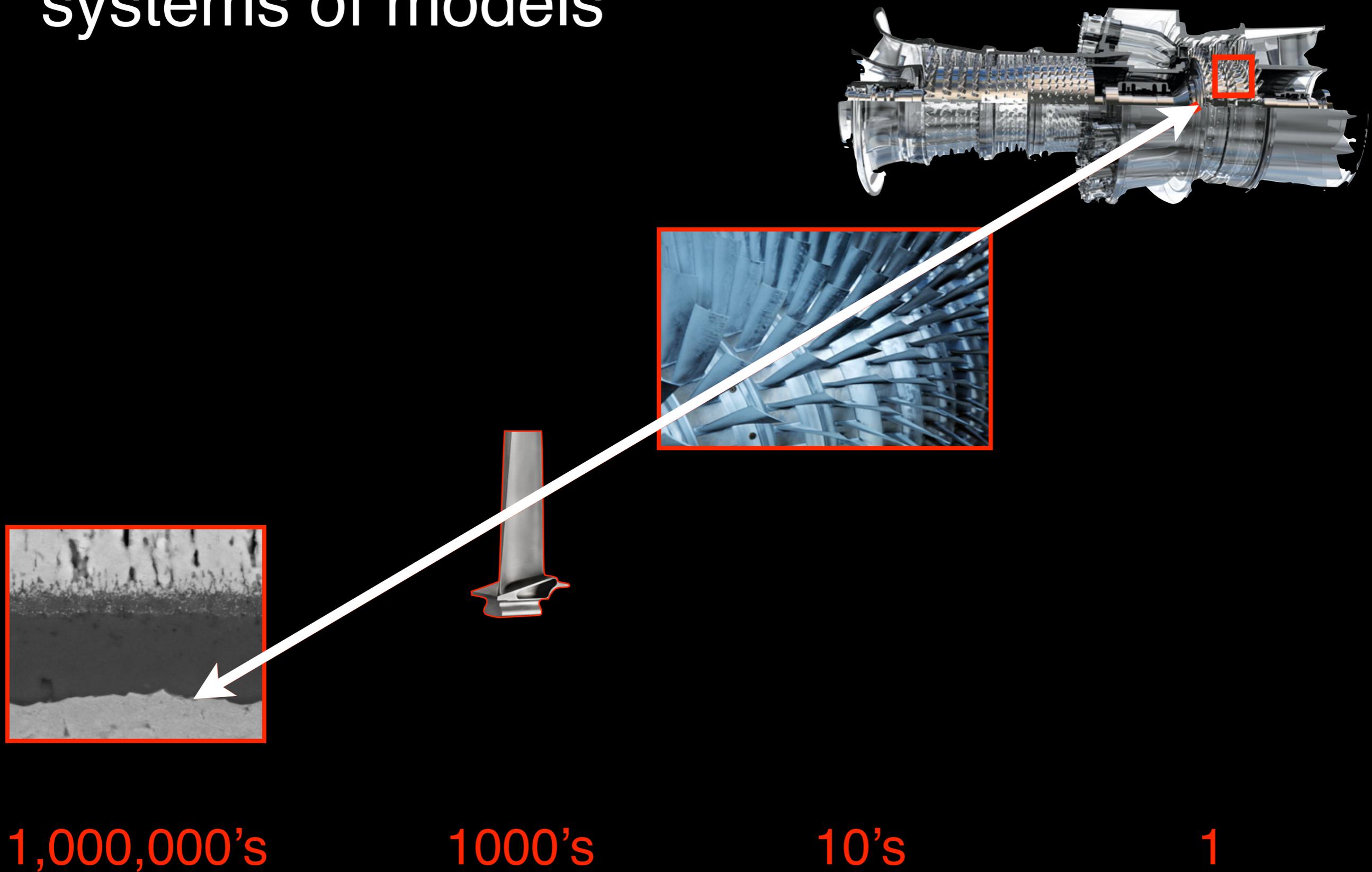
three big questions ...

- what is the nature of multiscale design?
- how we mathematically represent multiscale systems?
- how do we interact with multiscale systems?

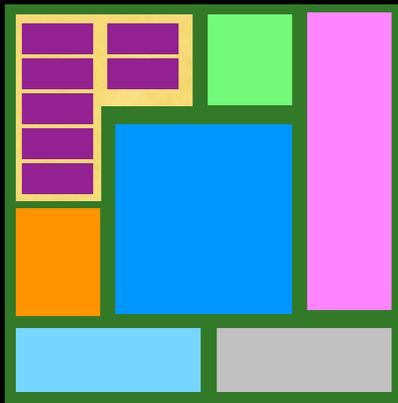


Today we cannot model the richness, fullness, or complexity of engineered, human, or natural systems.

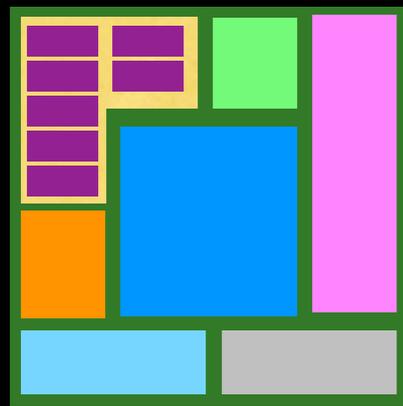
systems of models



our goal

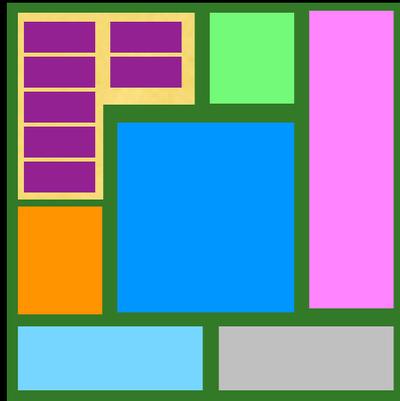


... to create a system of federated models representing the fidelity and complexity of a multiscale system



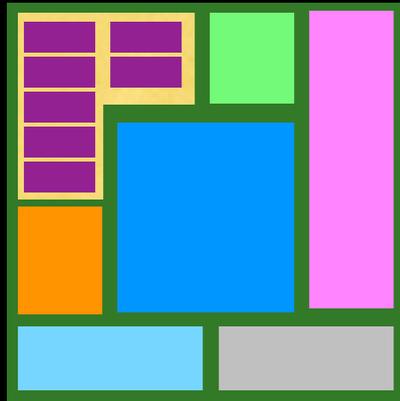
vision

-
- models are built at various scales by experts
 - the models are used without revision
 - models self identify and self assemble



self assembly

a structured organization of model systems as a consequence of local model-to-model information flow, without external direction



assumptions

-
- the models exist
 - the data needed by each model is known and consistent with the other models in the system

system of equations

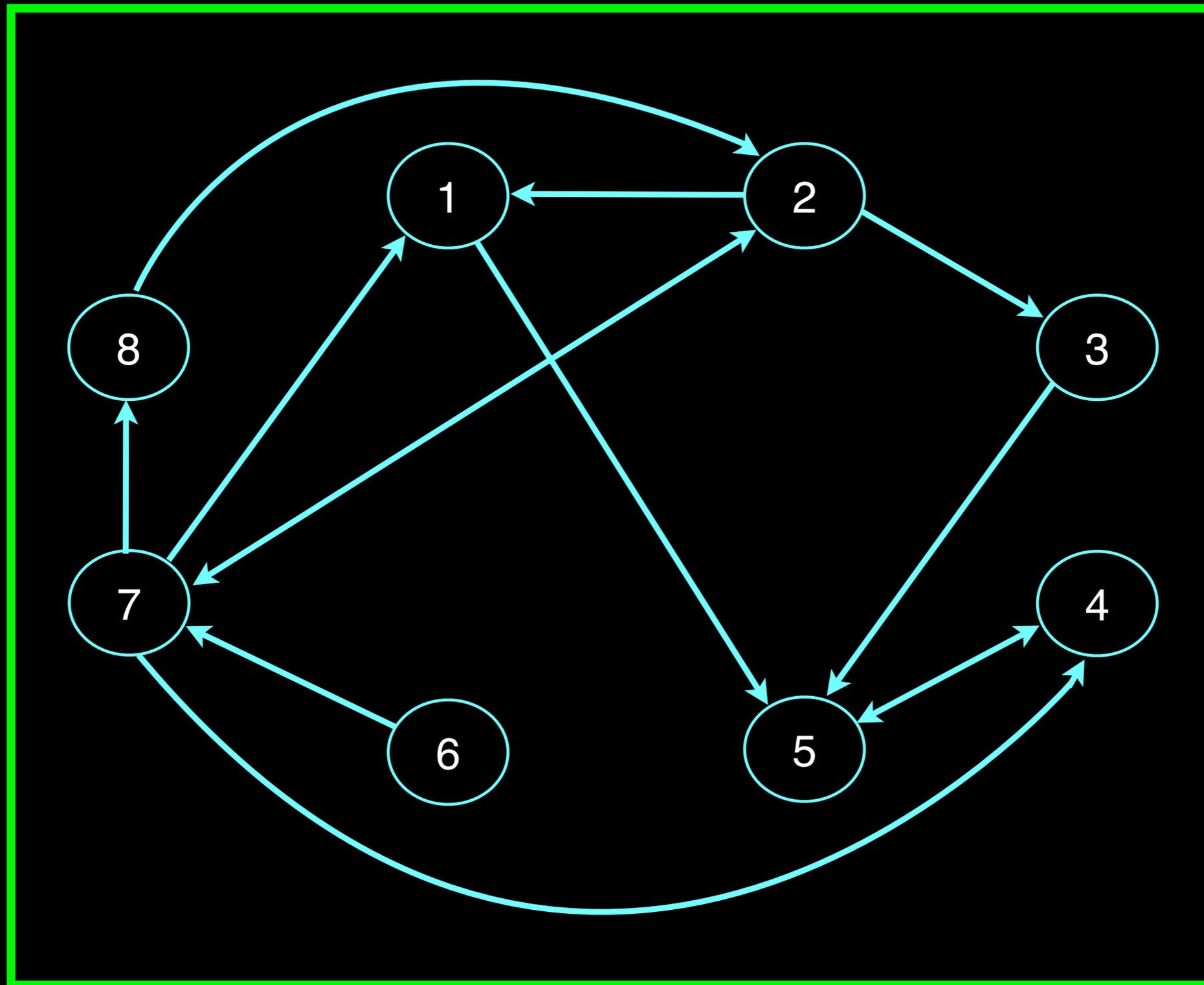
#	Equation
1	$a + b = -c$
2	$b + c = 4 - h$
3	$2b + d = 0$
4	$c + e = 10 + f$
5	$d - e + 5f = a + 3$
6	$g = 1$
7	$3b + c - g = -2h$
8	$h = c + 3$

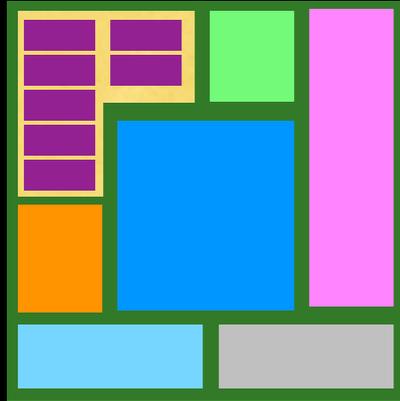
equation

matrix

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 6 \\ 10 \\ 3 \\ 1 \\ 0 \\ 3 \end{bmatrix}$$

directed graph





Tarjan's algorithm

- determines the flow of information in a system of equations
- identifies strongly connected components
- used to block and schedule the solution

¹R.E. Tarjan, "Depth-first Search and Linear Graph Algorithms," SIAM Journal of Computing 1:146-160 (1972)

system of equations

#	Equation
1	$a + b = -c$
2	$b + c = 4 - h$
3	$2b + d = 0$
4	$c + e = 10 + f$
5	$d - e + 5f = a + 3$
6	$g = 1$
7	$3b + c - g = -2h$
8	$h = c + 3$

sorted matrix

equation

matrix

$$\begin{array}{c} 1 \\ 2 \\ 7 \\ 3 \\ 4 \\ 5 \\ 6 \\ 8 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 3 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 0 \\ 6 \\ 10 \\ 3 \\ 1 \\ 3 \end{bmatrix}$$

connection matrix

equation

connection matrix

$$\begin{array}{l} 1 \\ 2 \\ 7 \\ 3 \\ 4 \\ 5 \\ 6 \\ 8 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix}$$

blocked connection matrix

equation

connection matrix

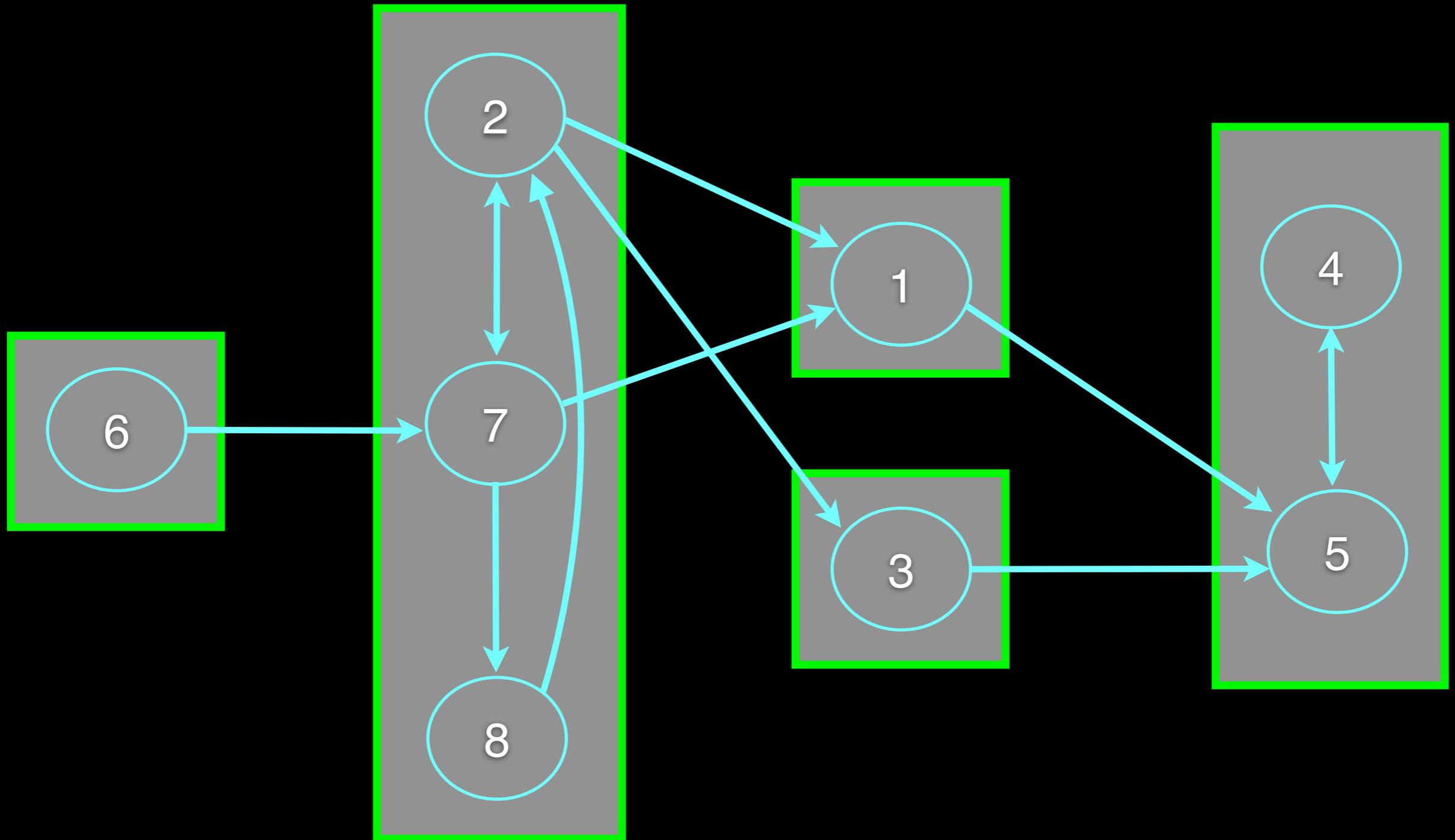
$$\begin{array}{c} 6 \\ 7 \\ 2 \\ 8 \\ 1 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} g \\ c \\ b \\ h \\ a \\ d \\ e \\ f \end{bmatrix}$$

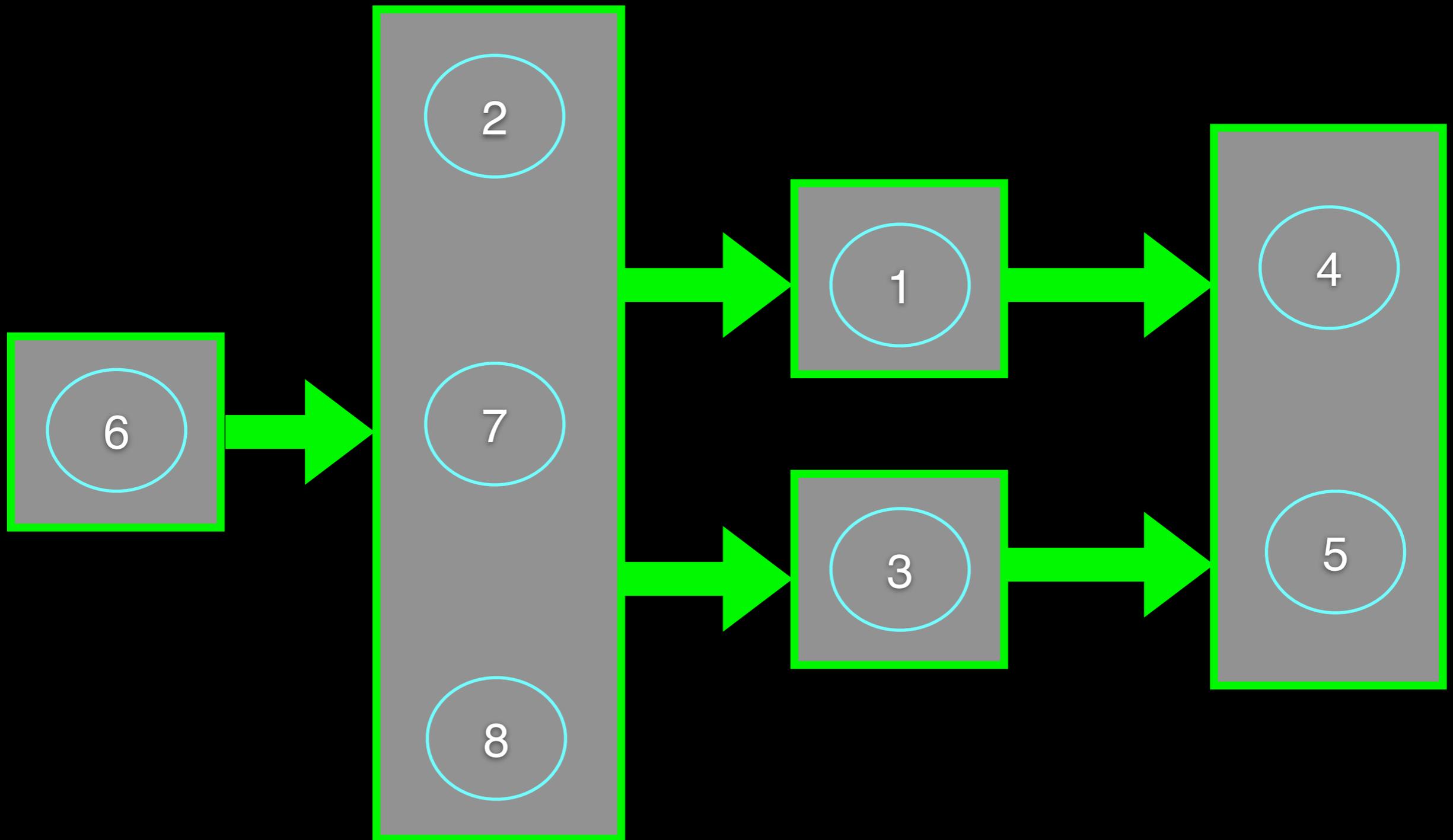
blocked connection matrix

equation

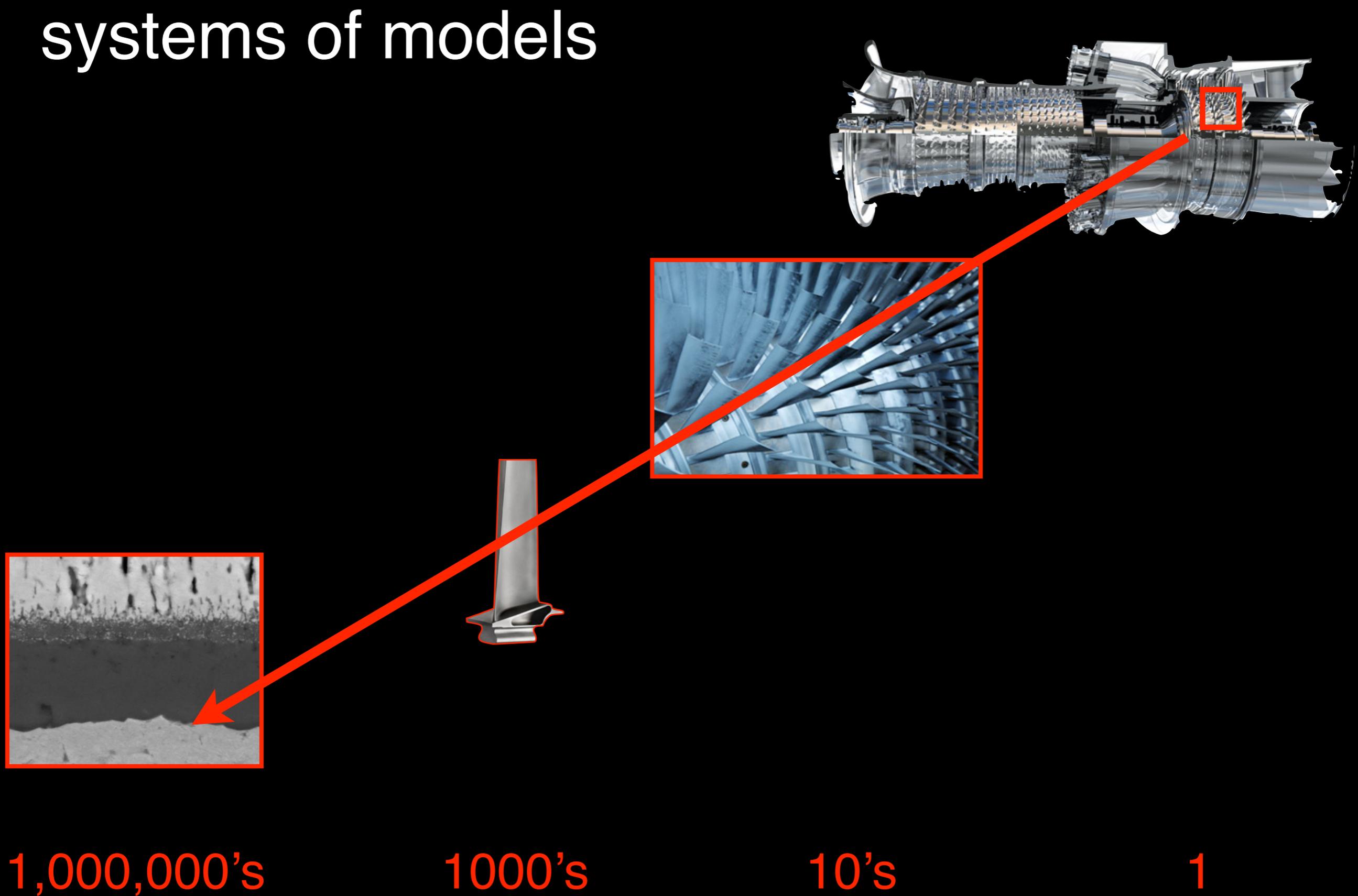
connection matrix

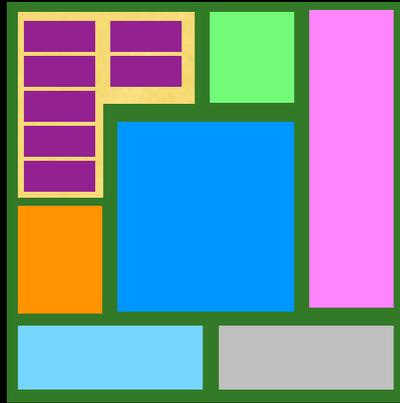
$$\begin{array}{r} 6 \\ 7 \\ 2 \\ 8 \\ 1 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} \boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \boxed{1} & \boxed{1} & \boxed{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & \boxed{1} & \boxed{1} & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & \boxed{1} & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & \boxed{1} & 0 & \boxed{1} & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 & \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 & 0 & \boxed{1} & \boxed{1} \\ 0 & 0 & 0 & 0 & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} \end{bmatrix} \begin{bmatrix} g \\ c \\ b \\ h \\ a \\ d \\ e \\ f \end{bmatrix}$$





systems of models





it's all just information

$$x_1 = 2$$



$$x_1 + x_2 = 3$$



$$x_2 = 1$$

input data



model



output data

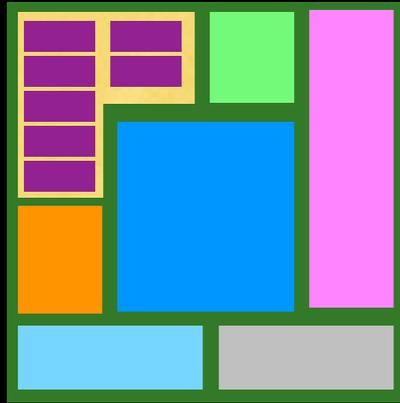
input data



database

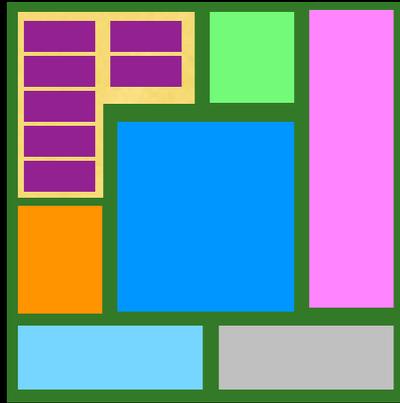


output data



information entities

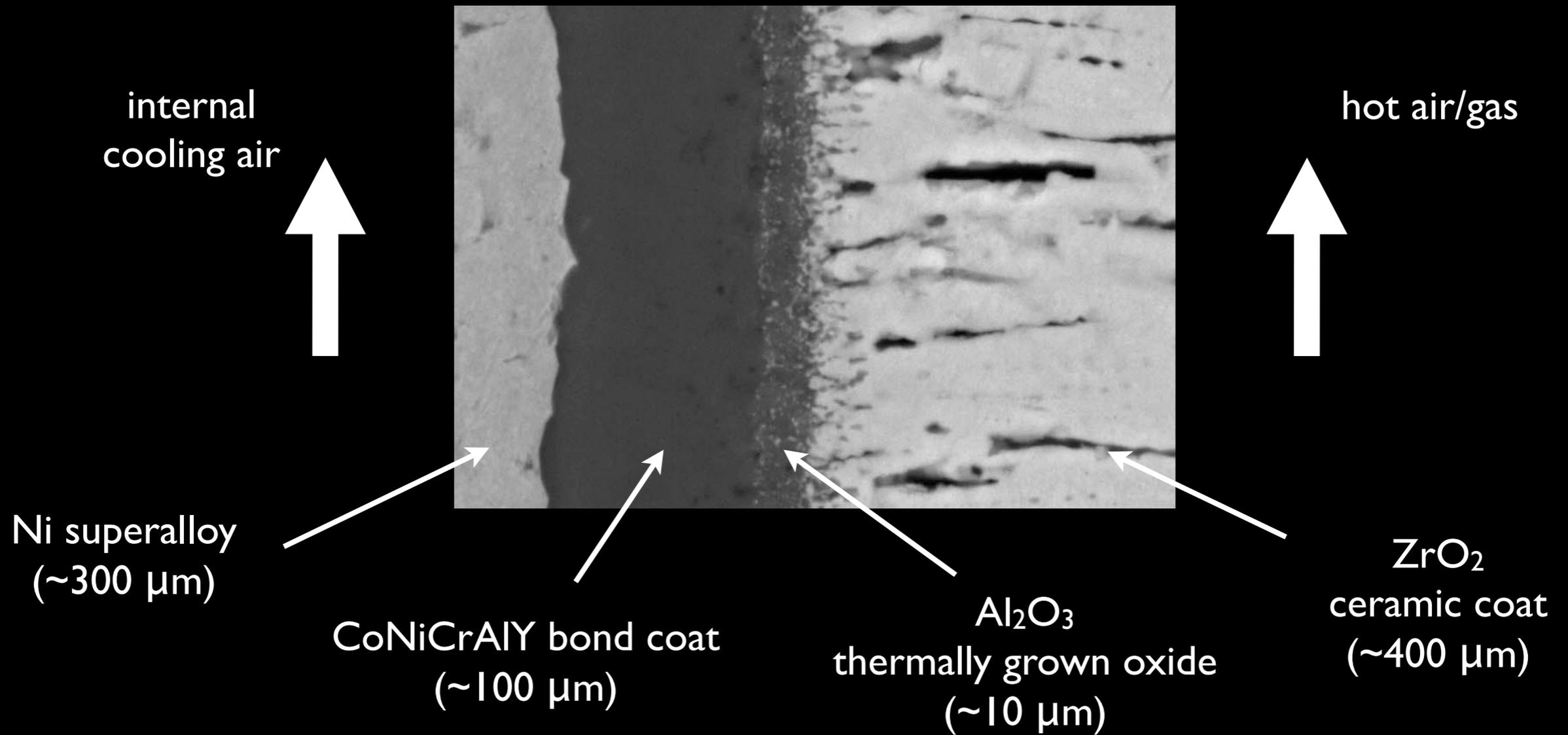
- models
- simulations
- databases
- sensors
- ...

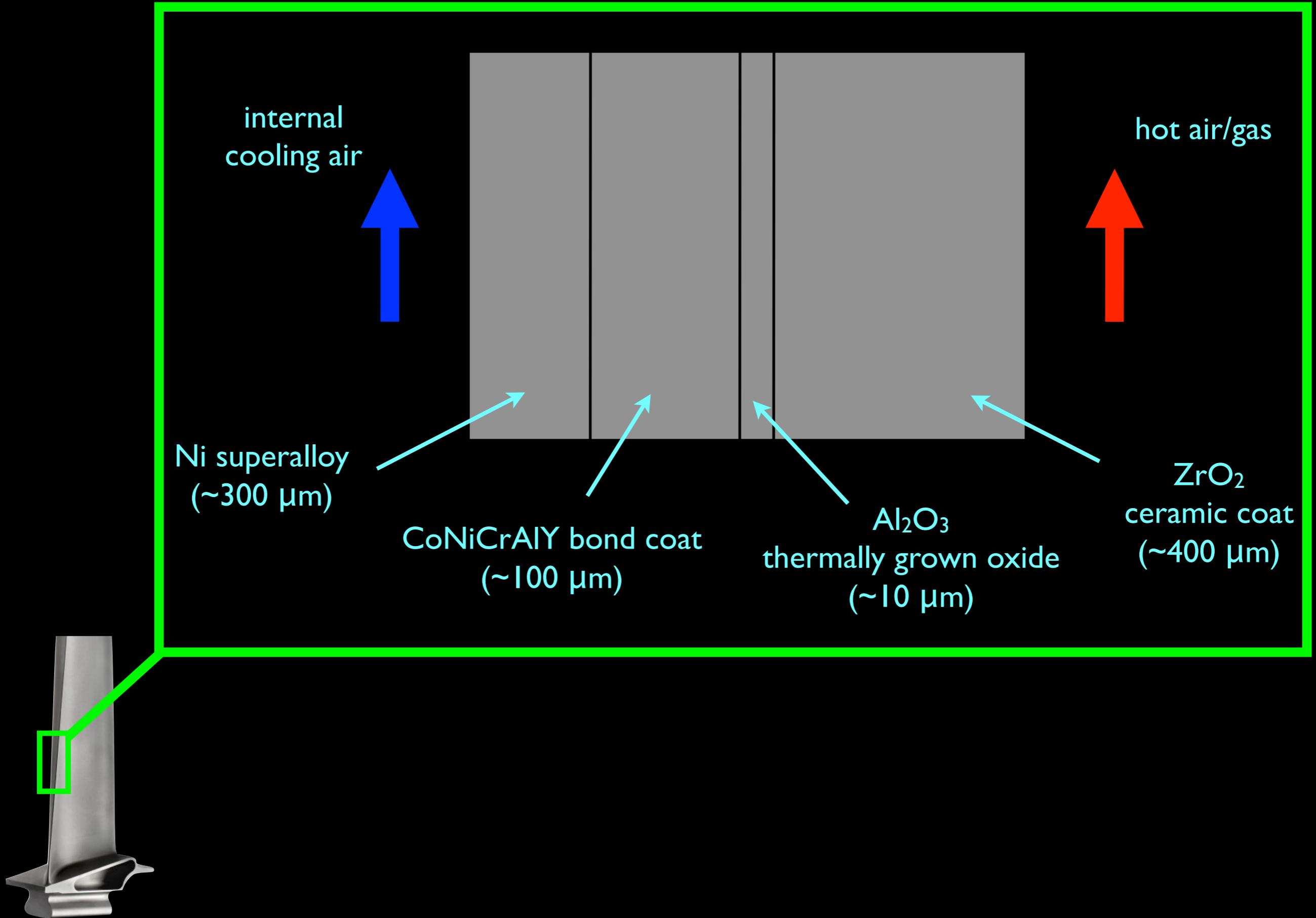


extended Tarjan's algorithm

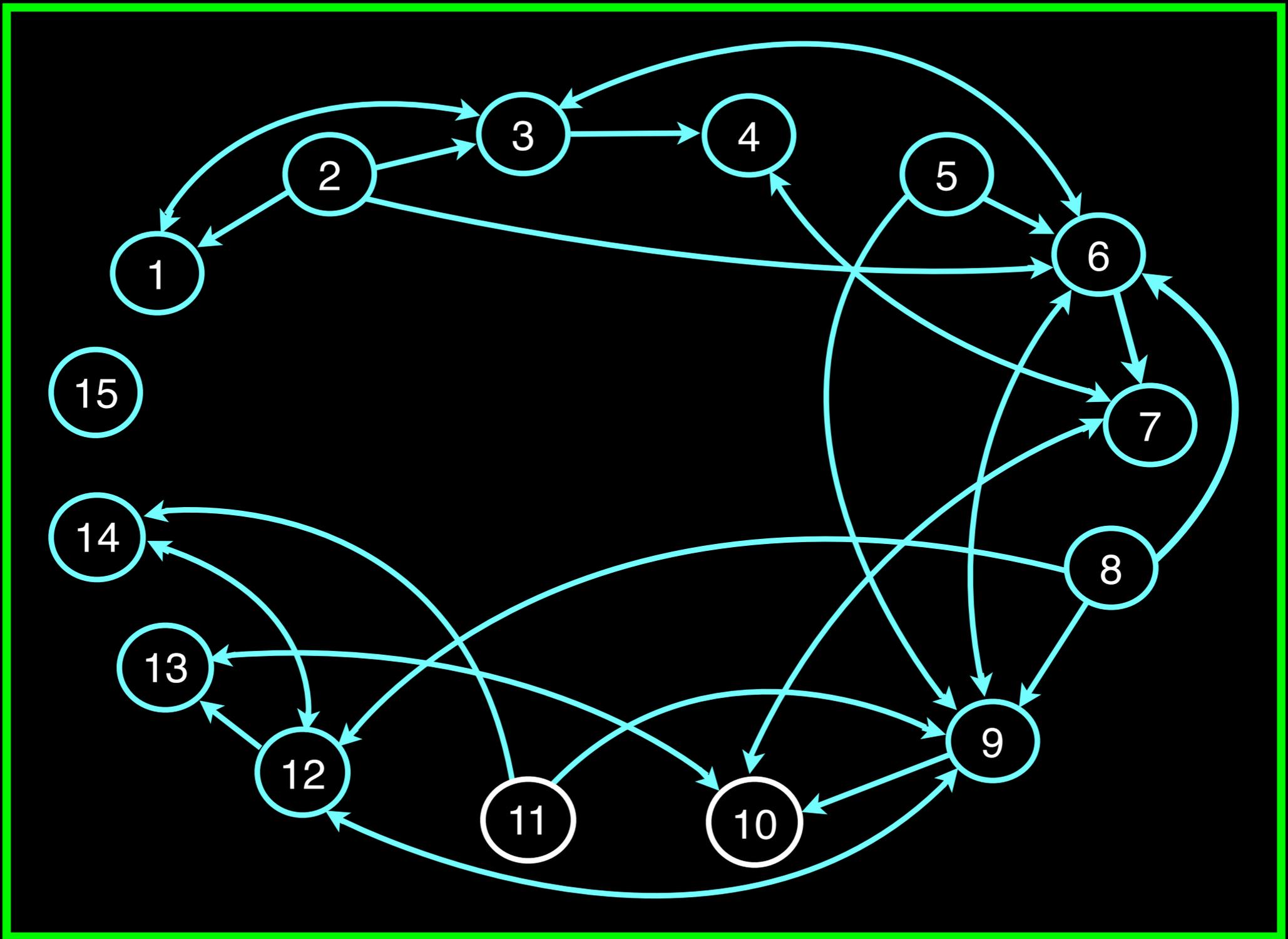
-
- identifies information flow in a complex computational system
 - identifies groupings
 - organizes and schedules order of operation

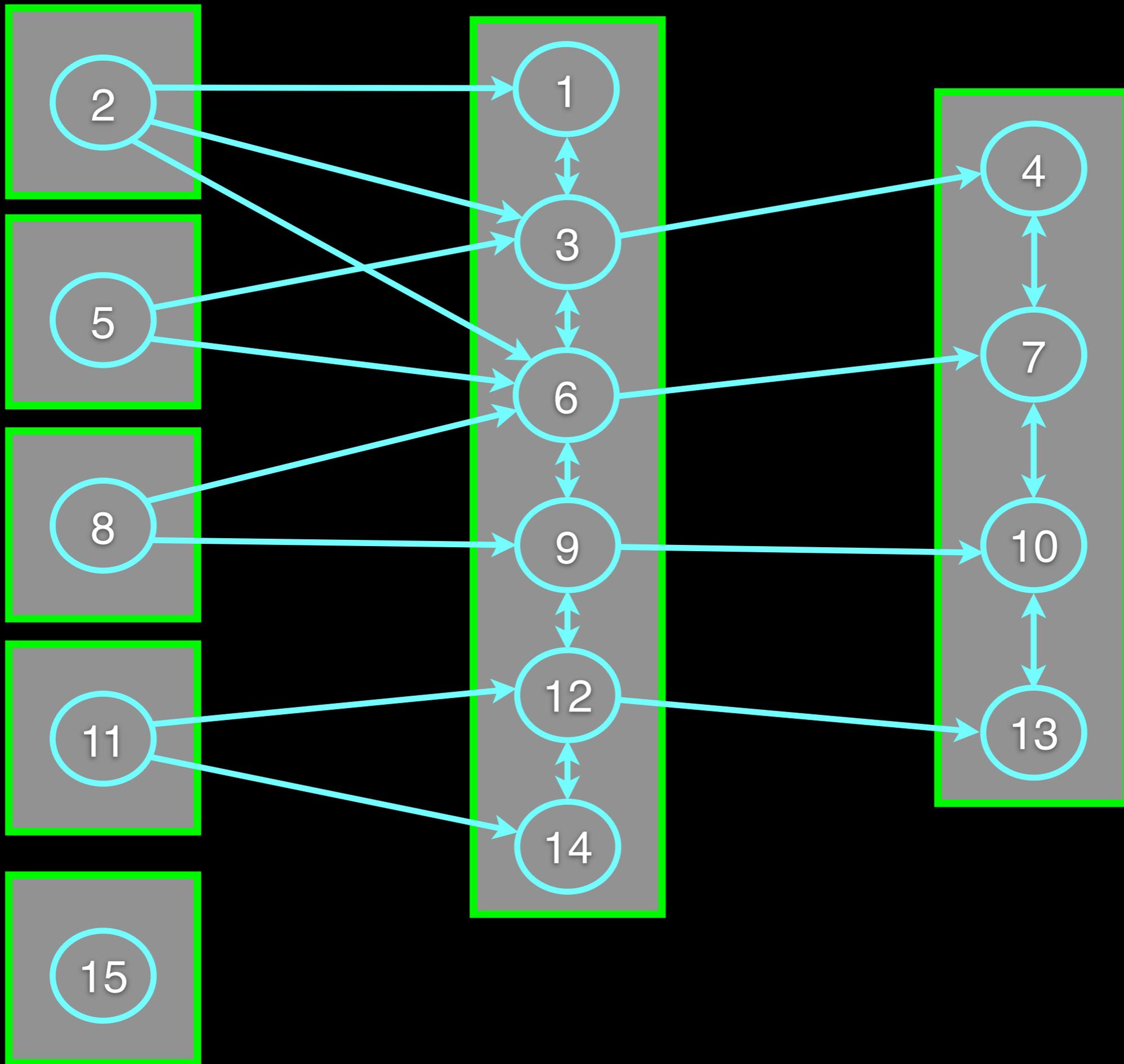
turbine blade

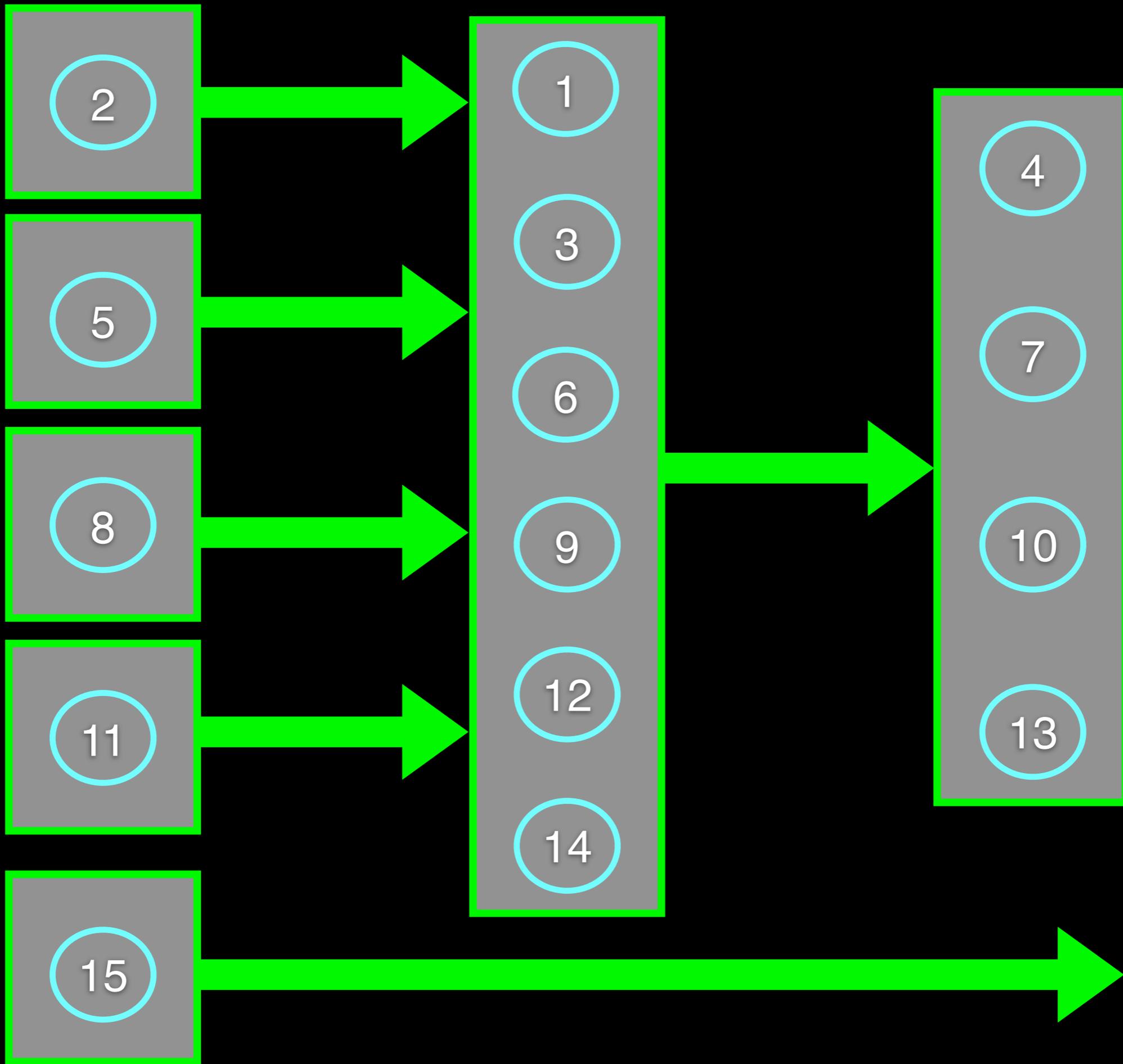




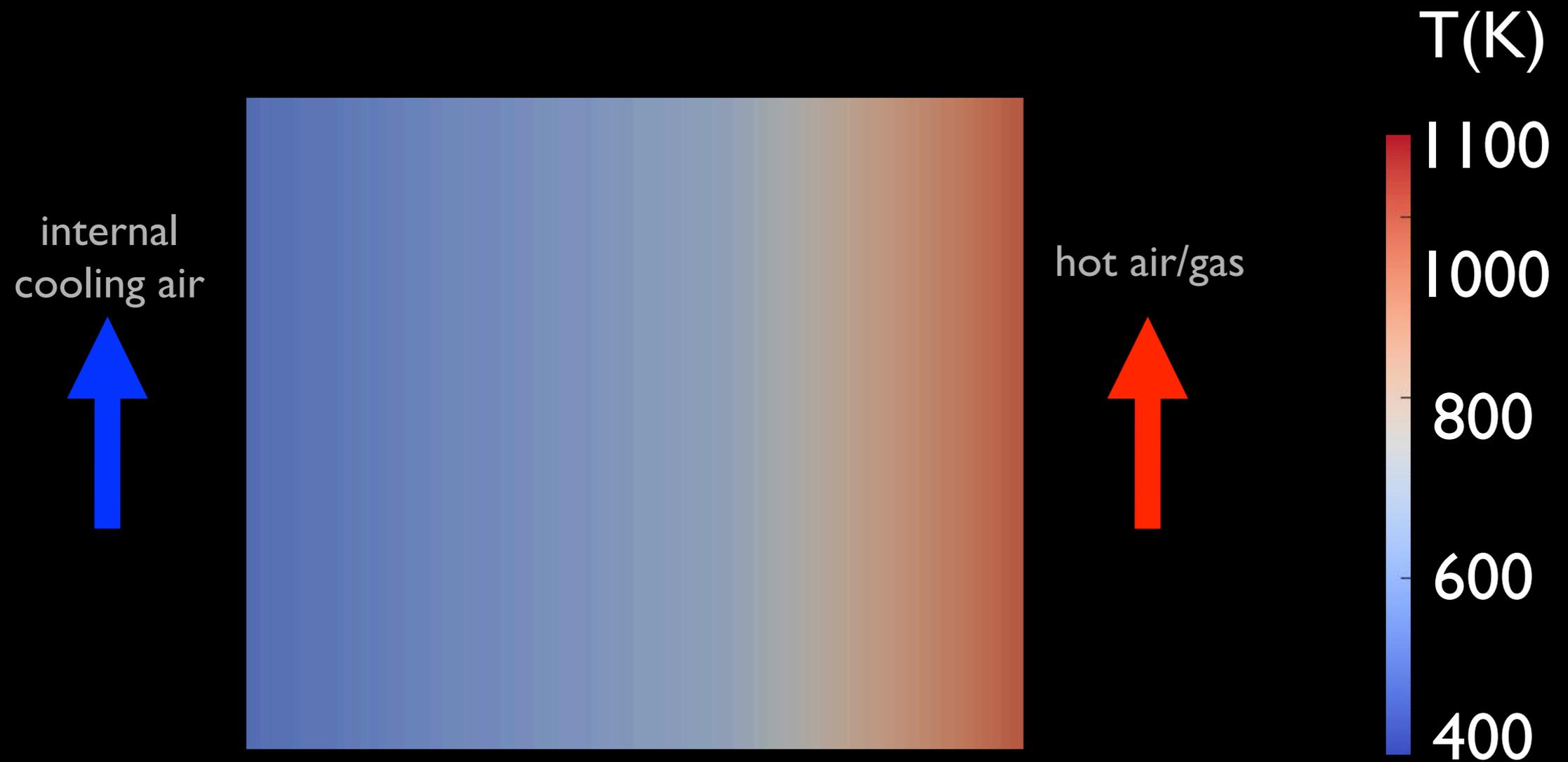
- 1 Convective heat transfer equation for (cooling air)
- 2 Substrate properties database
- 3 1D FVM solver for thermal diffusion in the substrate layer
- 4 1D FVM solver for thermal stress in the substrate layer
- 5 Bond coat material properties database
- 6 1D FVM solver for thermal diffusion in the bond coat
- 7 1D FVM solver for thermal stress in the bond coat
- 8 TGO layer material properties database
- 9 1D FVM solver for thermal diffusion in TGO layer
- 10 1D FVM solver for thermal stress in the bond coat
- 11 Ceramic topcoat material properties database
- 12 1D FVM solver for thermal diffusion in the ceramic topcoat
- 13 1D FVM solver for thermal stress in the ceramic topcoat
- 14 Convective heat transfer equation (hot air)
- 15 TBC material cost model



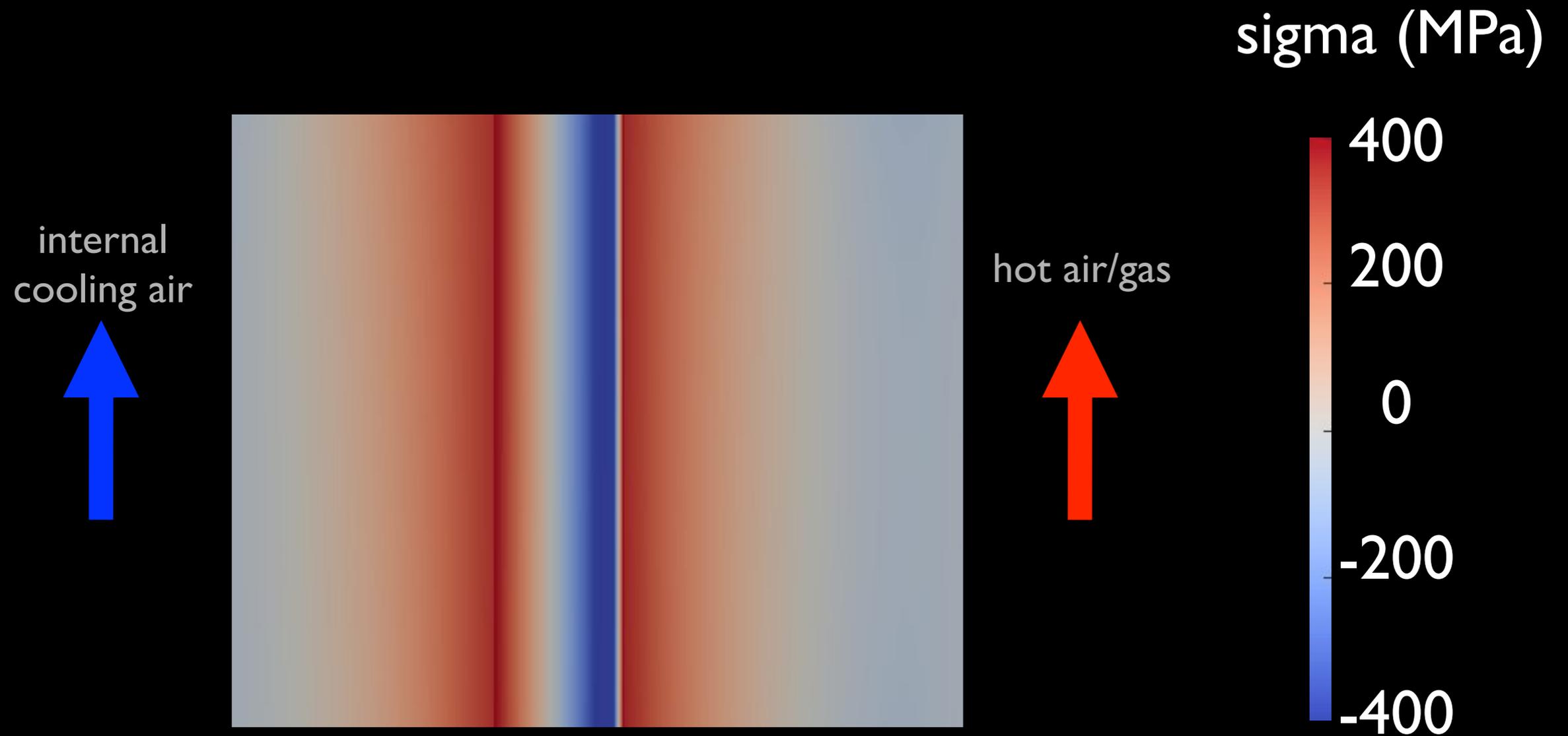


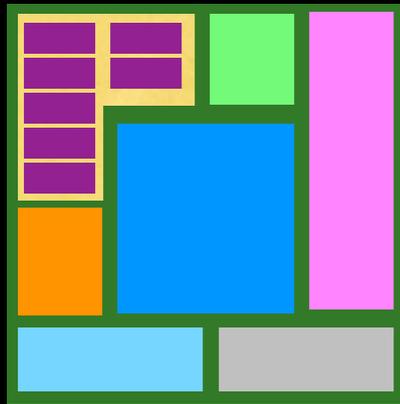


temperature distribution



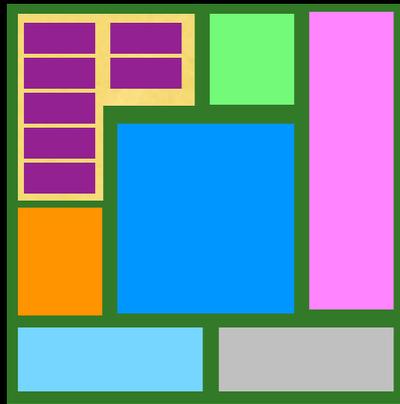
thermal shear stress





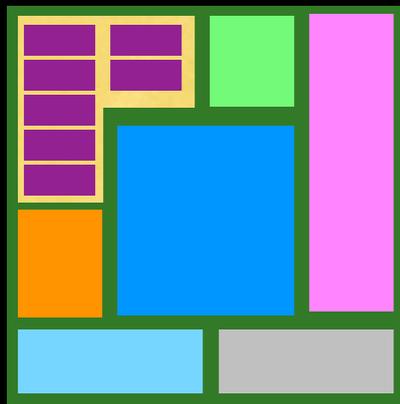
conclusions

-
- Tarjan's algorithm can be extended to identify the information flow in complex computational systems
 - extensible to multiscale modeling of gas turbines, steam turbines, boiler tubes, sensors ...



future work

-
- self identification
 - resource allocation
 - reduced-order models
 - integration into multiscale design process



questions?